An Application of First-Order Autoregressive Markov Regime Switching Models for Logarithmic Demand Forecasting in Backtesting "Paghi Poco" Supermarkets in Sicily

ALESSANDRO MURATORE, GIUSEPPE AIELLO, FILIPPO CAROLLO,
SALVATORE QUARANTA
Engineering Department,
University of Palermo,
Viale delle Scienze, Ed. 8,
ITALY

Abstract: - This work evaluates the forecasting performance of first-order autoregressive Markov Regime Switching (MRS) models to forecast demand in four retail units of the “Paghi Poco” supermarket chain, a national retail chain based in Sicily. In mind of the rapid pace of digitalization in retail and despite the increasing availability of point-of-sale (POS) data, we underline the value of analytical forecasting models that will aid operations management by modelling demand shifts. Specifically, we show that the MRS models will capture the regime shifts between low and high demand regimes that are present in the historical sales data by a back-testing methodology that draw upon an extended history of sales data, which contributed to a better understanding of market dynamics. The results show improvements in forecasting model performance when compared to traditional simpler models, demonstrating the probability of improved decision-making and enhanced agility and resilience to the retail supply chain via the MRS approach.

Key-Words: - Markov Regime Switching, Demand Forecasting, Logarithmic Model, Supply chain, Economic Dynamics, Transition Probabilities.

Received: June 13, 2025. Revised: September 11, 2025. Accepted: September 24, 2025. Published: October 17, 2025.

1 Introduction

Within retail settings demand forecasting is a strategically imperative force for aligning demand and supply inputs to the marketplace. The quality of demand forecasting has an immediate influence on supply chain management decisions, pricing, and promotional calls for action. However, traditional linear forecasting approaches typically do not encapsulate the discontinuities and non-linear dynamics that often characterize retail data especially with seasonality, macroeconomic shocks or customer preferences evolving over time. To address these limitations, the present study investigates the capacity of Markov Regime Switching (MRS) models as an alternative forecasting strategy to obtain latent structural transition of demand time series, specifically MRS models yield separate classifications of sales behavior across multiple regimes with distinctive statistical dynamics and the enhanced ability to respond and detect both sudden and cyclical fluctuations in consumer behavior. Although both have widely been used in macroeconomic and financial contexts, their utilization in retail environments, especially in decentralized grocery chains, is largely open. In this paper, we employ a first-order autoregressive regime-switching model to the “Paghi Poco” supermarket chain in Sicily. The stores chosen are the authors’ best available match for broadly differing demand patterns arising from local socio-economic conditions. This objective encompasses two specific downstream objectives: to evaluate the predictive performance of MRS in capturing regime-dependent demand fluctuations and to compare its operational forecasting performance with more traditional methods, such as ARIMA. This work aims to eventually assist in the design of more adaptive and robust retail planning systems in an unpredictable, dynamic environment.

2 Literature Review

Much research on retail economic and demand forecasting has been conducted using both linear and non-linear modeling. Regime-switch models, introduced in [1] for business cycle analysis, have been shown to be effective in representing structural breaks and phase shifts that may be hidden in time series data. MRS models improve interpretability and flexibility of forecasts in uncertain environments through the identification of regimes directly from the data. In line with this work, recent applications such as [2] and [3] have validated the robustness of regime-switch models against inflation and interest rate models. These works highlight the importance of regime identification in cases of high sensitivity to external perturbations and in contexts of behavioral change. Although used primarily in macroeconomic and financial settings, there has been a recent surge in the application of MRS analysis to the retail sector. [4] for example, found improved retail sales forecasting in emerging markets with a regime-dependent logic, while in other studies reinforced the benefits of nonlinear techniques to detect the presence of asymmetric demand patterns. Despite these challenges, there is both a theoretical and empirical lack of studies on the use of MRS models in the context of localized retail sales forecasting and in the case of multi-store environments where demand profiles tend to be sporadic and diverse, [5], [6], [7], [8], [9]. The present study aims to fill this gap by designing a regime-switching model that is aligned with the specific dynamics of the "Paghi Poco" supermarket network. In this way, it also contributes to the growing attention paid to data-driven decision support tools in the supply chain management literature, as well as to its practical implications on strengthening the resilience of store-level forecasts.

3 Methodology

3.1 Data Collection

The data set used in this study is based on the demand of the four different supermarkets named "Paghi Poco" which is registered in the meanwhile in four different cities in Sicily that are Palermo, Messina, Trapani, and Agrigento. The sampling interval is from November 28, 2023 to October 13, 2024, which is 319 d in a raw. Target areas were chosen to maximize coverage of Sicilian territory and to span a variety of consumer markets to enable a broad analysis of the behaviors of regional consumer. The raw demand values were transformed into logarithmic values, to reduce noise and enhance the transparency of the model. This prior preprocessing helps to stabilize the variance of the observations; this is particularly useful in modeling economic time series, as they usually have highly dispersed data. To verify whether the four stores exhibited significantly different demand dynamics, a preliminary one-way Analysis of Variance (ANOVA) was conducted on the log-transformed demand series. The results of the analysis ($F = 319.27$, $p < 2.2 \times 10^{-16}$) reveal substantial and statistically significant differences among the stores. This finding supports the decision to adopt an individualized modeling strategy, whereby separate regime-switching autoregressive models are estimated for each point of sale.

3.2 Model Specification

The first-order autoregressive Markov Regime Switching (MRS) model is specifically designed to reflect the complexities of demand processes by allowing for multiple regimes, with regime-specific statistical properties. The section reviews the model specification, including the mathematical representation of the model, regime definition, and underlying dynamics.

3.2.1 Mathematical Formulation

The basic form of the first-order autoregressive MRS model is expressed as:

$$Y_t = \mu_{s_t} + \emptyset_{s_t} Y_{t-1} + \varepsilon_t \tag{1}$$

where:

- μ_{s_t} intercept in the state s_t
- Y_t is the logarithm of demand at time t
- $\emptyset_{s_t}$ is the autoregressive coefficient, reflecting the impact of the previous period's demand on the current period in the state s_t
- ε_t is the error term, assumed to be normally distributed with zero mean and regime-specific variance: $\varepsilon_t \sim N(0, \sigma_{s_t}^2)$

This formulation captures the essence of autoregressive behavior while accounting for regime-specific dynamics.

3.2.2 Regime Definition

In the context of autoregressive-switching models, such as MS-AR(1), defining hidden states is important for identifying potential structural breaks in time series. These models are very effective at identifying sudden changes, for instance moving to different phases of a market or moving to a seasonal demand period. In a MS-AR(1) model, the observed process is defined and characterized as a driving autoregressive model. There is also an unobserved state variable driven by a discrete-time Markov chain. The model parameters written in Equation (1) is for the number of regime S that is a priori known for the model (or can be discovered from selection via good model selection; AIC, BIC). Estimation may rely on iterative approaches such as the EM algorithm or Bayesian approaches arriving at:

- Regime-specific autoregressive parameters;
- Transition probabilities between regimes;
- Smoothed regime probabilities indicating the likelihood of being in a given regime at time t

These smoothed probabilities allow for dynamic segmentation of the series into distinct regimes, offering insights into regime persistence and underlying drivers. Regime identification emerges endogenously from the data, without pre-imposed structural breaks. MS-AR models thus provide robust tools to analyze and interpret nonlinearities in economic time series, supporting advanced forecasting and decision-making processes. In this study, the application of an MS-AR(1) model leads to the identification of two distinct regimes that reflect contrasting demand conditions in the supermarket context:

- High-Demand Regime ($s_t = 1$): Sales are likely to increase due to favorable conditions such as effective promotions, high customer confidence, and seasonal peak.
- Low-Demand Regime ($s_t = 2$): This regime reflects periods of subdued sales possibly associated with economic downturns, intensified competitive pressure, or off-season dynamics.

The model's intrinsic capability to alternate between regimes in response to observed patterns allows it to adapt effectively to evolving market conditions and shifts in consumer behavior, thereby enhancing the forecasting precision and interpretability of demand trends.

3.2.3 Markov Process Dynamics

The regime-switching process is governed by a first-order Markov process, which means that the future state is entirely dependent on the current state. The transition probabilities between regimes are defined as follows:

$$P(S_t = j | S_{t-1} = i) = p_{ij} \tag{2}$$

where p_{ij} denotes the probability of transitioning from regime i to regime j. The transition matrix P is specified as follows:

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \tag{3}$$

The elements of the transition matrix provide insights into the likelihood of regime shifts. For instance:

- p_{11} indicates the probability of remaining in the high-demand regime
- p_{12} reflects the probability of transitioning from the high-demand to the low-demand regime
- p_{21} shows the probability of switching back to the high-demand regime from a low-demand
- p_{22} signifies the probability of staying in the low-demand regime.

In addition to parameter estimation, the MSM-AR(1) model computes regime probabilities essential for model classification. Filtered probabilities, calculated recursively via forward filtering [1], represent the posterior probability of being in a regime at time t given the information up to t. Smoothed probabilities, derived via backward smoothing [2], use the entire sample and provide a holistic view of regime transitions. Together, these probabilities enhance the interpretation of regime dynamics.

3.2.4 Parameter Estimation and Model Assumptions

The parameters of the MS AR(1) model are estimated using Maximum Likelihood Estimation supported by the Expectation Maximization algorithm. The model uses a likelihood function based on the observed data and unknown parameters. In the Expectation step, the algorithm calculates expected values of hidden state variables given current parameter estimates, reflecting the probability of being in a specific regime at time t. The Maximization step updates regime-specific means, variances, and transition probabilities by maximizing the expected log likelihood. This process is repeated until the maximum likelihood no longer increases, which is usually assessed by the stability of the log likelihood. Model adequacy is assessed using diagnostic checks, such as residuals analyses and model selection criteria, such as AIC and BIC. This framework allows for a better forecasting through an efficient modeling of the regime-specific dynamics. The MS AR(1) framework builds on a number of assumptions to ensure validity and reliability. Errors are assumed to be independent identically distributed normal and fit for any normal model, and heterogeneous within each state signifies high-dimensional model Ising model is typically mis-specified. The model is based on the Markov property, the changes of the regime of the system are based on the state of the system before, which promotes simplicity. It further assumes the statistical stationarity within each regime, the unchanging nature characterizing each regime for the statistical behaviour. Even to the extent these assumptions benefit reasonable models and forecasts, an error that is not normally distributed, or unmodeled an exogenous information will possibly impact model performance. Diagnostic checks shall be routinely carried out to support the assumptions made, and in assuring the robustness of the models in the "Paghi Poco" retail environment.

3.3 Backtesting Framework

To evaluate the predictive capabilities of the first-order MS-AR model of logarithmic demand for the Paghi Poco supermarket stores (the "Backtesting" process), a comprehensive back testing exercise was conceived. The backtesting process begins the exercise of assessing its performance in the model's forecasting and generalizability to new data, as well as being able to explain demand variation in a real-world context. A sample was made by splitting demand into training and testing parts. The training part is used to estimate the model parameters, while the testing phase is used solely to validate an out-of-sample forecast. More specifically, the training interval starts from the availability of the demand data through to the cutoff point where the cutoff is a predetermined point. The testing sample runs from November 28, 2023, to October 13, 2024. The back testing components are simply estimated coefficients from the training sample to construct rolling-path forecasts. These forecasts can then be directly compared to an average of the actual demand to quantify prediction accuracy. The measures of prediction accuracy were derived from traditional measures of error, specifically MAPE and RMSE error measures, which also lend information to the consistency and dispersion of the model. In addition, graphical representation of forecast accuracy was tested by exploring forecasted to observed values. This visual analysis enables the detection of systematic deviations, mis-specifications of the model, structural breaks, and/or regime shifts that have not been fully captured by the model. Overall, the back-testing framework provides essential validation of the power of the model's forecast and inspires future refinements. The insight gleaned from this analysis complements the usefulness of the MS-AR(1) model which could be utilized as a viable forecasting model for retail demand to adapt higher dynamics of the market.

4 Results and Discussion

4.1 Descriptive Statistics and Visualizations

The daily demand data are presented in logarithmic form, and we are examining four "Paghi Poco" supermarkets located in Sicily from November 28, 2023, to October 13, 2024. The high-frequency data comprise both short-term volatility and longer-term trends. The log transformation stabilizes the variance and the heteroscedasticity, which often appears in retail sales datasets, when promotional activity and seasonality are present. As can be seen from Table 1, the historical water consumption in Palermo and Agrigento are much more bidisperse, indicating that the number of regime shifts is more frequent. This justifies putting a regime-switching model into practice to handle demand dynamics and structural disparities among the outlets

Table 1. Descriptive Statistics

Super Market	Mean Log-Rev	Std. Dev. Log Rev	Min Log Rev	Max Log Rev
Trapani	9.361	0.2705	8.593	10.225
Messina	10.028	0.2327	9.449	10.742
Palermo	9.499	0.3160	8.405	10.197
Agrigento	9.506	0.3347	8.394	10.492

Source: created by the authors

Palermo and Agrigento supermarkets have higher standard deviations, which indicates that their daily demand is more volatile. In contrast, the stores in Messina and Trapani have more consistent income numbers, suggesting a reduced vulnerability to abrupt fluctuations in demand.

4.1.1 Demand Trends and Regime Shifts

Time series plots of the four supermarkets' logarithmic daily demands from November 28^{th} , 2023, to October 13^{th} , 2024, are shown in Figure 1, Figure 2 Figure 3 and Figure 4. There are noticeable trends of higher demand on weekends, public holidays, and during the busiest travel seasons (such the summer and Christmas), as well as evident daily changes in the data. Furthermore, sudden declines in demand during specific times point to the existence of separate regimes of high and low demand.

Figure 1, Figure 2, Figure 3 and Figure 4 illustrate varying demand dynamics across the four supermarkets. Trapani has a consistent demand, but Messina and Agrigento have a peak in summer, related to the tourism. Palermo is intermediate, with Agrigento being the most volatile. These patterns emphasize that there are high and low demand regimes and shows that distinct demand levels influenced by epoch, holiday and seasonal behaviour occur, which raise the profile of the regime switching approach as a more suitable method for modeling such nonlinear shifts.

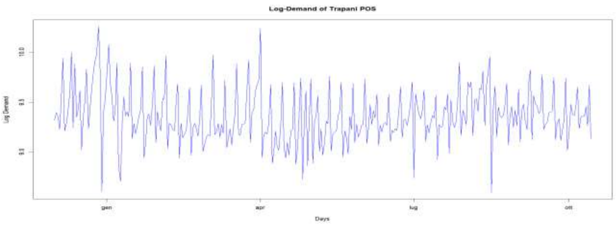

Fig. 1: Log Demand Trend of Trapani POS
Source: created by the authors

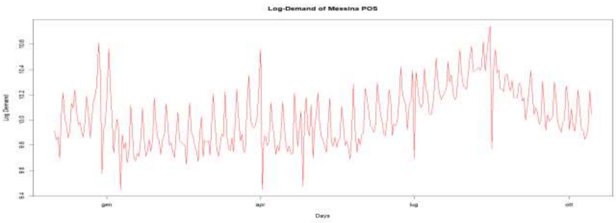

Fig. 2: Log Demand Trend of Messina POS
Source: created by the authors

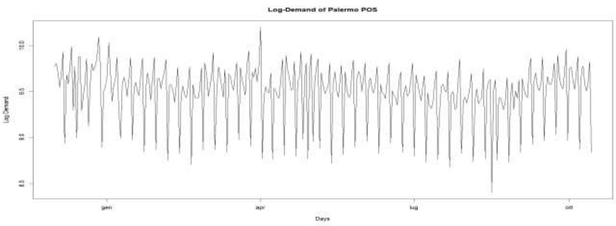

Fig. 3: Log Demand Trend of Palermo POS
Source: created by the authors

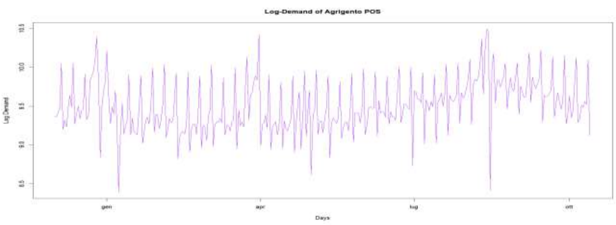

Fig. 4: Log Demand Trend of Agrigento POS
Source: created by the authors

4.2 Model Estimation and Transition Probabilities

In this section, we report the estimation results for the MSM-AR(1) model estimated on the logarithm of the demand data for the four "Paghi Poco" supermarkets. The model incorporates 2 regimes, high and low demand, each with its own mean and variance. Table 2 presents the parameter estimates, including significant coefficients at the 95% level. Larger autoregressive coefficients in each regime indicates a more persistent demand and lower exposure to the external shocks. These insights highlight the model's ability to reflect the retail demand structure and its usefulness for managerial decision making on inventory and promotions.

Table. 2 MSM-AR(1)'s coefficients

		Estimate	Std. Error	t value	Pr(>|t|)
Trapani	μ_{s_1}	1.7468	0.597	2.925	0.003445 *
	μ_{s_2}	12.2831	0.467	26.2572	2.2e-16 *
	ϕ_{s_1}	0.8306	0.064	12.958	2.2e-16 *
	ϕ_{s_2}	-0.3263	0.049	-6.6053	3.967e-11 *
	σ_{s_1}	0.183480	-	-	-
	σ_{s_2}	0.157556	-	-	-
	p_{11}	0.380053	-	-	-
	p_{12}	0.619946	-	-	-
	p_{21}	0.792893	-	-	-
	p_{22}	0.207106	-	-	-
Messina	μ_{s_1}	2.2806	0.414	5.4981	3.839e-08 *
	μ_{s_2}	12.4968	1.324	9.433	2.2e-16 *
	ϕ_{s_1}	0.7737	0.041	18.733	2.2e-16 *
	ϕ_{s_2}	-0.2596	0.132	-1.9548	0.05061
	σ_{s_1}	0.14394	-	-	-
	σ_{s_2}	0.204826	-	-	-
	p_{11}	0.936629	-	-	-
	p_{12}	0.063370	-	-	-
	p_{21}	0.366562	-	-	-
	p_{22}	0.633437	-	-	-
Palermo	μ_{s_1}	8.3462	0.295	28.282	2.2e-16 *
	μ_{s_2}	20.851	7.726	2.6987	0.006961 *
	ϕ_{s_1}	0.1363	0.031	4.3408	1.42e-05 *
	ϕ_{s_2}	-1.213	0.792	-1.5316	0.125621
	σ_{s_1}	0.151682	-	-	-
	σ_{s_2}	0.242144	-	-	-
	p_{11}	0.662601	-	-	-
	p_{12}	0.337398	-	-	-
	p_{21}	1	-	-	-
	p_{22}	0	-	-	-
Agrigento	μ_{s_1}	1.8412	0.756	2.4332	0.01497 *
	μ_{s_2}	11.2035	0.669	16.731	2.2e-16 *
	ϕ_{s_1}	0.8264	0.080	10.317	2.2e-16 *
	ϕ_{s_2}	-0.1964	0.070	-2.8017	0.005083 *
	σ_{s_1}	0.212359	-	-	-
	σ_{s_2}	0.268994	-	-	-
	p_{11}	0.436714	-	-	-
	p_{12}	0.563285	-	-	-
	p_{21}	0.752668	-	-	-
	p_{22}	0.247331	-	-	-

Source: created by the authors

The MSM-AR(1) model estimates reveal significant heterogeneity in parameter significance and behavior across the four supermarkets analyzed. In Trapani, all parameters, including intercepts and autoregressive coefficients, are significant at the 1 percent level, indicating strong regime separation. The autoregressive coefficient in the high-demand regime ($\emptyset_{s1}$ = 0.8306, $p < 0.001$) highlights substantial temporal dependence, while the negative coefficient in the low-demand regime ($\emptyset_{s2}$ = -0.3263) suggests potential volatility reversion. Messina also shows robust high-demand dynamics ($\emptyset_{s1}$ = 0.7737, $p < 0.001$), though the low-demand coefficient is only marginally significant (p = 0.0506), indicating less stable low-demand behavior. Palermo's estimates indicate weaker autoregressive effects overall, with the high-demand coefficient ($\emptyset_{s1}$ = 0.1363, $p < 0.001$) notably lower than in other stores, and the low-demand coefficient insignificant (p = 0.126), suggesting potential external shocks or local market anomalies. Agrigento demonstrates a strong high-demand autoregressive effect ($\emptyset_{s1}$ = 0.8264, $p < 0.001$) and a significantly negative low-demand coefficient ($\emptyset_{s2}$ = -0.1964, p = 0.005), confirming clear regime distinction. Residual standard deviations and transition probabilities further support the presence of distinct regimes with asymmetric persistence across stores. High probabilities of remaining in the high-demand state (e.g., 93.66 percent in Messina) point to strong regime persistence during peak periods, such as holidays or promotions. Conversely, low probabilities of remaining in the low-demand state, 0 percent in Palermo and 20.71 percent in Trapani, indicate that low-demand phases are typically short-lived, especially in urban contexts. Transition probabilities from high to low differ, with Trapani and Agrigento at 62 percent and 56 percent, respectively, reflecting local factors like peak promotion times or regional demand cycles. The risk of switching from low to high demand remains high, ranging from 75.26 percent in Agrigento to 100 percent in Palermo, often boosted by local economic incentives or advertising campaigns. In general, these findings highlight the dual characteristic of retail demand: during high-demand periods it is persistent and predictable, while for low-demand it is erratic and dominated by an exogenous process. The high frequency shifting regime identified is evidence against adequacy of static models with constant variance and that the MSM-AR(1) model has an edge over contemporaneous demand dynamics. This explicability increases the accuracy of forecasts and influences strategies for inventory management, pricing, and workforce planning that support profitability and service levels in a diverse retail environment.

4.2.1 Model Diagnostics by Regime and Location

A systematic diagnostic test was performed to evaluate the reliability and robustness of the MSM-AR(1) model under different demand and store location situations. The analysis encompasses both high-demand and low-demand regimes for each point of sale (POS), namely Messina, Palermo, Trapani, and Agrigento. This approach was designed to validate the residual properties post-estimation and to determine if the model satisfies the key assumptions of independence, homoscedasticity, and normality. The diagnostic analysis is based on the following graphical evaluations:

- Autocorrelation plots (ACF/PACF) for residuals and squared residuals, to verify the absence of serial correlation.
- Normal Q-Q plots, for visual inspection of residual distribution against the theoretical quantiles.
- Time series plots of standardized residuals, for detecting structural instabilities or regime-specific volatility clustering.

The diagnostic framework applied to the **Messina** store is illustrated in Figure 5. Panels (a) to (c) correspond to the high-demand regime, and panels (d) to (f) represent the low-demand regime.

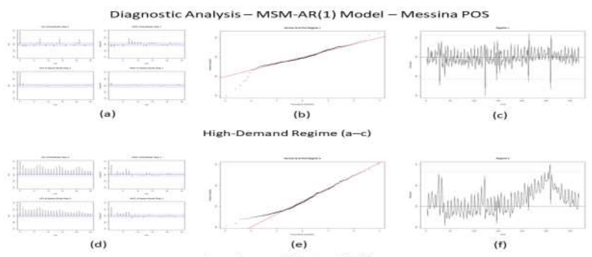

Fig. 5: Diagnostic Analysis – Messina POS
Source: created by the authors

In the high-demand regime (panels a–c), residuals and squared residuals (panel a) show no significant autocorrelation, the Q-Q plot (panel b) aligns closely with the Gaussian distribution, and the residual series (panel c) remains stable, indicating a consistent model fit. In contrast, in the low-demand regime (panels d–f), significant autocorrelation (panel d), slight departures from normality (panel e), and higher volatility (panel f) suggest potential unmodeled dynamics. These findings highlight the effectiveness of the MSM-

AR(1) model for high-demand periods and the need for enhancements, such as GARCH structures or additional variables, to capture the low-demand dynamics effectively.

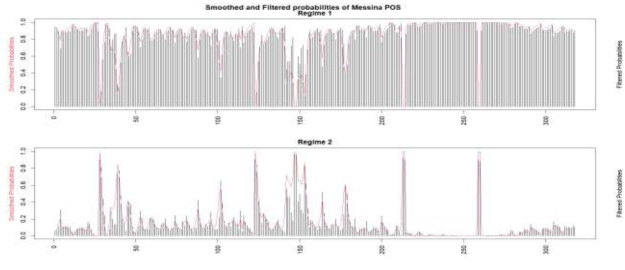

Fig. 6: Smoothed and Filtered Probabilities – Messina POS
Source: created by the authors

Figure 6 displays the smoothed and filtered probabilities for the MSM-AR(1) model applied to the Messina POS. In this case, Regime 1 is identified with the high-demand state, while Regime 2 captures the periods of low demand. The upper panel presents the probabilities associated with Regime 1, and the lower panel corresponds to Regime 2. The filtered probabilities (black) and smoothed probabilities (red) are closely aligned throughout the sample, reflecting consistency in real-time and retrospective classification of demand regimes. For Regime 1, the persistently high posterior probabilities reinforce the notion of a dominant high-demand regime. Meanwhile, transitions into Regime 2 are sparse and concentrated in specific episodes, as seen in the isolated probability spikes. This suggests that low-demand conditions are short-lived and episodic. The combined evidence provides further validation for the plausibility of regime classification within the estimated model.

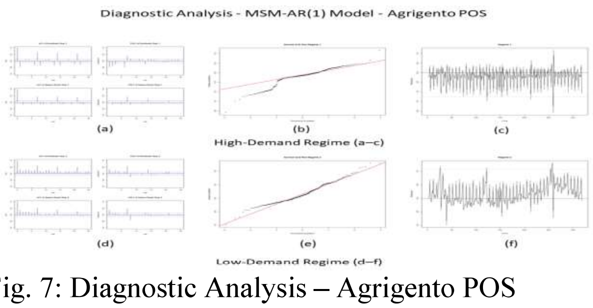

Fig. 7: Diagnostic Analysis – Agrigento POS
Source: created by the authors

Figure 7 presents residual diagnostics from the MSM-AR(1) model for Agrigento POS. In the high-demand regime (panels a–c), autocorrelations of residuals and squared residuals are within confidence bounds, residuals are approximately normally distributed, and standardized residuals show stable variance. In the low-demand regime (panels d–f), significant autocorrelations, slight non-normality, and clustered volatility suggest model limitations. These findings indicate that while the model fits the high-demand regime well, it may benefit from incorporating time-varying variance for the low-demand regime.

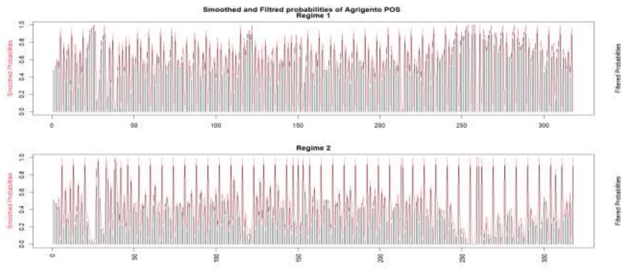

Fig. 8: Smoothed and Filtered Probabilities – Agrigento POS
Source: created by the authors

Figure 8 depicts the smoothed and filtered probabilities associated with the MSM-AR(1) model for the Agrigento POS. As in previous cases, Regime 1 is interpreted as the high-demand state, and Regime 2 corresponds to low-demand conditions. The upper and lower panels show the respective probabilities for each regime. The visual coherence between the smoothed and filtered probabilities across both panels suggests that the regime-switching mechanism is functioning in a stable and credible manner. In Regime 1, the probabilities demonstrate strong persistence, indicative of a prolonged and dominant high-demand regime. Conversely, the lower panel reveals that Regime 2 is characterized by shorter and more frequent excursions, often reverting quickly to the high-demand regime. These probabilistic dynamics align well with the sales data patterns and further validate the underlying switching behavior captured by the model.

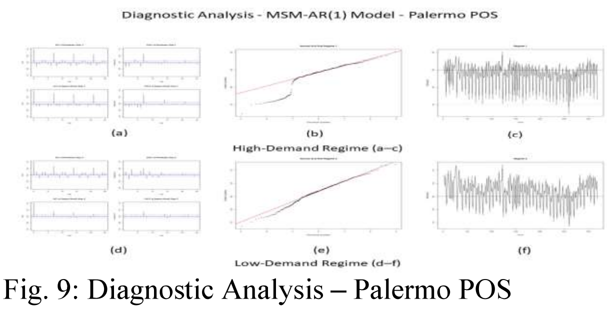

Fig. 9: Diagnostic Analysis – Palermo POS
Source: created by the authors

Figure 9 reports residual diagnostics for the MSM-AR(1) model at Palermo POS. In the high-demand regime (panels a–c), no significant autocorrelations are observed, though the Q-Q plot shows some lower-tail deviations, and the time plot indicates stable variance. In the low-demand regime (panels d–f), residuals meet white noise criteria, the Q-Q plot aligns closely with the normal distribution, and no signs of heteroscedasticity or volatility clustering are present. Overall, the diagnostics support the model's adequacy in capturing both demand regimes at Palermo POS, with residuals consistent with the model's assumptions.

Figure 10 shows the smoothed and filtered probabilities derived from the MSM-AR(1) model fitted to the Palermo POS. As with the previous locations, Regime 1 corresponds to high-demand conditions, while Regime 2 captures periods of relatively lower demand. The upper panel of the figure illustrates the probabilities for Regime 1, and the lower panel represents those for Regime 2. Filtered probabilities (in black) provide contemporaneous estimates of regime status, based on data available up to each time period. Smoothed probabilities (in red), on the other hand, incorporate the entire time series and hence yield more stable retrospective inferences. In Palermo, Regime 1 exhibits highly persistent posterior probabilities, which is consistent with a stable high-demand environment. Regime 2 appears only intermittently, with sharp yet infrequent probability spikes, suggesting transitory deviations from the dominant state. The consistency across filtered and smoothed estimates lends additional credibility to the underlying regime classification, affirming the model's effectiveness in capturing the true latent process.

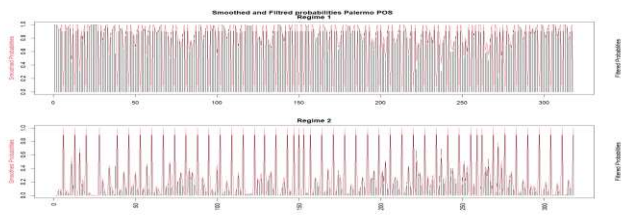

Fig. 10: Smoothed and Filtered Probabilities – Palermo POS
Source: created by the authors

Figure 11 presents residual diagnostics for the MSM-AR(1) model at the Trapani POS. In the high-demand regime (panels a–c), residuals show no significant autocorrelations, moderate deviations from normality in the lower tail, and stable variance over time. In the low-demand regime (panels d–f), residuals also exhibit no significant autocorrelations, align well with the normal distribution, and remain stable with no signs of heteroscedasticity. Overall, the diagnostics confirm the model's adequacy in capturing the dynamics at the Trapani POS across both regimes.

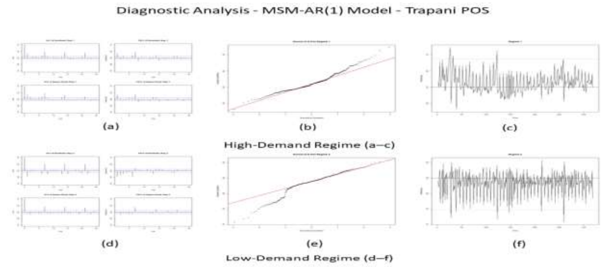

Fig. 11: Diagnostic Analysis – Trapani POS
Source: created by the authors

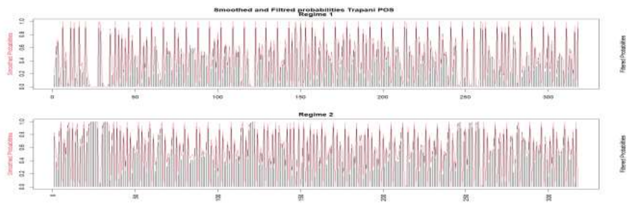

Fig. 12: Smoothed and Filtered Probabilities – Trapani POS
Source: created by the authors

Figure 12 shows the smoothed and filtered probabilities from the MSM-AR(1) model for the Trapani POS, with Regime 1 capturing high-demand and Regime 2 representing low-demand periods. Filtered and smoothed probabilities align closely, indicating consistent regime classification. Transitions occur frequently enough to justify the regime-switching approach, with stable inference supporting the identification of distinct demand phases. Residual diagnostics confirm model adequacy, with residuals aligning with theoretical assumptions. Overall, the MSM-AR(1) model effectively captures demand dynamics in the all POS, supporting its practical use in forecasting.

4.3 Forecast Accuracy

4.3.1 Methodology for Forecast Evaluation

Evaluating forecast accuracy is crucial in validating any predictive model, especially in retail where precise demand forecasts influence operational and strategic decisions. This study assesses the predictive performance of the first-order Markov Regime Switching Autoregressive model (MSM-AR(1)) using a structured back testing framework on daily demand data from four "Paghi Poco" supermarkets in Sicily, covering the period from November 28, 2023, to October 13, 2024. To create a realistic forecasting scenario, the last 33 daily observations were set aside as a holdout sample, while the remaining data were used to estimate regime-specific autoregressive coefficients and transition probabilities. Forecasts were generated for each point in the holdout period using these estimated parameters. The model's performance was then evaluated using three key error metrics. Mean Absolute Error (MAE) measures the average absolute difference between predicted and observed demand, offering a straightforward measure of forecast accuracy. Root Mean Squared Error (RMSE) gives more weight to larger deviations, making it useful for detecting occasional large forecast errors. Mean Absolute Percentage Error (MAPE) expresses errors as a percentage of actual demand, enabling comparisons across stores and time periods despite differences in scale. These metrics, calculated over the holdout period, demonstrate the MSM-AR(1) model's capacity to deliver reliable forecasts even in retail environments characterized by volatility and regime shifts. By combining autoregressive structure with regime-dependent dynamics, the model adapts effectively to underlying demand variations, enhancing its robustness compared to simpler linear models. These findings set the stage for discussing the model's managerial implications and potential deployment in retail operations.

4.3.2 Performance Comparison with Other Models

The predictive capability of the MSM-AR(1) model was compared with that of the conventional AR(1). Table 3 indicates that the performance of MSM-AR(1) is better than AR(1) in terms of forecasting accuracy measures as MAE, RMSE, and MAPE, showing that it possesses better capabilities of capturing the fluctuations between high and low demand seasons. For instance, Agrigento's MAE decreases from 0.2666 under AR(1) to 0.0251 under MSM-AR(1).

Table. 3. Forecasting Performance Indicators

	Model	MAE	RMSE	MAPE
Trapani	AR(1)	0.2082	0.2697	2.2188
	MSM-AR(1)	0.0168	0.0722	0.1795
Messina	AR(1)	0.1409	0.1863	1.4050
	MSM-AR(1)	0.0108	0.0407	0.1070
Palermo	AR(1)	0.2317	0.3055	2.4770
	MSM-AR(1)	0.0282	0.1206	0.3000
Agrigento	AR(1)	0.2666	0.3404	2.8040
	MSM-AR(1)	0.0251	0.1021	0.2600

Source: created by the authors

Figure 13, Figure 14, Figure 15, Figure 16, Figure 17, Figure 18, Figure 19 and Figure 20 provide graphical representations of these forecasts over the evaluation window from September 11 until October 13, 2024, and reveal that the MSM-AR(1) model is able to better replicate important turning points and regime-specific fluctuations. In general, the MSM-AR(1) model's ability to account for changes in regimes makes it more suitable for predicting retail demand, although its performance relies on model assumptions, data availability, and adequate transition probability specification, [10].

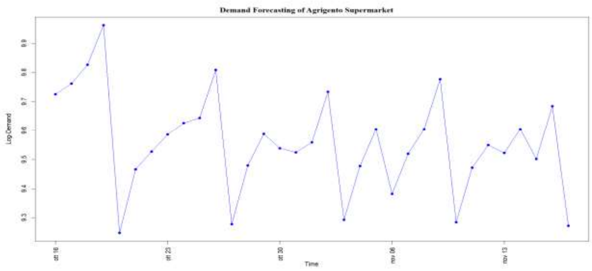

Fig. 13: Demand Forecasting of Agrigento POS
Source: created by the authors

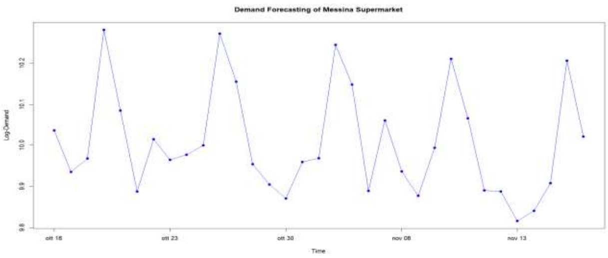

Fig. 14: Demand Forecasting of Messina POS
Source: created by the authors

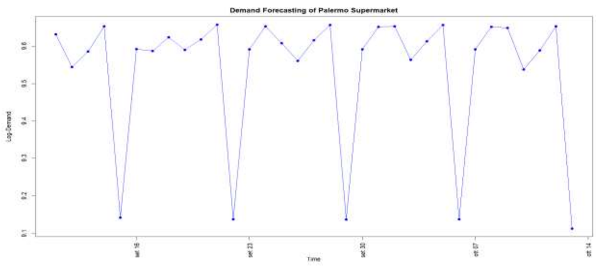

Fig. 15: Demand Forecasting of Palermo POS
Source: created by the authors

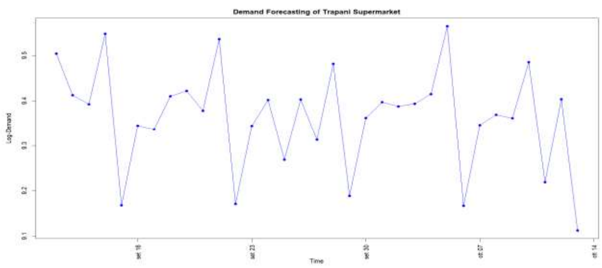

Fig. 16: Demand Forecasting of Trapani POS
Source: created by the authors

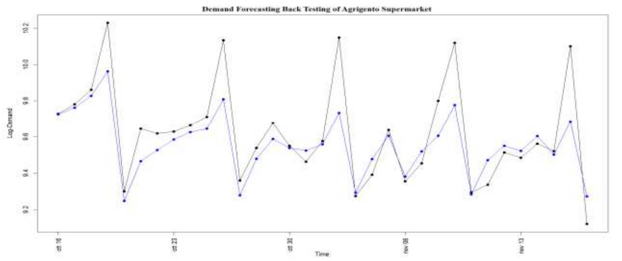

Fig. 17: Back Testing Forecasting of Agrigento POS
Source: created by the authors

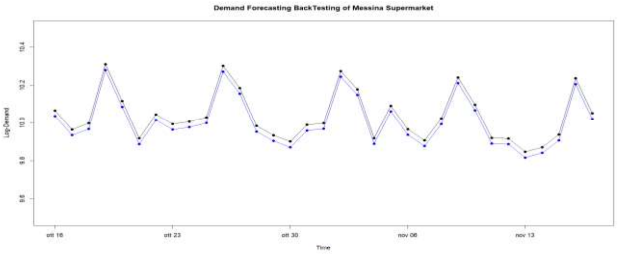

Fig.18: Back Testing Forecasting of Messina POS
Source: created by the authors

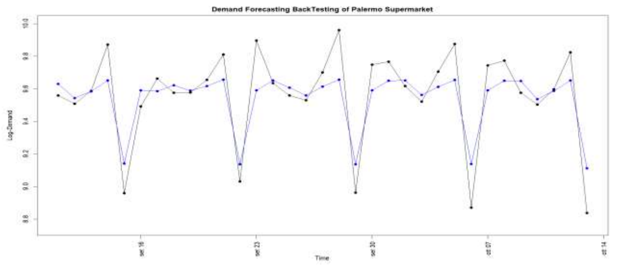

Fig.19: Back Testing Forecasting of Palermo POS
Source: created by the authors

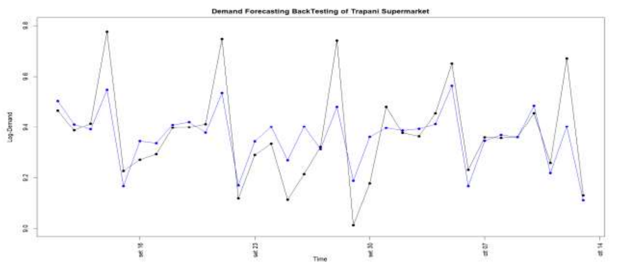

Fig. 20: Back Testing Forecasting of Trapani POS
Source: created by the authors

Although the forecast is able to simulate the cyclical demand pattern, slight differences are observed between the log-scale actual demand and the forecast. This therefore suggests others factors might be worth to be incorporated into the model in the future. For example, leveraging dummy variables to capture which day in the week the demand comes in might improve the forecast. In addition, the model could be expanded to include other variables as external determinants (i.e., local event, economic-related figures), that might lead it to better predict the demand and the consumer's pattern of behavior.

5 Conclusion

This study develops a forecasting method for retail demand in grocery distribution, dedicated to the analysis of "Paghi Poco" supermarkets in Sicily, which is based on a first-order MSM-AR(1) approach. Incorporating the latent regime recognition, our model is able to capture the nonlinear daily demand fluctuation well and outperforms the AR model in prediction. The stochastic decision model detecting when the system will depart high-/low-demand states can still be used in dynamic operational planning. Live deployment in many stores enable growth and adaptation with your business. To better fit the context, some extensions being investigated include the use of higher level lags and time-varying transition probabilities that can be functions of certain covariates. In a broader context, the study may be particularly useful in intelligent forecasting systems, and it points to future directions for data-driven decision-support in retail logistics.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

The authors wrote, reviewed and edited the content as needed and verifies that none utilized artificial intelligence (AI) tools were used. The authors take full responsibility for the content of the publication.

References:

[1] Hamilton J.D., Econometrica, *A new approach to the economic analysis of nonstationary time series and the business cycle*, Vol.57, No.2, 1989, pp. 357-384. doi: 10.2307/1912559.

[2] Kim C. J., Journal of Econometrics, *Dynamic Linear Models with Markov Switching*, Vol.60, N.1-2, 1994, pp.1-22. doi: 10.1016/0304-4076(94)90036-1.

[3] Ang. A. & Bekaert G., Journal of Business & Economic Statistics, *Regime switching in interest rates*, Vol.20, N.2, 2002, pp.163-182 doi: 10.1198/073500102317351930.

[4] Chikwe J., et al., Scentific Report, *Real-time Metagenomic Analysis of Undiagnosed Fever Cases Unveils a Yellow Fever Outbreak in Edo State*, Vol. 10, N.1, pp. 3180. doi: 10.1038/s41598-020-59880-w.

[5] Box G.E.P. & Jenkins G.M., *Time Series Analysis: Forecasting and Control.* San Francisco: Holden-Day. 1976.
[6] Lu, J., Feng, T., Timmermans, H., & Yang, Z., Transportmetrica A: Transport Science, *An integrated Markov decision process and nested logit consumer response model of air ticket pricing*, Vol.13, N.6, pp. 544-567. doi: 10.1080/23249935.2017.1306727.
[7] Yang S., Wang L., Wang J., Li C., Zhou H. *Multi-Agent Deep Reinforcement Learning for Integrated Demand Forecasting and Inventory Optimization in Sensor-Enabled Retail Supply Chains*, Sensors, Vol.25, No.8, 2025, Art. 2428. doi: 10.3390/s25082428.
[8] Riachy, Chirine & He, Mengda & Joneidy, Sina & Qin, Shengchao & Payne, Tim & Boulton, Graeme & Occhipinti, Annalisa & Angione, Claudio. (2024). *Enhancing deep learning for demand forecasting to address large data gaps*. Expert Systems with Applications. 268. 126200. doi: 10.1016/j.eswa.2024.126200.
[9] Jahin, Md Abrar & Shahriar, Asef & Al Amin, Md. (2025). *MCDFN: supply chain demand forecasting via an explainable multi-channel data fusion network model*. Evolutionary Intelligence. 18. doi: 10.1007/s12065-025-01053-7.
[10] Wang, J., Chong, W. K., Lin, J., & Hedenstierna, C. P. T. (2023). *Retail Demand Forecasting Using Spatial-Temporal Gradient Boosting Methods*. Journal of Computer Information Systems, 64(5), 652–664. doi: 10.1080/08874417.2023.2240753

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.