A Low-Cost Smartphone Polarimeter and Malus's Law

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The use of smartphones in the introductory physics laboratory has recently received attention since it provides the possibility to perform a variety of didactic experiments. Smartphones equipped with several built-in sensors, controlled by appropriate software, allow students to explore physical phenomena and carry out various measurements of physical quantities. In this article, after a brief historical introduction about the discovery of light polarization and the development of the polariscope/polarimeter, we describe a low-cost and easily constructed smartphone polarimeter that provides a quantitative way of experimenting with light polarization. In particular, we discuss a didactic activity concerning the use of the smartphone ambient-light sensor to measure the intensity of light coming through two properly oriented polarizing filters that follows the well known Malus's law.

A historical perspective of light polarization

In Icelandic medieval legends, to detect the direction of the Sun on cloudy or foggy days during navigation, Vikings used the sunstone (sólarsteinn), probably the Iceland spar (calcite, crystallized calcium carbonate), for its light-polarizing property. The polarization of sunlight in the Arctic can indeed be detected through clouds; the direction of the Sun can be identified by moving the sunstone across the visual field to reveal a yellow pattern [1]. However, scientific studies of the refraction in transparent crystals of Iceland spar date back to 1669, when the Danish physician, mathematician, and physicist Erasmus Bartholinus (Rasmus Bartholin, 1625 - 1698) reported about the discovery of the double refraction by these crystals [2]. Bartholinus observed that images seen by eyes through the crystal were doubled; furthermore, when the crystal was rotated, one image remained stationary while the other rotated according to the crystal rotation. Perceiving that light passing through the crystal was split into two different rays, he called the stationary image the *ordinary beam* (*solita*) and the moving image the *extraordinary beam* (*insolita*). This property is called *birefringence* or *double refraction.* Although he was not yet aware, the history of light polarization began with this discovery.

In 1808, the French physicist Étienne-Louis Malus (1775 - 1812), while observing the rays of the Sun reflected by the windows at Luxembourg Palace in Paris through a crystal of Iceland spar, discovered that the light reflected by a glass surface could be extinguished when viewed through the crystal. In particular, he discovered that this intrinsic property did not necessarily require a crystal to manifest but could also be produced by reflection at a proper angle from any transparent body or a polished surface [3]. Since the intensity of the reflected light varied from a maximum to a minimum as the crystal was rotated, Malus proposed that the amplitude, $A(\theta)$, of the reflected beam varies as $A(\theta) = A_0 \cos(\theta)$, where θ is the angle of rotation of the crystal axis from the position for which the intensity of transmitted light is a maximum, I_{max}. To obtain the intensity, $I(\theta)$, of the reflected polarized light, Malus squared the amplitude, obtaining $I(\theta) = I_{max} \cos^2(\theta)$, which is known as *Malus's law*.

Malus assumed that light consisted of incandescent corpuscular dipoles which could be iso-oriented upon refraction by birefringent crystals or upon reflection at particular angles. He related the orientation of the mirrors and the ray of light to the poles of the compass and for this he introduced the term *polarization* [3]. He then proposed that natural light consisted of the s-polarization (perpendicular to the plane of incidence) and p-polarization (parallel to the plane of incidence), which are perpendicular to each other and perpendicular to the direction of propagation of light.

In 1815, the Scottish physicist Sir David Brewster (1781 - 1868) empirically determined that the polarizing angle is $\theta_B = \arctan(n_2/n_1)$, where n_1 is the refractive index of the initial medium, through which the light propagates, and n_2 the refractive index of the other medium [4], which is known as *Brewster's law*, and the angle θ_B defined by it is called *Brewster's angle*. The reflected ray is completely polarized if the angle that the ray makes with the plane of the mirror is θ_B (for a wave passing from air to glass $\theta_B \approx 57\,^\circ$).

The French physicists Dominique-François Arago (1786 - 1853) and Augustin-Jean Fresnel (1788 - 1827) established the experimental fact that oppositely polarized beams did not interfere with one another and did not produce fringes, as expected by two beams of ordinary light in interference experiments. With the discovery of polarization, the presumed analogy between the propagation of light and sound waves fell away, since longitudinal sound waves cannot be polarized.

In 1861 and 1862, the Scottish physicist James Clerk Maxwell (1831 - 1879) published a theory that was the first theory to describe electricity, magnetism and light as different manifestations of the same phenomenon. Maxwell's theory established that light is an electromagnetic wave, identified the electric field as the polarization vector, and provided a unified framework for the analysis of polarization phenomena.

Soon after the discovery of polarization by Malus, new instruments were designed to investigate these new phenomena [5]. By arranging for a beam of sunlight reflected by a first glass plate, called *polarizer*, at the polarizing angle to impinge on a second polarizing glass plate, called *analyzer*, that can be rotated to vary the angle between the second and first plane of incidence, Malus devised the first polarizer-analyzer reflection polariscope. This instrument helped in convincing Malus of the vector nature of light vibrations and led him to his well-known cosine-squared law of transmitted light intensity.

Using the ideas suggested by Malus, in 1816 the French physicist Jean-Baptiste Biot (1774 - 1862) described the construction of the reflection polariscope [6,7], shown in Fig. 1. Further improvements that have been made are described in the Appendix.

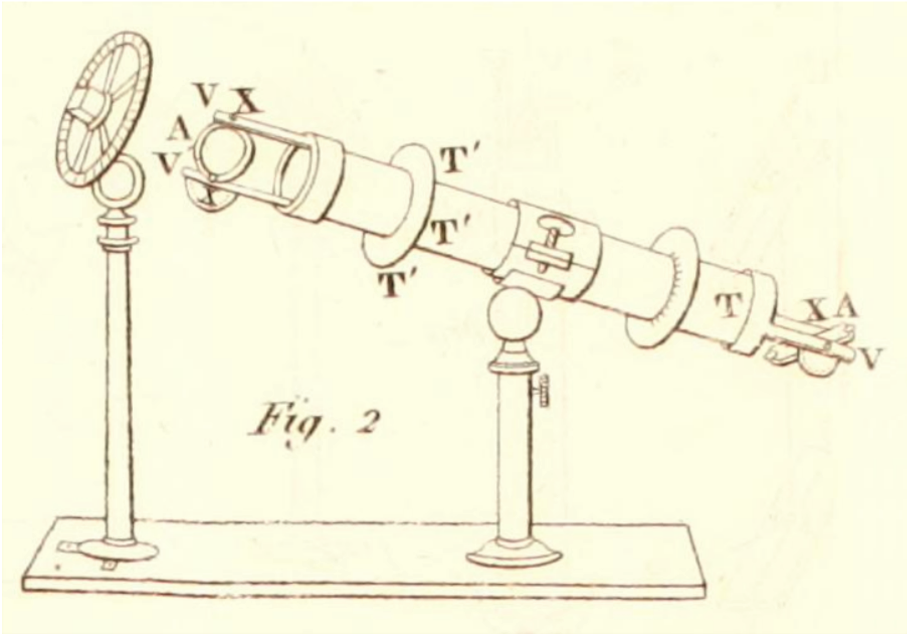

Figure 1: Biot's polariscope. Image adapted from Ref. [6].

Experimental setup and results

Experiments aimed at verifying Malus's law are traditionally carried out by using two consecutive linear-polarizing filters working as polarizer and analyzer, respectively. In these experiments, unpolarized white light is generated by a lamp and it is analyzed, after passing through the two filters, by a detector, such as the high-sensitivity light sensor provided by PASCO Scientific or that provided by Vernier, connected to a computer. Experimental data are thus recorded and visualized by appropriate software. Recently, Malus's law has successfully and conveniently been investigated by using smartphones equipped with the ambient-light sensor [8].

Here we describe an experimental setup we have developed for the investigation of the Malus's law based on low-cost off-the-shelf polarizers and a smartphone (Fig. 2). Two pieces of linear polarizing filters, acting respectively as polarizer and analyzer, are placed in two plastic mounts, of about 2.5 cm diameter, obtained by properly cutting out the neck and the cap of two plastic bottles (one for each filter). A paper protractor (a polar coordinate paper should work as well) is glued on a plastic support fixed on a home-made cardboard box (approximate dimension of ≈ 20 cm $\times\, 10$ cm $\times\, 5$ cm), in which a hole, large enough to permit easy turning of the filter mount, has been made to hold the analyzer (inset of Fig. 2). A second hole in the cardboard box allows one to read the smartphone display.

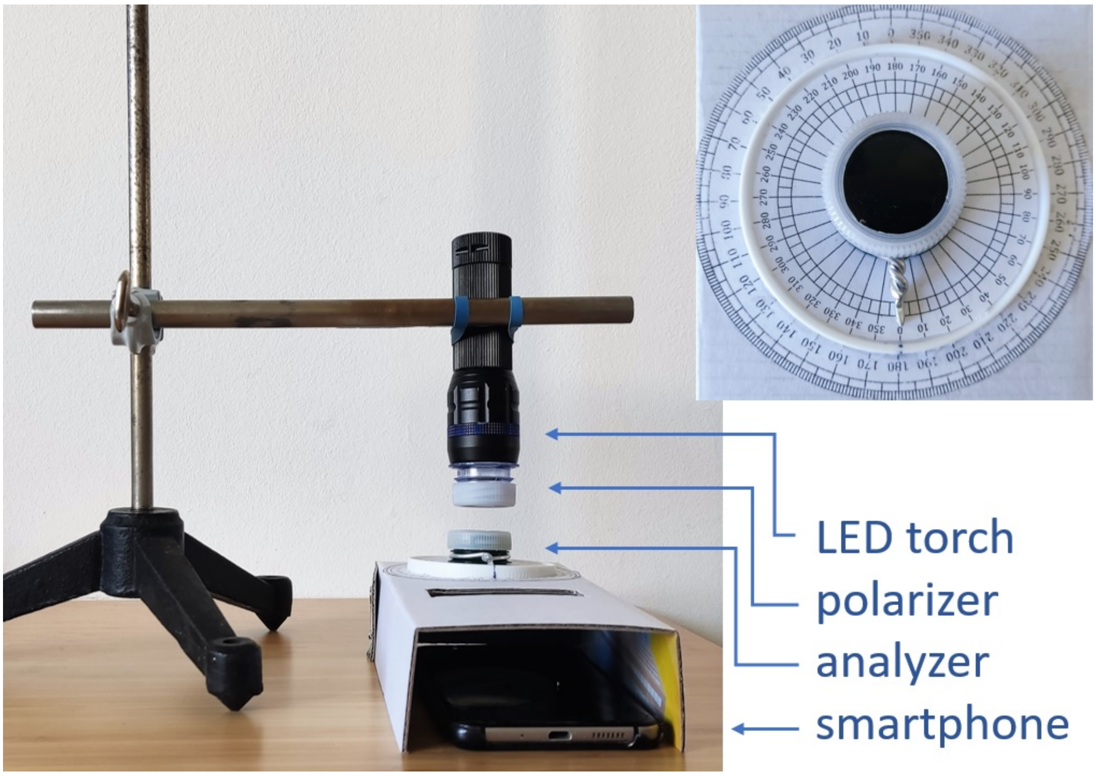

Figure 2: Experimental setup. The inset shows the analyzer top view with pointer and protractor.

A portable single-LED flashligh is used as an unpolarized-white-light source. The polarizer is directly mounted onto the lamp, which is fixed to a horizontal shaft. A pointer is tied to the analyzer mount for convenient reading of the orientation of the analyzer with respect to the polarizer axis. The smartphone is placed beneath the cardboard box, which serves also to reduce the ambient illumination. The built-in ambient-light sensor of the smartphone is used to detect and measure the light intensity transmitted through the two polarizing filters. The sensor is controlled by the free app phyphox (www.phyphox.com), which allows one also to remotely control, and observe real-time experimental data, from any different network-connected computer [9]. The sensor measures the illuminance [10], which is proportional to the light intensity, with a resolution of about 1 lx. Readings show three significative digits, corresponding to an uncertainty of about 1% of the measured value.

On changing the relative angle, θ, between the two linear-polarizing-filter axes, we measure the intensity of the light, $I(\theta)$, passing through the two filters vs. the angle θ. It is worth noting that for uniformly unpolarized light the intensity after passing through a polarizing filter is reduced by a factor of 2, since it is the average intensity, from $\theta = 0$ to $\theta = 2\pi$, of the components along the polarizing axis.

Fig. 3 shows the values of the measured illuminance, $I(\theta)$, using as polarizer and analyzer two slightly-curved-lens pieces of a Polaroid sunglass [11]. Measurements have been performed by keeping fixed the polarizer at an arbitrary position and rotating the analyzer by steps of 5 degrees, with an uncertainty of about 2 degrees. The experimental results of $I(\theta)$ reported in Fig. 3 clearly show a good agreement with Malus's law. The slightly different intensity at $\theta = 0$ from that at $\theta = 180\,^\circ$ might be due to an imperfect planar alignment of the polarizer and analyzer. Furthermore, possible causes of error come from the ambient light, as indicated by the illuminance of about 10 lx observed at $\theta = 90\,^\circ$.

As a comparison, we performed the same experiment by using, for polarizer and analyzer, a commercial linear polarizing filter Screen-Tech type ST-38-20 (www.screen-tech.eu). This filter is a 0.2 mm thick film having relative light intensity transmission of $42\% \pm 2\%$ (the relative light intensity transmission through two parallel filters is $36\% \pm 2\%$, whereas through two cross-oriented filters is $< 0.01\%$) and 99.98% degree of polarization with a spectral range 380 – 780 nm. The experimental results of $I(\theta)$ reported in Fig. 4 show a very good agreement with Malus's law.

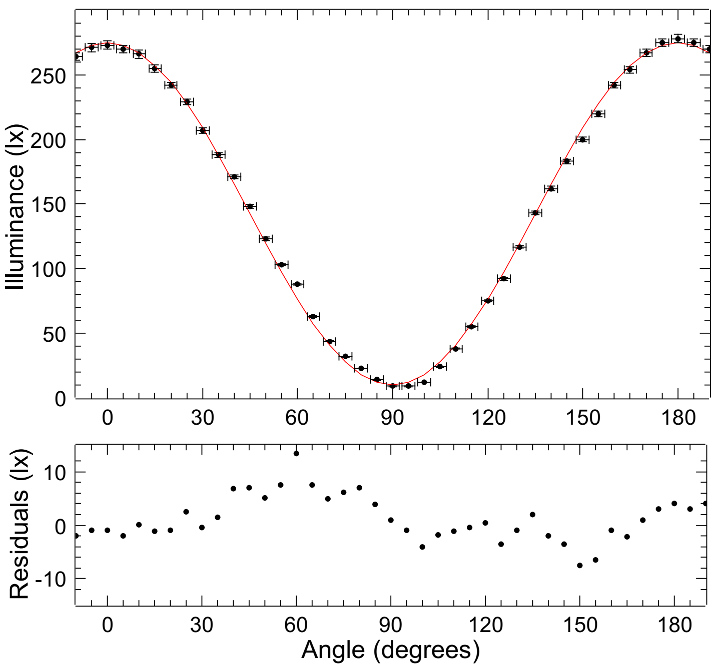

Figure 3: (top) Illuminance detected by using Polaroid sunglass. Symbols are experimental values; the continuous line is a plot of $I(\theta) = I_{amb} + (I_{max} - I_{amb})\cos^2(\theta)$, with $I_{max} = 275$ lx and $I_{amb} \approx 10$ lx. Lamp illuminance is $I_{lamp} \approx 23000$ lx. Illuminance detected after one filter $I_1 \approx 2540$ lx ($\approx 11\%$). Illuminance detected after two parallel filters $I_2 \approx 270$ lx ($\approx 1.2\%$). (bottom) Plot of residuals.

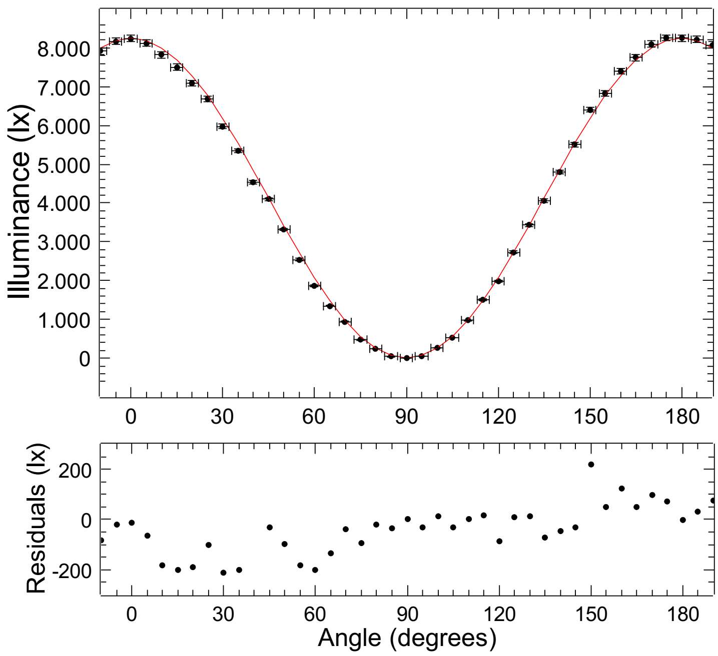

Figure 4: (top) Illuminance detected by using commercial linear-polarizing filter. Symbols are experimental values; the continuous line is a plot of $I(\theta) = I_{amb} + (I_{max} - I_{amb})\cos^2(\theta)$, with $I_{max} = 8250$ lx and $I_{amb} \approx 10$ lx. Lamp illuminance is $I_{lamp} \approx 23000$ lx. Illuminance detected after one filter $I_1 \approx 9900$ lx ($\approx 43\%$). Illuminance detected after two parallel filters $I_2 \approx 8250$ lx ($\approx 36\%$). (bottom) Plot of residuals.

Discussion and conclusion

Light polarization is a difficult topic in classroom teaching. It can be explained by the wave nature of light as the property of electromagnetic waves that describes the orientation of electric and magnetic field oscillations. Addressing these topics with the aid of experiments helps students to better understand these phenomena. To help students in understanding the polarization properties of light, many demonstration experiments have been proposed [7, 8, 12]. Malus's law experiment is helpful to demonstrate the transverse nature of electromagnetic waves by establishing the link between optics and electromagnetism.

The use of the smartphone in physics experiments provides a precious educational resource, contributing to students' learning of science concepts and stimulating their interest in physics [13–15]. On the other hand, the use of smartphones as measurement devices opens the possibility of arranging low-cost setups. Furthermore, the students' interest in smartphones may foster students' active learning in the classroom [16].

In conclusion, we have discussed an educational activity aimed at the understanding of the working principle of the polariscope/polarimeter and how it was used, also from a historical perspective, to explain the phenomenon of light polarization. We have further described a quantitative way of experimenting with light polarization and Malus's law with a low-cost and easily constructed smartphone polarimeter, by using the smartphone ambient-light sensor to measure the intensity of light coming through two properly oriented polarizing filters. The experimental results of the intensity of light, obtained in two different types of polarizing filters, polaroid lenses and commercial polarizing film, are accounted for very well by Malus's law. Furthermore, they allow students to compare and discuss optical properties of different materials. The proposed simple experiments on light polarization can be carried out in the classroom to introduce students to the study of optics. Simple models of these instruments can also be easily built by students, actively engaging them in the discussion of light polarization phenomena.

Acknowledgments

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References

[1] D. Száz and G. Horváth, "Success of sky-polarimetric Viking navigation: revealing the chance Viking sailors could reach Greenland from Norway," *R. Soc. open sci.* **5**, 172187 (2018), doi: 10.1098/rsos.172187.

[2] Erasmi Bartholini, *Experimenta Crystalli Islandici Disdiaclastici Qvibus Mira & Insolita Refractio Detegitur* (Danielis Paulli, Copenhagen 1670), books.google.it/books?id=B7tdAAAAcAAJ [English translation: *Experiments with the double refracting Iceland crystal which led to the discovery of a marvelous and strange refraction*, translated by Werner Brandt (Westtown, PA, 1959)].

[3] E. L. Malus, "Sur une propriété de la lumière réfléchie," *Mém. Phys. Chim. Soc. d'Arcueil* **2**, 145–158 (1809); *Théorie de la double réfraction de la lumière dans les sustances cristallisèes* (Baudouin, Imprimeur de l'Institut de France, Paris, 1810), books.google.it/books?id=Wb3EHev07w0C.

[4] D. Brewster, "On the laws which regulate the polarisation of light by reflexion from transparent bodies," *Philos. Trans. Royal Soc.*, 125–159 (1815), doi: 10.1098/rstl.1815.0010.

[5] F. J. Cheshire, "The early history of the polariscope and the polarizing microscope," *J. R. Microsc. Soc.* **282**, 1–18 (1923).

[6] J. B. Biot, *Traité de Physique Expérimentale et Mathématique* (Tom. iv, Chez Deterville, Paris, 1816), archive.org/details/traitdephysiquee04biot.

[7] W. A. Hilton, "Biot's Polariscope," *Phys. Teach.* **11**, 312–313 (1973), doi: 10.1119/1.2350019.

[8] M. Monteiro, C. Stari, C. Cabeza, and A. C. Martí, "The polarization of light and Malus's law using smartphones," *Phys. Teach.* **55**, 264–266 (2017).

[9] S. Staacks, S. Hütz, H. Heinke, and C. Stampfer, "Advanced tools for smartphone-based experiments: phyphox," *Phys. Educ.* **53**, 045009–6, doi: 10.1088/1361-6552/aac05e.

[10] Illuminance is the total luminous flux incident on a surface, per unit area, which indicates of how much the incident light illuminates the surface, wavelength-weighted by the luminosity function to correlate with human brightness perception.

[11] A polaroid is a type of synthetic plastic sheet polarizer devised in 1928 by the American physicist Edwin Herbert Land (1909 - 1991) and patented in 1929. See: Harvard Business School, Boston (MA), Baker Library Historical Collections, *The invention of the polarizer*, www.library.hbs.edu/hc/polaroid/emergence-of-a-new-technology/invention-of-the-polarizer/.

[12] A. F. Leung, "The law of Malus using Polaroid polarizers," *Phys. Teach.* **18**, 612–615 (1980), doi: 10.1119/1.2340629.

[13] A. Agliolo Gallitto and L. Lupo, "A mechanical model of the smartphone's accelerometer," *Phys. Educ.* **50**, 646–647 (2015), doi: 10.1088/0031-9120/50/6/646.

[14] M. A. Gonzalez, A. Gonzalez Manuel, M. E. Martin, C. Llamas, O. Martinez, J. Vegas, M. Herguedas, and C. Hernandez, "Teaching and Learning Physics with Smartphones," *J. Cases Inf. Technol.* **17**, 31–50 (2015), doi: 10.4018/JCIT.2015010103.

[15] K. Hochberg, J. Kuhn, and A. Müller, "Using Smartphones as Experimental Tools - Effects on Interest, Curiosity, and Learning in Physics Education," *J. Sci. Educ. Technol.* **27**, 385–403 (2018), doi: 10.1007/s10956-018-9731-7.

[16] D. Lindquist, T.Denning, M. Kelly, R. Malani, W. G. Griswold, and B. Simon, "Exploring the potential of mobile phones for active learning in the classroom," *ACM SIGCSE Bulletin* **39**, 384–388 (2007).

Appendix: Tourmaline crystal and Nicol's prism

In 1814, the French physicist Jean-Baptiste Biot (1774 - 1862) discovered that birefringent crystals of tourmaline (a complex borosilicate mineral) have the remarkable property of absorbing the ordinary ray and transmitting the extraordinary ray, consequently a thin section of a crystal, cut parallel to the optic axis, transmits only polarized light, and may be used either as a polarizer or analyzer [A1]. Fig. A1 shows a pair of tourmalines from the Historical Collection of Physics Instruments of Palermo University [A2]. When superposed with their axes parallel, they transmit light quite freely, conversely when one is rotated through a right angle, the superposed parts becomes opaque, since the polarized light transmitted by one is absorbed by the other, as shown in Fig. A1 (left). This led to the construction of the tourmaline-tong polariscope, which was described in 1826 by the German chemist and physicist Carl Michael Marx (1794 - 1864) [A3]. It consists of a pair of crystal sections, cut parallel to the optic axis, mounted in a pair of wire tongs in such a way that one of them can be rotated in front of the other [A4,A5]. The object to be examined is placed between the two tourmaline crystals and observed directly by naked eye [A6]. However, the deep color of the tourmaline crystals prohibited its use for many purposes so many attempts were made to replace it by an optical element able to transmit a beam of polarized white light.

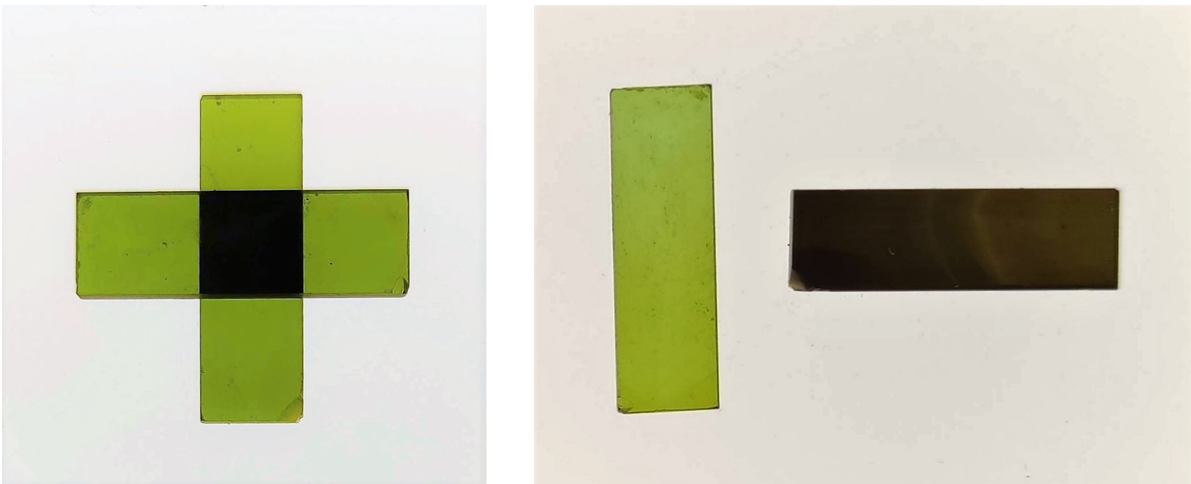

Figure A1: A pair of tourmalines, from the Historical Collection of Physics Instruments of Palermo University [A2], observed in unpolarized (left) and polarized light (right).

In 1828, the Scottish geologist William Nicol (1770 - 1851) devised a particular prism, known as *Nicol's prism*, that deflects (and dumps) one of the two beams produced by double refraction using total internal reflection at the separation layer of Canada balsam between two calcite prism halves [A7, A8]. Since the refractive index for Canada balsam

($n_b = 1.54 - 1.55$) is in between the refractive index for the ordinary ($n_o = 1.658$) and extraordinary ($n_e = 1.486$) rays, the ordinary ray will be totally internally reflected as illustrated in Fig. A2 (right).

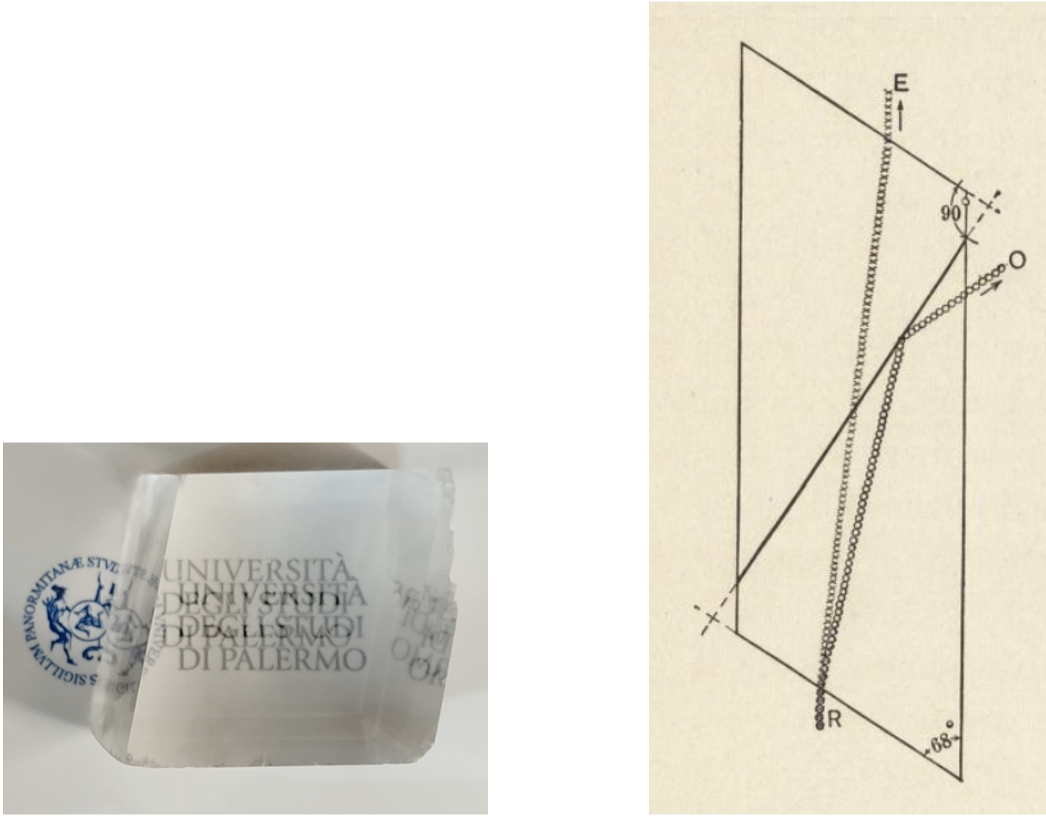

Figure A2: (left) The double image seen through a calcite crystal (about 45 mm wide and thick) from the Historical Collection of Physics Instruments of Palermo University [A2]. (right) Scheme of the Nicol's prism. A ray of unpolarized light R entering the prism is resolved into two component rays vibrating at right angles to each other. The ordinary ray O is deflected slightly more than the other and strikes the balsam cement at such an angle as to be totally internally reflected; the extraordinary ray E passes through the prism and emerges completely polarized. Image adapted from Ref. [A8].

The use of Nicol's prisms as polarizer and analyzer simplified the construction of single-beam polariscopes and polarimeters [A9, A10]. The first instrument, consisting merely of a stand on which were mounted two Nicol's prisms and a graduated circle, with the analyzing Nicol's prism rotating with the circle, was produced by Biot in 1840 [A11]. Further improvements have been made to allow for a more accurate reading of the extinction angle of the light, however we will not address these topics as they go beyond the scope of the present article. A detailed description of the historical development of polariscopes and polarimeters can be found in Refs. [A9, A10].

References

[A1] J. B. Biot, "Sur un mode particulier de polarisation qui s'observe dans la Tourmaline," *Ann. Chim.* **94**, 191–199 (1815), books.google.it/books?id=tZU5AAAAcAAJ, reprinted in *Journal des Mines*, 221 (1815), pags. 387–388, www.annales.org/archives/annales/1815-1/207-208.pdf.

[A2] Historical Collection of Physics Instruments of Palermo University, sites.google.com/site/aurelioagliologallitto/collezione-storica.

[A3] C. M. Marx, "Beiträge zur mineralogischen Optik," *Ann. Phys.* **8**, 243–252 (1826), babel.hathitrust.org/cgi/pt?id=umn.31951d00316716r.

[A4] R. W. Wood, *Physical Optics* (III Ed. The Macmillan Company, New York, 1935), archive.org/details/ost-physics-physicaloptics031166mbp.

[A5] P. Brenni, *Polarimeter and Polariscope*, in R. Bud and D. J. Warner, Eds., *Instruments of Science: An Historical Encyclopedia* (Garland: New York, 1998) pags. 475–477.

[A6] Fondazione Scienza e Tecnica, Firenze, *Nörremberg polariscope and tourmaline tongs* (P. Brenni and A. Giatti Eds.), youtu.be/3lzyfvnJ4yk.

[A7] W. Nicol, "On a Method of so far increasing the Divergency of the two Rays in Calcarcous-spar, that only one Image may be seen at a time," *Edinburgh New Philos. J.*, 83–84 (1829), www.biodiversitylibrary.org/item/20135.

[A8] E. M. Chamot, *Elementary Chemical Microscopy* (II Ed., John Wiley & Sons, Inc. New York, 1921), pags. 50–55, archive.org/details/gri_33125010596100.

[A9] E. R. Lyle and G. G. Lyle, "A brief history of polarimetry," *J. Chem. Educ.* **41(6)**, 308–313 (1964), doi: 110.1021/ed041p308.

[A10] W. B. Jensen, *Polarimeters* (Oesper Museum Booklets on the History of Chemical Apparatus No. 1, University of Cincinnati, 2014), drc.libraries.uc.edu/handle/2374.UC/733811.

[A11] G. W. Rolfe, *The polariscope in the chemical laboratory. An introduction to polarimetry and related methods* (Macmillan & Co., London, 1905), archive.org/details/polariscopechemlab00rolfrich.

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