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## Magnetostochastic resonance under colored noise condition

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6 Stochastic resonance (SR) is an amplification of the system output in correspondence of  
7 well-defined finite values of the noise strength that is injected into the system [Gammaitoni *et al.*,  
8 *Rev. Mod. Phys.* **70**, 223 (1998), Grigorenko *et al.*, *IEEE Trans. Magn.* **31**, 2491 (1995),  
9 Mantegna *et al.*, *J. Appl. Phys.* **97**, 10E519 (2005)]. In order to clarify the influence of a colored  
10 noise, in this paper magnetostochastic resonance (MSR) in magnetic systems described by the  
11 dynamic Preisach model is numerically investigated in the presence of colored noise. It is shown  
12 that noise spectrum affects MSR, white noise,  $1/f$  and  $1/f^2$  noise induce in magnetic systems  
13 described by the dynamic Preisach model MSR, the maximum level of signal-to-noise ratio  
14 (SNR) obtained by using white noise but  $1/f$  noise presented a range where SNR value is higher  
15 than the case of white noise; maximum signal amplification is obtained for white noise. © 2012  
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### 16 INTRODUCTION

17 Stochastic resonance (SR) is a well-known phenomenon  
18 characterized by an amplification of the system response for  
19 certain finite values of the noise strength injected into the  
20 system.<sup>1,2</sup> In particular, the signal-to-noise ratio (SNR) and  
21 the signal amplification show nonmonotonic behaviors with  
22 a maximum as a function of the noise intensity. SR has been  
23 experimentally observed<sup>3</sup> in many physical systems and also  
24 in magnetic systems.<sup>4</sup> Some theoretical approaches have  
25 been developed to describe SR (for a theory of SR in mag-  
26 netic systems see Ref. 4 and for a review see Ref. 3) for  
27 bistable systems, but no theoretical approach has been devel-  
28 oped to describe SR in systems that present a magnetic-like  
29 hysteresis area (i.e., an entire area of accessible states, which  
30 is surrounded by a major loop). This effect is usually named  
31 magnetic stochastic resonance (MSR). MSR has been  
32 numerically described using both the classical Preisach  
33 model (CPM) and dynamic Preisach model (DPM).<sup>5,6</sup> In  
34 these investigations real noise has been numerically simu-  
35 lated. Real noise has a frequency spectrum that depends on  
36 various factors, however in theoretical analysis some stand-  
37 arized models are used. The typical models are named  
38 white and colored noise: white noise is a random process  
39 with a flat power spectrum density and colored noise is a pro-  
40 cess with a power spectrum density that has a frequency de-  
41 pendence. In all the above-recalled approaches, MSR has  
42 been investigated in white noise condition. SR in the pres-  
43 ence of colored noise has been investigated only in bistable  
44 systems and no attempt to include colored noise in magnetic  
45 systems has yet been done.

46 In order to clarify the influence of the type of noise in  
47 magnetic systems, in this paper MSR in magnetic systems  
48 described by DPM is numerically investigated under colored

noise condition. The colored noise models used assume a  $1/f$  49  
and  $1/f^2$  dependency on frequency. The use of DPM allows 50  
one to study the features of the SR in connection with the 51  
dynamic features of the magnetic systems and various types 52  
of noise.<sup>7,8,10</sup> 53

### THE DYNAMIC PREISACH MODEL 54

DPM was introduced to grasp dynamic characteristics of 55  
magnetic materials. A complete description of the model can 56  
be found in Ref. 9. In the following, only the details impor- 57  
tant for the comprehension of this paper will be outlined. 58

In DPM, the magnetization  $M(t)$  at the generic time  $t$  is 59  
given by 60

$$M(t) = M_s \int_0^\infty dh_c \int_{-\infty}^\infty p(h_c, h_u) \cdot \varphi(h_c, h_u, t) dh_u, \quad (1)$$

where  $M_s$  is the saturation magnetization,  $p(h_c, h_u)$  is the Prei- 61  
sach model density function, and  $\varphi(h_c, h_u, t)$  describes the 62  
state of each elementary Preisach model loop at the time  $t$ . 63  
 $\varphi(h_c, h_u, t)$  varies according to 64

$$\frac{\partial \varphi(h_u, h_c, t)}{\partial t} = \begin{cases} k[H(t) - (h_u + h_c)] & \text{if } H(t) < (h_u + h_c) \\ k[H(t) - (h_u - h_c)] & \text{if } H(t) < (h_u - h_c) \end{cases}, \quad (2)$$

where  $k$  is an unknown parameter. The dynamic model 65  
becomes equivalent to CPM if the parameter  $k$  becomes infi- 66  
nite, because, in this case, the function  $\varphi(h_c, h_u, t)$  can assume 67  
only the values  $-1$  and  $+1$ . The parameter  $k$  quantifies the 68  
finite rate of the switching of the hysterons of DPM. 69

### THE NUMERICAL APPROACH 70

In this paper, the external magnetic field ( $h_{ext}$ ) applied to 71  
a magnetic material has two components, one small sinuiso- 72  
dal component added to a colored noise component: 73

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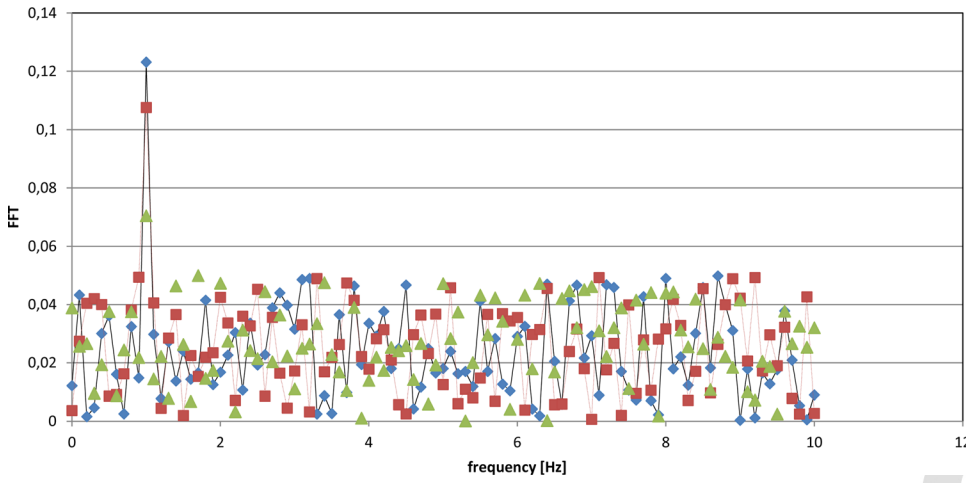


FIG. 1. (Color online) FFT of the output of the system. Diamonds represent white noise, squares  $1/f$  noise, and triangles  $1/f^2$  noise.

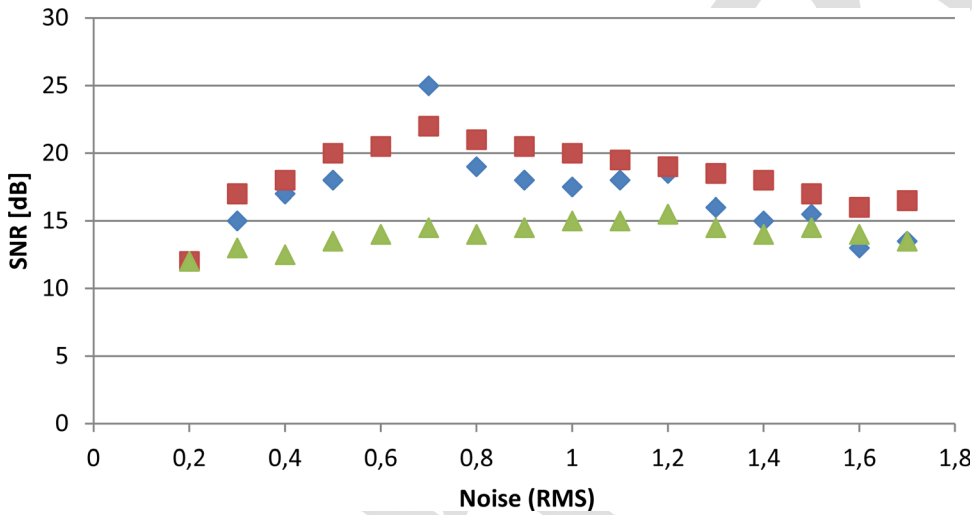


FIG. 2. (Color online) SNR vs noise intensity. Diamonds represent white noise, squares  $1/f$  noise, and triangles  $1/f^2$  noise. The maximum SNR is reached in the case of white noise but  $1/f$  noise presents a broader maximum, and an area where SNR values are higher than white noise case.

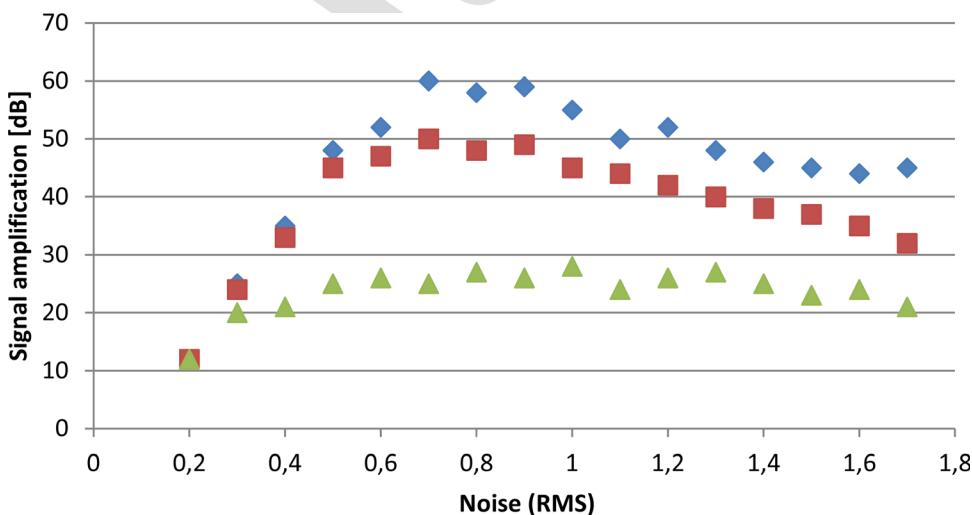


FIG. 3. (Color online) Signal amplification vs noise intensity. Diamonds represent white noise, squares  $1/f$  noise, and triangles  $1/f^2$  noise. The maximum signal amplification is reached in the case of white noise.

$$h_{\text{ext}} = H_s \sin t + D(t), \quad (3)$$

74 where  $t$  is the time and  $D$  is the colored noise.  $D$  was generated  
75 by a suitable generator in which the type of noise and its root  
76 mean square was controllable. The frequency of the sinusoidal  
77 component was kept constant at the value of 1 in all the nu-  
78 merical simulations here presented and the dynamic features  
79 of the system were changed by letting  $k$  vary in the DPM and  
80 the correlation time in the noise generator. The value of  $h_{\text{ext}}$   
81 was computed at several time steps. As a result, the time evo-  
82 lution of the magnetization of the system could be computed  
83 by inserting Eq. (3) in DPM [Eq. (1) and (2)]. A Lorentzian  
84 Preisach distribution function was used in Eq. (1). Its expres-  
85 sion is given in Ref. 5. The two parameters  $\sigma_c$  and  $H_0$ , which  
86 define the Lorentzian, were set equal to 0.1 and 1 respectively.  
87 This distribution generates a major loop of the static hysteresis  
88 that has a coercive field equal to 1 (see Ref. 5).

89 The magnetization was computed by discretizing the in-  
90 tegral in Eq. (1) on a suitable grid. The grid on the Preisach  
91 plane is rectangular with  $0 \leq h_c \leq 4$  and  $-3 \leq h_u \leq 3$  and it is  
92 made by at maximum  $1000 \times 1000$  points and the set of dif-  
93 ferential equations in Eq. (2) were solved by standard numer-  
94 ical techniques.

95 To compute the SNR and the power amplification, the  
96 fast Fourier transforms (FFT) of the magnetization were  
97 computed and the value of the component of the FFT for the  
98 frequency of the signal was used.

99 The SNR was calculated as

$$\text{SNR} = 10 \log_{10} \left( \frac{P_1}{N_1} \right) \quad (4)$$

100 and the power amplification as

$$\eta = 2 \left( \frac{|M_1|}{M_s} \right)^2, \quad (5)$$

101 where  $P_1$  is the output signal power level obtained from the  
102 FFT of the resulting magnetization at the frequency of the si-  
103 nusoidal component,  $N_1$  is the noise level obtained from the  
104 same FFT at the frequency of the sinusoidal component,  $M_1$   
105 is the component of the FFT at the frequency of the sinusoi-  
106 dal component, and  $M_s$  is the amplitude of the magnetization  
107 obtained with no noise pumped in the system.

108 The SNR, the power amplification and the behavior of  
109 the magnetization for several  $H_s$  and  $D$  and in correspon-  
110 dence of white, 1/f and  $1/f^2$  as a function of the parameter  $k$   
111 have been computed.

112 In Fig. 1 the FFT of the time varying magnetization for  
113 an amplitude of  $H_s = 0.5$  in the case of presence of noise  
114 with a value of  $H_{\text{rms}} = 0.8$  and for  $k = 1000$  is shown for the  
115 three types of noise. The amplitude of the harmonic of the  
116 FFT of the time varying magnetization at the frequency of  
117 the applied signal for a signal amplitude of  $H_s = 0.5$  in the  
118 case of absence of noise is much smaller ( $1/1000$ )<sup>5</sup> than the

amplitude of the same harmonic when an external noise is 119  
applied. That means that the addition of noise amplifies the 120  
harmonic value at the frequency of the signal. This, together 121  
with the nonmonotonic behavior of both SNR and  $\eta$ , is the 122  
fingerprint of SR. Figure 1 shows the amplification of the 123  
harmonic value at the frequency of the signal and how white 124  
noise guarantees the maximum signal amplification; 1/f 125  
noise amplification is 20% less than white noise and  $1/f^2$  126  
amplification is 40% less. 127

In Fig. 2 SNR is shown as a function of  $H_{\text{rms}}$  for the 128  
three types of noise at  $H_s = 0.5$ . 129

The maximum SNR is obtained for white noise, but 1/f 130  
noise seems to have a broader range where it is larger than 131  
white noise. This is due to the fact that in 1/f noise the noise 132  
reduction plays a role in the SNR by enhancing its value for 133  
a broad range, this tendency is confirmed in the  $1/f^2$  case 134  
where noise reduction plays a role in a broader range than 135  
the 1/f case. In the  $1/f^2$  case the maximum in SNR is reached 136  
at a noise rms (root mean square) value higher than the other 137  
cases (a noise RMS equal to 1.2). In Fig. 3  $\eta$ [dB] is shown as 138  
function of  $H_{\text{rms}}$  for the three types of noise at  $H_s = 0.5$ . Also 139  
in this case, white noise guarantees a higher signal amplifica- 140  
tion. However, in this case there is no range where 1/f noise 141  
presents a higher level of amplification. This is due to the 142  
fact that in signal amplification the level of noise is not 143  
included in the calculation. 144

## CONCLUSIONS 145

In this paper, magnetostochastic resonance in the 146  
presence of colored noise has been investigated. It has been 147  
shown that: 148

- (1) Noise spectrum affects MSR. 149
- (2) White noise, 1/f and  $1/f^2$  noise induce in magnetic sys- 152  
tems described by the dynamic Preisach model MSR. 153
- (3) Maximum level of SNR has been obtained by using 154  
white noise but 1/f noise presents a range where SNR 156  
value is higher than the case of white noise. 157
- (4) Maximum signal amplification is obtained for white 158  
noise. 160

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