An innovative only-output method to identify a structural system

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Abstract. Structural Health Monitoring (SHM) is nowadays common in many branches of engineering since it allows to have a continuous or periodic report of the structural conditions and therefore to promptly intervene if there are incipient damages. The first step to perform a SHM is the identification of the dynamic parameters, i.e. natural frequencies, damping ratios and modal shapes, and it is a crucial step since a modification of the structural parameters can be a direct consequence of structural damages. Among the structural identification methods, Operational Modal Analysis (OMA) methods have received increasing attention from the researchers since they do not require the knowledge of the structural excitation that is due to ambient vibrations and that is usually modeled as a white noise. This aspect makes this kind of methods cheaper and simpler than the classical Experimental Modal Analysis (EMA) methods.

In this paper an innovative OMA method is proposed. It is a semi – automated method that allows to identify natural frequencies, damping ratios and modal shapes of a structural system and that can be used also from users that have not knowledge in stochastic dynamics and signal analysis. First of all, the modal shapes are estimated through the use of signal filtering techniques applied on the stochastic properties of the output process and then natural frequencies and damping ratios can be estimated from the mono – component analytical signals obtained by performing a decomposition of the analytical signals matrix. The proposed method has been used to perform the dynamic identification of a real historic building situated in Palermo, i.e. Chiaramonte palace, and the results obtained have been compared with those obtained by using other OMA methods.

1. Introduction

The estimation of natural frequencies, damping ratios and the modal shapes of a structural system takes the name of dynamic identification and it plays an important role in many areas of engineering. There are two main types of methods through which it is possible to perform a dynamic identification of a system, respectively, Experimental Modal Analysis (EMA) methods and Operational Modal Analysis (OMA) methods [1]. EMA methods are generally applied to systems that have linear or non-linear behavior, but it requires knowledge of both the structural excitation and the structural response and, therefore, this results in a complicated and expensive set-up of the tests to be performed in-situ [1]. OMA methods do not require the knowledge of the structural excitation, and that makes them preferable to be used thanks to the easier and cheaper set-up for the in-situ tests [2]. Another positive aspect of using OMA methods is based on the fact that the identification of the structural system takes place when it is in operational conditions, i.e. when the structural input is given by ambient vibrations that are modeled as a white noise process [3-5]. Therefore, OMA methods have a stochastic framework. However, the usage of OMA methods is not limited to the identification of structural systems, but it also includes other purposes such as performing a Structural Health Monitoring (SHM), the calibration of finite elements models and the detection of structural damages. OMA methods turned out to be extremely useful for real cases like historical buildings [6-8], bridges [8-11] and other type of structures.

OMA methods can be divided into methods that operate in the frequency domain, in which the dynamic parameters are usually estimated from the Power Spectral Density (PSD) of the structural output; and methods that operate in the time domain, in which the same parameters are usually estimated from the correlation function of the structural output process. Among the most appreciated OMA methods developed in frequency domain, it is worth mentioning the Frequency Domain Decomposition method (FDD) [12,13] and the Peak Picking method (PP) [14], which is usually applied in combination with the Half Power bandwidth method (HP) [14]. Due to their deterministic framework, PP and HP have been initially classified as EMA methods because they were normally applied on the Frequency Response Function of a system (FRF); later on they have been applied on the PSD and have been also considered OMA methods since then. The main characteristic of FDD [12,13] is that it allows the identification of the natural frequencies and the modal shapes of a structural system from the Singular Value Decomposition (SVD) [4] of the PSDs matrix. An updated version of FDD, called Enhanced Frequency Domain Decomposition (EFDD) [15], has been developed in order to estimate also the damping ratios [1]. However, the exact estimation of the damping ratios by using FDD techniques is still an open issue [1]. There are numerous OMA methods that operate in time domain, such as Natural Excitation Technique (NExT) [16], Auto Regressive Moving Average (ARMA) [17], Time Domain Decomposition (TDD) [18], and Stochastic Space Identification (SSI) [19-22]; however, the most popular is SSI that can be implemented in two different forms: SSI data-driven and SSI covariance-driven.

Due to their stochastic framework, OMA methods in general result to be of very difficult usage for people that are not familiar with signal analysis and stochastic dynamics. For this reason, other methods based on the analytical signal and on the modal decomposition of the correlation functions' matrix have been recently proposed [23,24]. However, these methods can be applied only if the matrix containing the identified modal shapes is a square matrix, i.e. if the number of sensors is equal to the number of identified modal shapes. In the practical applications, the number of sensors is often not equal to the number of identified modal shapes and this is the reason that led to the proposal of a semi-automated OMA procedure of this paper. Through the use of this method it is possible to estimate natural frequencies, modal shapes and damping ratios of a structural system. Since the dynamic identification performed by using the analytical signal has been firstly used in a deterministic framework [25-27] and then successfully extended to OMA [23,24,28], the proposed method is based on filtering techniques and on the decomposition of the matrix that contains the analytical signals of the output process' correlation functions. A practical application consisting in the dynamic identification of Chiaramonte palace in Palermo is presented and the results obtained are compared with those obtained by using EFDD and SSI.

2. Proposed method

In this section, all the steps of the proposed method are described in detail. Considering a MDOF system having mass matrix $\mathbf{M}$, dissipation matrix $\mathbf{C}$ and stiffness matrix $\mathbf{K}$, its equation of motion, in the case in which it is excited by a zero-mean white noise (ground acceleration) $W(t)$, is

$$\begin{cases} \mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=-\mathbf{M}\mathbf{V}W(t) \\ \mathbf{X}(0)=\mathbf{0} \qquad \text{w.p.1} \\ \dot{\mathbf{X}}(0)=\mathbf{0} \qquad \text{w.p.1} \end{cases}. \tag{1}$$

In equation (1), $\ddot{\mathbf{X}}(t)$, $\dot{\mathbf{X}}(t)$ and $\mathbf{X}(t)$ represent the response process expressed, respectively, in terms of acceleration, velocity and displacement, while $\mathbf{V}$ is the forcing location vector containing only unitary values. The correlation functions' matrix of the response process $\ddot{\mathbf{X}}(t)$ is labeled as $\mathbf{R}_{\ddot{\mathbf{X}}}(\tau)$ and its component can be calculated as [4]

$$R_{\ddot{X}_i \ddot{X}_j}(\tau) = E\left[\ddot{X}_i(t)\ddot{X}_j(t+\tau)\right] \tag{2}$$

being $E[\bullet]$ the stochastic average.

By using the Wiener-Khinchine relationships, the PSDs' matrix can be easily calculated considering that its components are expressed as [4]

$$S_{\ddot{X}_i \ddot{X}_j}(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} e^{-i\omega\tau} R_{\ddot{X}_i \ddot{X}_j}(\tau)\, d\tau\,. \tag{3}$$

Each component of the PSDs' matrix presents peaks in correspondence of the structural frequencies, and thus, by extracting the abscissa of each peak, a first estimation of the natural frequencies can be performed. Through the use of band-pass filters having very little bandwidth, the structural output process can be filtered around each natural frequency. Once the output process has been filtered, the correlation function can be calculated and its components can be expressed in the form

$$R_{\ddot{X}_i^{(j)} \ddot{X}_1^{(j)}}(\tau) = E\left[\ddot{X}_i^{(j)}(t)\ddot{X}_1^{(j)}(t+\tau)\right] \tag{4}$$

in which $\ddot{X}_i^{(j)}(t)$ represents the i-th component of the output process filtered around the j-th natural frequency. For structural systems that have well-separated frequencies, the modal shapes can be approximated as

$$\frac{R_{\ddot{X}_i^{(j)} \ddot{X}_1^{(j)}}(0)}{R_{\ddot{X}_1^{(j)} \ddot{X}_1^{(j)}}(0)} = \frac{\sigma_{\ddot{X}_i^{(j)} \ddot{X}_1^{(j)}}}{\sigma^2_{\ddot{X}_1^{(j)}}} \approx \frac{\phi_{ij}}{\phi_{1j}} \tag{5}$$

in which $\sigma^2_{\ddot{X}_1^{(j)}}$ is the variance of $\ddot{X}_1^{(j)}(t)$ while $\sigma_{\ddot{X}_i^{(j)} \ddot{X}_1^{(j)}}$ represents the covariance between $\ddot{X}_i^{(j)}(t)$ and $\ddot{X}_1^{(j)}(t)$.

The analytical signal of the components in equation (2) is a complex signal in which the real part is $R_{\ddot{X}_i \ddot{X}_j}(\tau)$ while the imaginary part is the Hilbert transform of $R_{\ddot{X}_i \ddot{X}_j}(\tau)$ and thus it can be expressed as [23,24]

$$Z_{R_{\ddot{X}_i \ddot{X}_j}}(\tau) = R_{\ddot{X}_i \ddot{X}_j}(\tau) + i\hat{R}_{\ddot{X}_i \ddot{X}_j}(\tau) \tag{6}$$

being $\hat{R}_{\ddot{X}_i \ddot{X}_j}(\tau)$ the Hilbert transform of $R_{\ddot{X}_i \ddot{X}_j}(\tau)$ that is defined as [23,24]

$$\hat{R}_{\ddot{X}_i \ddot{X}_j}(\tau) = \pi^{-1} \wp \int_{-\infty}^{\infty} (\tau - \bar{\tau})^{-1} R_{\ddot{X}_i \ddot{X}_j}(\bar{\tau})\, d\bar{\tau} \tag{7}$$

in which $\wp$ represents the principal value. The matrix containing all the analytical signals, labeled as $\mathbf{Z}_{\mathbf{R}_{\ddot{\mathbf{X}}}}(\tau)$, can be therefore determined but its components contain more than one contribution in frequency domain. In order to obtain analytical signals that have only one contribution in frequency domain, a decomposition of $\mathbf{Z}_{\mathbf{R}_{\ddot{\mathbf{X}}}}(\tau)$ can be performed as

$$\mathbf{Z}_{\mathbf{R}_{\ddot{\mathbf{Y}}}}(\tau) = \mathbf{\Phi}^{-1} \mathbf{Z}_{\mathbf{R}_{\ddot{\mathbf{X}}}}(\tau) \mathbf{\Phi}^{-T}\,. \tag{8}$$

Since the inverse of $\mathbf{\Phi}$ and $\mathbf{\Phi}^T$ can be calculated only in the case in which the number of identified modal shapes is equal to the number of sensors (i.e. when $\mathbf{\Phi}$ is a square matrix), the decomposition in equation (8) cannot always be used. To overcome this limit, the inverse matrix is substituted with the pseudoinverse matrix that is defined as [29]

$$\mathbf{\Phi}^{+} = \mathbf{B}\mathbf{C}^{+}\mathbf{A}^{T} \tag{9}$$

in which $\mathbf{A}$ and $\mathbf{B}$ represent, respectively, the left and the right eigenvectors of $\mathbf{\Phi}$, the apex + denotes the pseudoinverse, while $\mathbf{C}$ is the matrix containing the singular values of $\mathbf{\Phi}$. $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be easily calculated by performing a SVD, while the pseudoinverse of $\mathbf{C}$ can be calculated by substituting the components on its diagonal with their reciprocals. The matrix $\mathbf{Z}_{\mathbf{R}_{\ddot{X}}}(\tau)$, therefore, can be decomposed as

$$\mathbf{Z}_{\mathbf{R}_{\ddot{Y}}}(\tau) = \mathbf{\Phi}^{+}\mathbf{Z}_{\mathbf{R}_{\ddot{X}}}(\tau)\left(\mathbf{\Phi}^{T}\right)^{+}. \tag{10}$$

Each component on the principal diagonal of $\mathbf{Z}_{\mathbf{R}_{\ddot{Y}}}(\tau)$ has only one significant contribution in frequency domain, therefore the structural frequencies and damping ratios can be estimated. Particularly, the j-th component on the diagonal of $\mathbf{Z}_{\mathbf{R}_{\ddot{Y}}}(\tau)$, labeled as $Z_j(\tau)$, can be expressed in polar form as

$$Z_j(\tau) = D_j(\tau)e^{i\vartheta_j(\tau)} \tag{11}$$

in which $D_j(\tau)$ represents the envelope and $\vartheta_j(\tau)$ is the phase. These quantities can be calculated starting from equation (11) as

$$D_j(\tau) = \sqrt{\left(\mathrm{Re}\left[Z_j(\tau)\right]\right)^2 + \left(\mathrm{Im}\left[Z_j(\tau)\right]\right)^2} \approx \sigma_{\ddot{Y}}^2 e^{-2\pi f_j \zeta_j \tau} \tag{12}$$

$$\vartheta_j(\tau) = \arctan\left(\mathrm{Im}\left[Z_j(\tau)\right] \big/ \mathrm{Re}\left[Z_j(\tau)\right]\right) \approx 2\pi\tau f_j\sqrt{1-\zeta_j^2} \tag{13}$$

in which f_j and ζ_j represent, respectively, the j-th structural frequency and the j-th damping ratio, while $\sigma_{\ddot{Y}}^2$ is the variance of the process $\ddot{Y}(t)$. The instantaneous damped frequencies, i.e. $\bar{f}_j(\tau)$, can be calculated from the first derivative of the phase in the form

$$\bar{f}_j(\tau) = \frac{1}{2\pi}\frac{d}{d\tau}\left(\vartheta_j(\tau)\right) \tag{14}$$

and, by performing the stochastic average of equation (14), the j-th can be immediately estimated.

By performing the natural logarithm of equation (12), the following linear form can be obtained

$$\ln\left(D_j(\tau)\right) = a_j + b_j\tau \tag{15}$$

in which $a_j = \ln\left(\sigma_{\ddot{Y}}^2\right)$ and $b_j = -2\pi f_j \zeta_j \tau$. Therefore, the damping ratios can be estimated as

$$\zeta_j = \sqrt{\frac{\bar{b}_j^2}{1+\bar{b}_j^2}} \tag{16}$$

being $\bar{b}_j = b_j \big/ \left(2\pi E\left[\bar{f}_j(\tau)\right]\right)$.

In the next section, the proposed method is applied to a real case study, i.e. the dynamic identification of Chiaramonte palace in Palermo, and the results obtained are compared with those obtained by using other popular OMA methods.

3. Real case study: Chiaramonte palace

Palazzo Chiaramonte (figure 1), also known as Palazzo Steri, is a historic building dating back to the 14th century located in the marine area of Palermo. The structure, completed in 1307, was commissioned by Giovanni Chiaramonte, a leading exponent of one of the most powerful Palermitan families of that historical period [30]. Over the centuries, the structure has had various uses, and, today, it is the headquarters of the University of Palermo. Clear example of Arab-Norman influences, Palazzo Chiaramonte is an imposing three-story masonry building with a square plan of about 40x40 m. The central courtyard (figure 2), object of this study, is distributed over an area of about 400 m^2, has a total height of about 20 m and consists of a double arcade with ogival arches resting on columns.

The structural output process has been acquired through the use of piezoelectric accelerometers whose characteristics are reported in table 1. Particularly, six sensors have been placed in the points indicated in figure 3 [31].

Figure 1. Chiaramonte palace.

Figure 2. Central courtyard.

Figure 3. Position of the sensors.

Table 1. Characteristics of the piezoelectric sensors.

Producer	PCB Piezotronics
Model	PCB 393B04
Sensitivity	1000 mV/g
Measuring range	+/- 5 g
Frequency range	From 0.06 Hz to 450 Hz
Broadband resolution	$3x10^{-6}$ g rms
Mass	50 g

The piezoelectric sensors have been connected, through the use of BNC cables, to a PXIe 1028 acquisition unit equipped with a 16-channels PXIe 4497 acquisition card. The structural output process has been acquired for 600 s by using a sampling time step $\Delta t = 0.01$ s . Once that the output process has been acquired, the correlation functions' matrix has been estimated by using equation (2) while, by using equation (3), the components PSDs' matrix have been calculated. The auto-PSDs in direction u and v are reported, respectively, in figure 4 and figure 5.

From the peaks of the PSDs, a first estimation of the natural frequencies has been performed and then, the output process has been filtered through the use of Butterworth band-pass filters of 8-th order having bandwidth [2.7616 – 2.7682] Hz, [3.5611 – 3.5678] Hz, [3.8541 – 3.8608] Hz, [4.7269 – 4.7336] Hz and [5.3189 – 5.3256] Hz.

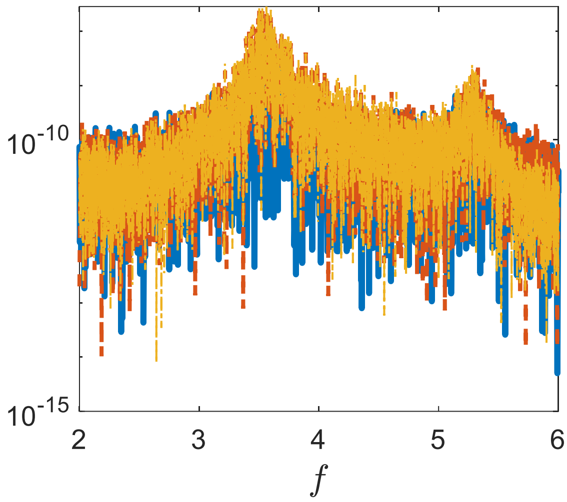

Figure 4. Auto-PSD in direction u: X_1 (continuous line); X_2 (dashed line) and X_3 (dash-dotted line).

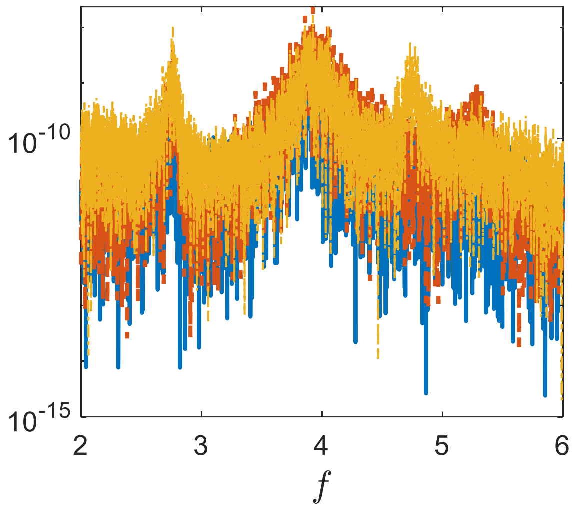

Figure 5. Auto-PSD in direction v: X_4 (continuous line); X_5 (dashed line) and X_6 (dash-dotted line).

The correlation functions' matrix of the filtered process has been calculated by using equation (4) and the modal shapes identified through the use of equation (5) are compared, in figures 6-10, with those identified by using EFDD and SSI.

From figures 6-10 it is clear that the proposed method is able to identify the modal shapes of the structural system with a precision similar to that of EFDD and SSI. However, SSI identifies only the second, the third and the fifth modal shape.

Once that the modal shapes have been estimated, the matrix containing all the analytical signals of the correlation functions, calculated as reported in equation (6), have been decomposed as in equation (10). The components on the diagonal of $\mathbf{Z}_{\mathbf{R}_\mathbf{Y}}(\tau)$ are reported in figure 11.

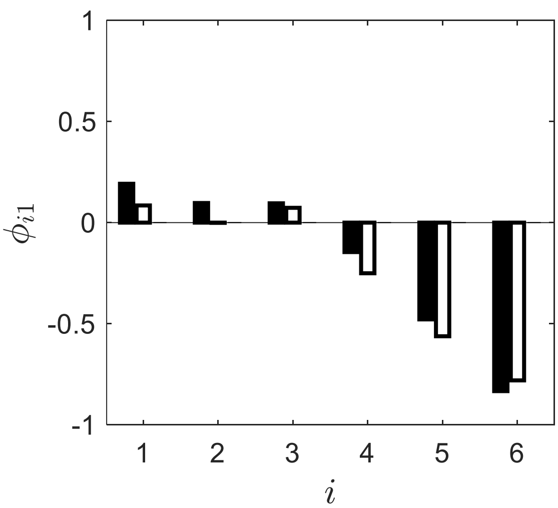

Figure 6. First modal shape: Proposed method (black); EFDD (white, continuous edge); SSI (white, dash-dotted edge).

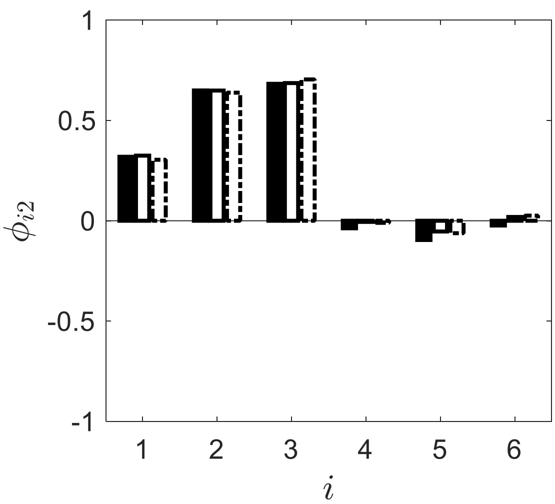

Figure 7. Second modal shape: Proposed method (black); EFDD (white, continuous edge); SSI (white, dash-dotted edge).

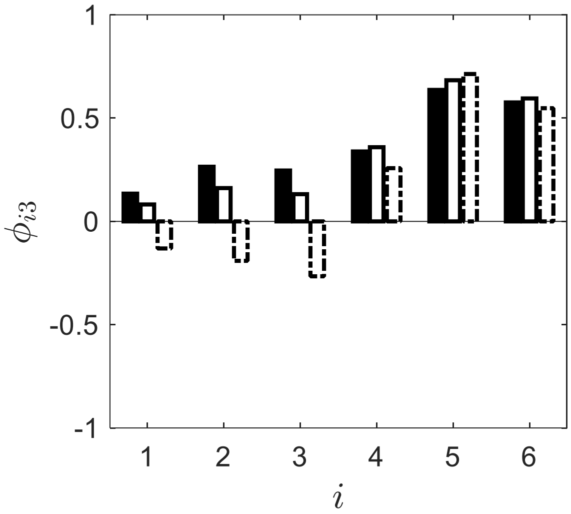

Figure 8. Third modal shape: Proposed method (black); EFDD (white, continuous edge); SSI (white, dash-dotted edge).

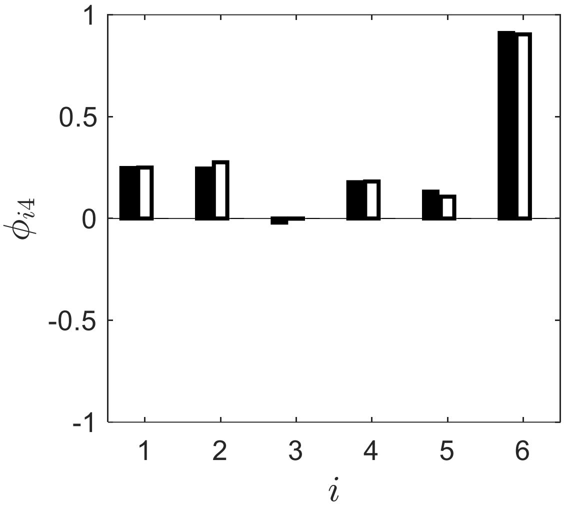

Figure 9. Fourth modal shape: Proposed method (black); EFDD (white, continuous edge); SSI (white, dash-dotted edge).

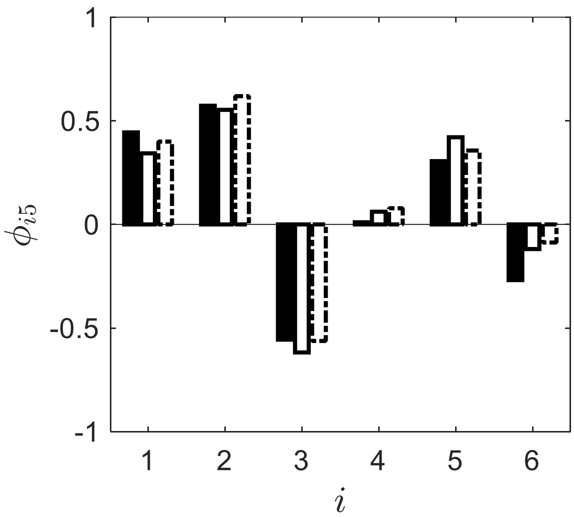

Figure 10. Fifth modal shape: Proposed method (black); EFDD (white, continuous edge); SSI (white, dash-dotted edge).

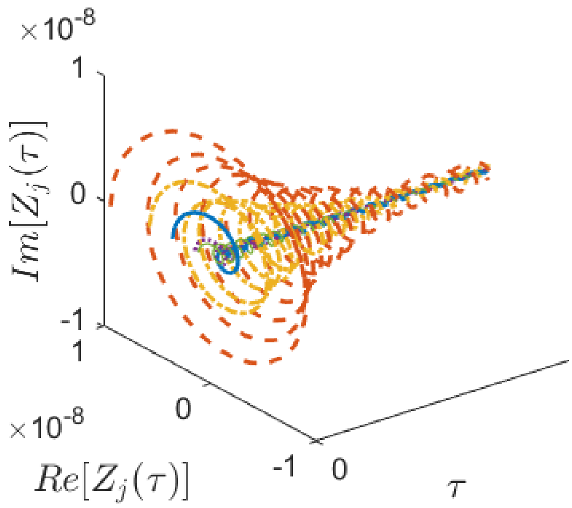

Figure 11. Components on the principal diagonal of the matrix $\mathbf{Z}_{\mathbf{R}_{\mathbf{Y}}}(\tau)$: First mode (continuous thick line); Second mode (dashed line); Third mode (dash-dotted line); Fourth mode (dotted line) and Fifth mode (continuous thin line).

From the analytical signals reported in figure 11, by using equations (12-16), the structural frequencies and the damping ratios have been estimated. The results obtained in terms of frequencies and damping ratios are compared with those obtained by using EFDD and SSI, respectively, in table 2 and table 3.

Table 2. Structural frequencies estimated with the proposed method, EFDD and SSI.

Method	f_1	f_2	f_3	f_4	f_5
EFDD	2.7649	3.5645	3.8574	4.7302	5.3223
SSI	-	3.5574	3.9211	-	5.2951
Proposed	2.7213	3.5519	3.8628	4.7481	5.2941

Table 3. Damping ratios estimated with the proposed method and EFDD.

Method	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
EFDD	0.96%	2.59%	1.91%	1.73%	1.28%
SSI	-	2.56%	2.35%	-	1.26%
Proposed	1.33%	2.40%	2.45%	1.23%	1.31%

From the results reported in table 2 it is clear that all the used methods lead to results that are very similar to each other but SSI identifies only the second, the third and the fifth frequencies. From the results reported in table 3 it can be observed that the first and the fourth damping ratios are not identified by SSI and that, for the same modes, the results obtained by using the proposed method are slightly different from the damping ratios identified by using EFDD. All the used methods lead to similar results for the second and the fifth modes while, for the third mode, EFDD underestimates the damping ratio.

4. Conclusions

In this paper a novel semi-automated identification method for the Structural Health Monitoring is proposed. It is very simple to use and it is based on filtering techniques and on the modal decomposition of the matrix containing the analytical signals of the correlation functions. Thanks to the use of pseudoinverse matrix in the modal decomposition, it allows to perform the dynamic identification also in the case in which the number of sensors is different from the number of identified modes. The proposed method has been used to identify natural frequencies, damping ratios and modal shapes of a historic building located in Palermo and the results obtained have been compared with those obtained by using EFDD and SSI that are two of the most popular OMA methods. The results obtained shows that the proposed method can identify the modal parameters with a precision similar to that of EFDD and SSI and thus it can be considered as a reliable tool to perform the structural dynamic identification.

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