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A Dynamic Model of Ballasted Rail Track with Bituminous Sub-Ballast Layer

Di Mino G.<sup>a,\*</sup>, Di Liberto M.<sup>a</sup>, Maggiore C.<sup>b</sup>, Noto S.<sup>a</sup>

<sup>a</sup> University of Palermo, Piazza Marina 61, Palermo 90133, Italy

<sup>b</sup> University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom

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**Abstract**

The bituminous sub-ballast layer within the ballasted rail track allows some mechanical and environmental advantages. An analytical model of a ballasted rail track with sub-ballast layer has been proposed by considering the rail as viscous-elastic continuous beam discretely supported, at four levels of elasticity.

The model was used to compare the mechanical performance of both ballasted track with and without bituminous sub-ballast layer. The results confirmed that the bituminous sub-ballast layer reduces the dynamic forces on the ground and achieves the technical objectives as reduction of ground borne vibrations and increase of the design life of the rail track.

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*Keywords:* Bituminous sub-ballast layer; ballasted rail track; rail track model; vehicle-track interaction.

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\* Corresponding author. Tel.: +39-091-23899722; fax: +39-091-6657749.

E-mail address: [gaetano.dimino@unipa.it](mailto:gaetano.dimino@unipa.it)

## 1. Introduction

It is widely known that placing a bituminous sub-ballast layer under a ballast layer improves the mechanical behaviour of rail track; some advantages such as the reduction of the ballast tension stresses and thus the reduction of fatigue phenomena are usually observed in this case ([1], [2], [3]). These benefits imply: less deterioration of the rail track and sleepers can lay on a more stable and less degraded ballast layer. For those reasons, many countries usually choose to place a bituminous sub-ballast layer under the ballast.

The first experimental works were carried out at the end of 1960 in USA [4]; however, the use of bituminous sub-ballast layers, instead of crushed stone sub-ballast layers, developed in 1980; nowadays it is well spread due to less demand for maintenance of the rail track [5], [6].

In Japan, since 1970, the use of bituminous sub-ballast in the rail tracks is widespread for the construction of both normal and high speed rail traffic [7].

In Italy, for more than three decades, the high speed rail track has been composed of a layer thickness of 12 cm [8]. Some considerations relating to the environmental aspects have been taken into account. Experimental surveys on high speed rail track, undertaken by the Italian national railway (R.F.I. - Rete Ferroviaria Italiana), has shown that a bituminous sub-ballast layer reduces the ground borne vibrations [9].

In Germany, since 1970, bituminous ballast track beds have been used in order to reduce maintenance costs and preserve environmental resources. The main German asphalt track consists of concrete ties placed on a 26÷30 cm thick asphalt layer.

In France, the high speed rail (extended for over 1800 Km with double rail track) is characterised by the classic cross section consist of 30 cm thick ballast resting on 20 cm of granular sub-ballast; the ballast and granular sub-ballast rest on a 50 cm thick layer of limestone aggregate. The France National Railway (SNCF) is carrying out several tests on the TGV-East line connecting Paris to Strasbourg, to determine if asphalt sub-ballast should be a considered as an alternative material for use in future high speed rail infrastructure projects.

In the French experimental survey, the 50 cm limestone layer is substituted for a bituminous sub-ballast layer of 14 cm thick; in this way it is possible to obtain a reduction of the total thickness of the railway (36 cm). Thus a reduction in material of 5000 m<sup>3</sup> per Kilometre can be achieved [10].

In Spain, many researchers have started several experimental tests in order to better understand the behaviour of bituminous sub-ballast layers instead of crushed stone sub-ballast layers [11], [12].

Therefore, the international body of evidence shows that a bituminous sub-ballast layer could be a good option to improve railways in order to cope with future bigger traffic volume, heavier loads and higher speed velocity rail, preserving the cost of maintenance and management of the permanent way.

In the last decade, the behaviour of the modern rail tracks has been analysed by several researcher with different mathematical models. The first models were based on the mathematical hypothesis of a beam on an elastic support [13], [14], [15], [16], [17], [18].

A more detailed analysis, involving the interaction between wheel and rail track, was carried out by K[19]. In many models, the rail track is modelled as a Bernoulli beam, also known as Timoshenko beam theory, with a non dissipative boundary feedback; the rail track support, continuous or discontinuous, can have different elasticity levels depending on the level of analysis of the model; some researchers have shown that a discontinuous support better describes the real behaviour of the rail track under a wide range of frequencies [20].

In the simplest mathematical models, the sleepers support is modelled with only one elasticity level by means of vertical springs-dash-pots system; the sleeper is modelled as a rigid concentrated mass.

More developed models, for the analysis of the interaction wheel-rail track, consider a sleeper support with concentrated masses and two elasticity levels. In this way it is possible to better analyse the different mechanical behaviour of the ballast and the subgrade [21], [22], [23], [24]; interconnection between different adjacent ballast blocks by means of spring-damper elements takes into account the behaviour of shear stresses in the ballast.

Sun and Dhanasekar (2002) [25] suggested a model characterised by three elasticity levels able to analyse also granular sub-ballast layers; in this case, the shear stress was also included by means of using vertical spring-dashpots elements between adjacent ballast blocks.

Recently, Lei and Rose (2008) [26] have suggested a model characterised by four elasticity levels able to simulate the flexural behaviour of the bituminous sub-ballast by means of a Bernoulli beam. However, the model is not able to simulate the rail track when a load is applied to every sleeper.

In this paper, a 2D model for rail track with four elasticity levels was developed. It is able to model both the shear stress behaviour of the ballast layer and the flexural behaviour of the bituminous sub-ballast layer. A continuous Bernoulli beam with dissipative boundary feedback was used to model the rail track and the sub-ballast layer.

This model evaluates the possible use of asphaltic material with recycled rubber (dry process) in order to construct sub-ballast layers for rail tracks. A comparison of the mechanical behaviour and deformation between two different permanent ways is made for both traditional bituminous material and bituminous material with 1.5% of rubber content. The results are compared with a railway made of a granular compacted sub-base under the ballast layer.

## 2. Lumped mass model for ballasted track with asphalt subballast

Figure 1 shows a 2D lumped masses model of the rail track system with four levels of elasticity, developed for this work.

The rail, modelled as a viscoelastic continuous beam, is considered connected, with periodicity equal to the distance between the sleepers, to a series of spring-damping elements ( $K_p$ ,  $C_p$ ) which reproduce the dynamic behaviour of the fastening system.

These elements represent the first level of elasticity and are connected to the concentrated masses  $M_s$  representing the vertical inertia of the sleepers. The inertial behaviour of the ballast portions beneath each sleeper is modelled with the masses  $M_b$ ; consequently the second level of elasticity is obtained by interjecting a series of spring-damping elements ( $K_b$ ,  $C_b$ ) arranged vertically between the concentrated masses  $M_s$  and  $M_b$ .

Also, each mass  $M_b$  is connected elastically ( $K_w$ ,  $C_w$ ) to the next in order to simulate the shear behaviour of the ballast.

The bending behaviour of the asphaltic sub-ballast is modelled by placing a viscoelastic continuous beam which is connected to the upper levels by means of spring-damping elements ( $K_{sb}$ ,  $C_{sb}$ ) which take in account the vertical displacement of the layer.

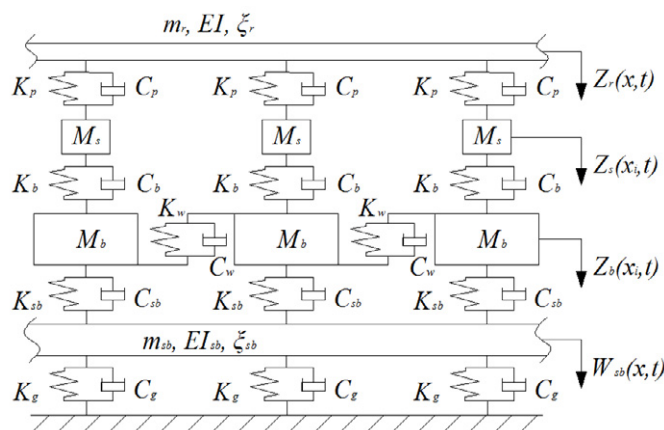
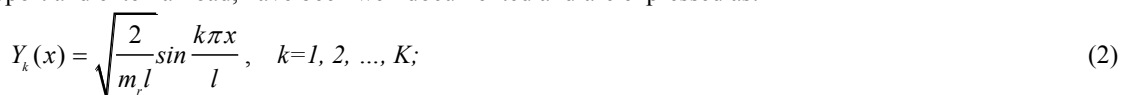


Fig. 1. Lumped masses model of the rail track with asphalt sub-ballast

$x_{Gj}$  is the abscissa of the barycentre of the  $j^{\text{th}}$  wheel.



$$\omega_k = \left( \frac{k\pi}{l} \right)^2 \sqrt{\frac{EI}{m_r}}, \quad k=1, 2, \dots, K; \quad (3)$$

where  $l$  is the beam length, which corresponds to the length of the track model, and  $K$  is the number of models considered.

In the modal approach the rail deflection  $Z_r(x, t)$  can be obtained as superposition of  $K$  rail deflection modes  $Y_k(x)$  whose weight are given by the functions  $q_k(t)$ :

$$Z_r(x, t) = \sum_{k=1}^K Y_k(x) q_k(t). \quad (4)$$

Substituting Equation 2 into the equilibrium equation of the rail (see Equation 1) and considering that the rail-sleeper force  $F_{rsi}$  can be determined by means of the expression

$$F_{rsi}(t) = K_{pi}[Z_r(x_i, t) - Z_{si}(t)] + C_{pi}[\dot{Z}_r(x_i, t) - \dot{Z}_{si}(t)], \quad (5)$$

Equation 1 can be expressed as follows:

$$\begin{aligned} \ddot{q}_k(t) + \sum_{i=1}^N C_{pi} Y_k(x_i) \sum_{k=1}^K Y_k(x_i) \dot{q}_k(t) + 2\xi\omega_k \dot{q}_k(t) + \frac{EI}{m_r} \left( \frac{k\pi}{l} \right)^4 q_k(t) + \sum_{i=1}^N K_{pi} Y_k(x_i) \sum_{k=1}^K Y_k(x_i) q_k(t) + \\ - \sum_{i=1}^N C_{pi} Y_k(x_i) \dot{Z}_{si}(t) - \sum_{k=1}^K K_{pi} Y_k(x_i) q_k(t) Z_{si}(t) = \sum_{j=1}^4 P_j(t) Y_k(x_{Gj}), \quad k = 1, 2, \dots, K. \end{aligned} \quad (6)$$

where  $Z_{si}(t)$  and  $\dot{Z}_{si}(t)$  are, respectively, the vertical displacement and the vertical velocity of the  $i^{\text{th}}$  sleeper, and  $\xi$  is the damping ratio of the rail. The factor  $2\xi\omega_k$  can be expressed as follows:  $2\xi\omega_k = \int_0^1 c_r Y_k(x) Y_k(x) dx$ .

### 3.2. The sleepers and ballast blocks

The motion equations of both sleepers and sub-ballast blocks are obtained from the dynamic equilibrium. Thus, for the  $i^{\text{th}}$  sleeper:

$$\begin{aligned} M_s \ddot{Z}_{s,i} + (C_p + C_b) \dot{Z}_{s,i} - C_b \dot{Z}_{b,i} - C_r \sum_{k=1}^K Y_k(x_i) \dot{q}_k(t) + \\ + (K_p + K_b) Z_{s,i} - K_b Z_{b,i} - K_r \sum_{k=1}^K Y_k(x_i) q_k(t) = M_s g \end{aligned} \quad (7)$$

for the  $i^{\text{th}}$  ballast blocks:

$$\begin{aligned} M_b \ddot{Z}_{bi} - C_b \dot{Z}_{s,i} - C_w \dot{Z}_{b,i-1} + (C_p + 2C_w + C_b) \dot{Z}_{bi} - C_w \dot{Z}_{b,i+1} - C_s \dot{W}_{sb}(x_i) + \\ - K_b Z_{s,i} - K_w Z_{b,i-1} + (K_p + 2K_w + K_b) Z_{bi} - K_w Z_{b,i+1} - K_s W_{sb}(x_i) = M_b g \end{aligned} \quad (8)$$

### 3.3. The sub-ballast layer

The asphaltic sub-ballast layer was modelled as a Bernoulli viscoelastic continuous beam characterised by the following equilibrium equation:

$$\begin{aligned} EI_{sb} \frac{\partial^4 W_{sb}(x, t)}{\partial x^4} + m_{sb} \frac{\partial^2 W_{sb}(x, t)}{\partial t^2} + \sum_{i=1}^N \left[ C_{sb} (\dot{W}_{sb}(x_i, t) - \dot{Z}_{b,i}) + C_g \dot{W}_{sb}(x_i, t) \right] \delta(x - x_i) + \\ + \sum_{i=1}^N \left[ K_{sb} (W_{sb}(x_i, t) - Z_{b,i}) + K_g W_{sb}(x_i, t) \right] \delta(x - x_i) = 0 \end{aligned} \quad (9)$$

Where  $EI_{sb}$  is the stiffness of the sub-ballast and  $m_{sb}$  is the mass per unit of length of the sub-ballast.

For the rail track, the modal approach was used in order to solve Equation 9; thus the deflection of the sub-ballast can be written considering the following transformation:

$$W_{sb}(x, t) = \sum_{k=1}^{\tilde{K}} \tilde{Y}_k(x) \tilde{q}_k(t), \quad (10)$$

$$\text{where } \tilde{Y}_k(x) = \sqrt{\frac{2}{m_{sb}l}} \sin \frac{k\pi x}{l} \quad k=1, 2, \dots, \tilde{K} \quad (11)$$

where  $\tilde{Y}_k(x)$  is the  $k^{th}$  deflection mode of the sub-ballast,  $\tilde{q}_k(t)$  is the  $k^{th}$  modal coordinate and  $\tilde{K}$  is the complex number of modes considered in the approach.

Substituting the Equation 11 in the Equation 9, the dynamic equilibrium of the sub-ballast is expressed by the following equations:

$$\begin{aligned} \ddot{\tilde{q}}_k(t) + \left[ (C_{sb} + C_g) \sum_{i=1}^N \tilde{Y}_k(x_i) \sum_{k=1}^{\tilde{K}} \tilde{Y}_k(x_i) + 2\xi_{sb,k} \tilde{\omega}_{sb} \right] \dot{\tilde{q}}_k(t) - \sum_{i=1}^N C_{sb} \tilde{Y}_k(x_i) \dot{Z}_{b,i} + \\ + \sum_{i=1}^N (K_{sb} + K_g) \tilde{Y}_k(x_i) \sum_{k=1}^{\tilde{K}} \tilde{Y}_k(x_i) \tilde{q}_k(t) - \sum_{i=1}^N K_{sb} \tilde{Y}_k(x_i) Z_{b,i} + \frac{EI_{sb}}{m_{sb}} \left( \frac{k\pi}{l} \right)^4 \tilde{q}_k(t) = 0 \end{aligned} \quad (12)$$

$k = 1, 2, \dots, \tilde{K},$

where  $\xi_{sb,k}$  is the damping ratio of the asphaltic sub-ballast layer and  $\omega_{sb,k}$  is the natural frequency of the  $k^{th}$  deflection mode described by the following:

$$\omega_{sb,k} = \left( \frac{k\pi}{l} \right)^2 \sqrt{\frac{EI_{sb}}{m_{sb}}} . \quad (13)$$

#### 4. Train vehicle-track interaction

The considered load consists of a single vehicle which was modelled with a lumped mass system with 10 degrees of freedom [27], [28].

The train-rail track interaction takes place through the “wheel-rail contact force”, which can be calculated using the Hertzian theory:

$$P_j(t) = C_H \cdot [Z_{wj}(t) - Z_r(x_{Gj}, t) - Z_0(V \cdot t)]^{3/2}, \quad j = 1, \dots, 4 \quad (14)$$

where  $C_H$  is the “Coefficient of Hertz contact”, determined using the expression suggested by Sun and Dhanasekar (2002) [25],  $Z_{wj}(t)$  is the vertical deformation of the  $j^{th}$  wheel at the instant  $t$ ,  $Z_r(x_{Gj}, t)$  is the vertical deformation that the rail has at the instant  $t$  at the point of contact with  $j^{th}$  wheel, and  $Z_0(V \cdot t)$  is the value of the vertical rail track profile at the  $j^{th}$  wheel-rail contact point.

The quantity  $\Delta Z(t) = Z_w(t) - Z_r(x_{Gj}, t) - Z_0(V \cdot t)$  represents the vertical deformation of the wheel-rail contact.

The deflection  $Z_0(x)$  of the track can be considered as a stationary ergodic Gauss random process with expectation zero [29].

Indicated by  $S_z(\Omega)$  the power spectral density of the vertical irregularity, the  $r^{th}$  sample of the process is obtained by means of the following equation [30]:

$$z_0^{(r)}(x) = \sum_{k=1}^N \sqrt{2 \times S_z(\Omega_k) \Delta \Omega} \times \sin(\Omega_k x + \varphi_k^{(r)}) \quad (15)$$

where  $\varphi_k^{(r)}$  is a random variable with a constant density of probability (equal to  $1/2\pi$ ) in the range  $[0, 2\pi]$ , and  $\Omega$  is the space angular frequency.

The quantity  $\Delta \Omega$  represents the sampling range of the  $S_z(\Omega)$ , and, indicated by  $\Omega_{sup}$  and  $\Omega_{inf}$  the upper and lower bounds of the range of  $\Omega$ , the  $\Omega_k$  is:

$$\Omega_k = \Omega_{inf} + \left(k - \frac{1}{2}\right) \Delta \Omega \quad (16)$$

with  $\Delta \Omega = (\Omega_{sup} - \Omega_{inf})/2$ .

The variables  $\varphi_k^{(r)}$  are independent of each other and they were obtained using the Monte Carlo method.

Equation 16 allows arising numerically the space trend of the vertical irregularities, once known the relative PSD. In order to evaluate the vertical deflection of the track, a spectrum from other publications such as [27] and [31] can be used.

The global equilibrium equation of the train-rail track system is obtained by assembling the dynamic equations of the train and the rail track into a single relationship as:

$$\underline{\underline{M}} \ddot{\underline{Z}} + \underline{\underline{C}} \dot{\underline{Z}} + \underline{\underline{K}} \underline{Z} = \underline{F}(t, Z_w, Z_r, Z_0), \quad (15)$$

where  $\underline{Z}$ ,  $\dot{\underline{Z}}$  and  $\ddot{\underline{Z}}$  are the generalised vectors of displacement, speed and acceleration,  $\underline{F}$  is the vector of the external forces, and  $\underline{\underline{M}}$ ,  $\underline{\underline{K}}$ ,  $\underline{\underline{C}}$  are mass, stiffness and damping matrices of the vehicle-rail track system.

The response of the vehicle-rail track system is obtained by integrating Equation 15.

To take into account the unilateral constraint of the wheel-rail contact, a control on the  $\Delta Z_j(z)$  was made:  $P_j(t)=0$  when  $\Delta Z_j(t)<0$  (detachment condition) at the generic instant  $t$ .

## 5. Numerical Analysis

The developed model was used to analyse the behaviour of a dry asphalt rubber (DAR) mixture for a sub-ballast layer in a rail track.

The numerical analysis was developed comparing the mechanical behaviour of the schematic rail track shown in Figure 3-a (B category) in two different cases: in the first one, the asphalt sub-ballast layer (12 cm) is made of a traditional asphalt mixture according to the Italian standards RFI (named HMA-RFI in the experimental work); in the latter one, the sub-ballast layer is made of dry asphalt rubber mixture characterised by 1.5% of rubber content obtained by means of the substitution of limestone aggregates with crumbed rubber in volumetric proportion (named DAR1.5 in the experimental work). The recycled crumb rubber comes from the waste tires of trucks.

The final results have been compared also to a rail track characterised by an over compacted layer (30 cm), named OCL in the experimental work, under the ballast layer (see Figure 3-b).

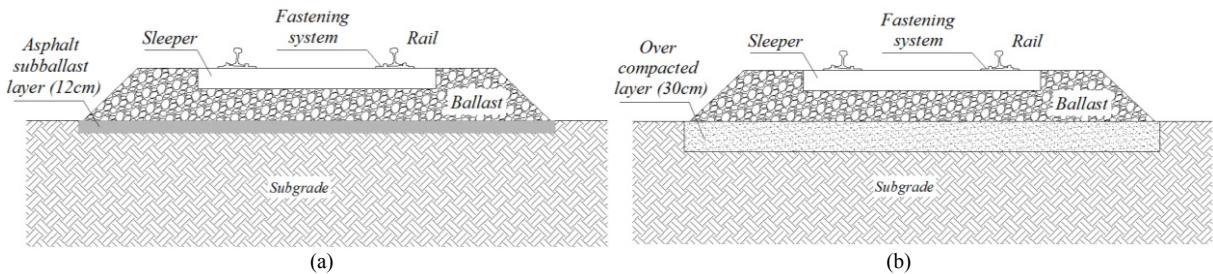


Fig. 3. Typical Italian ballasted rail track (B category): (a) with asphalt sub-ballast; (b) with over compacted layer

In table 3 the main volumetric parameters of the two asphalt materials are shown.

Table 1. Volumetric parameters of the asphalt sub-ballast

Mix identification	Binder content (%)	Rubber content (%)	Air voids percent (%)	Specific Gravity ( $\text{kg/m}^3$ )
HMA-RFI	4,5	0	4,4	2534
DAR1.5	5,5	1,5	6,6	2375



Several four point bending tests on prismatic specimens [32] were carried out in order to evaluate the mechanical behaviour and the damping properties of the two bituminous mixtures [33].

The tests were undertaken at the temperature of 10°C, 17°C e 25°C, which are typical values of the average seasonal temperature in the South of Italy [34]. The loading frequencies chosen were 0.1 Hz, 0.5 Hz, 1 Hz, 2 Hz, 5 Hz, 10 Hz, 15 Hz and 20 Hz.

Figure 4 shows isothermal curves of the stiffness modulus (see Figure 4-a) and of the damping ratio (see Figure 4-b) obtained considering a strain level of 30  $\mu\text{m/m}$ .

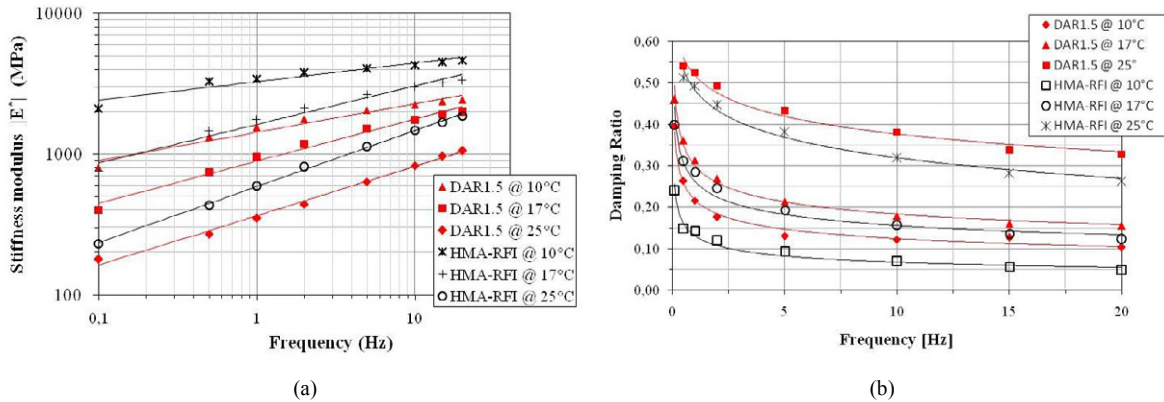


Fig. 4. Isothermal curves of the stiffness modulus (a) and of the damping ratio (b) @  $\varepsilon_o = 30 \mu\text{m/m}$

A soft sub-grade with a Young's modulus  $E_{sg}$  equal to 20 MPa in order to evaluate the mechanical behaviour of both rail tracks under restrictive conditions.

The analysis of the ballasted rail track with an over compacted layer (see Figure 3b) was developed with a lumped mass model (see Figure 5) because the over compacted layer is not characterised by a bending behaviour. In fact, the third level of elasticity is represented by discrete masses ( $M_{oc}$ ), springs ( $K_{oc}$ ) and dampers ( $C_{oc}$ ).

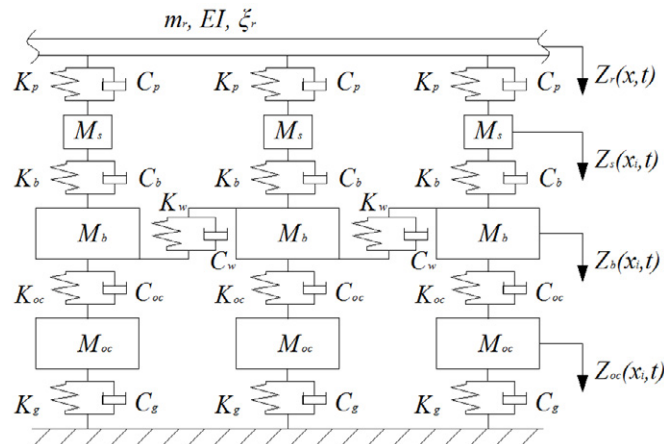


Fig. 5. Lumped mass model for ballasted track with super-compacted layer

## 6. Modeling data

In this paper, a rail UNI60 and concrete sleepers (distance between the sleepers equal to 65 cm) was considered.

The modelling data is reported in Table 2. The values of masses, springs and dampers were calculated based on the experimental results showed in Figure 4 [25].

In the case of the over compacted layer, the same values were obtained considering a dynamic modulus equal to 5000 MPa.

The stiffness of the subgrade was calculated considering an elastic modulus equal to 20 MPa [26].

Table 2 Track parameters

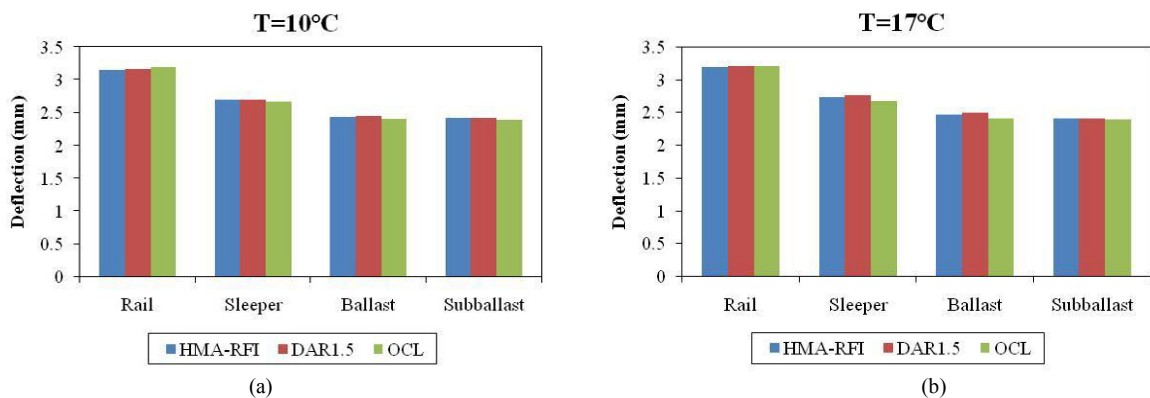
Track components	Mass per unit length (kg/m)	Damping ratio	Mass (kg)	Stiffness (MN/m)	Damping (kN s/m)
Rail	$m_r$ : 60	$\zeta_r$ : 0,02	-	-	-
Sleeper	-	-	$M_s$ : 260	-	-
Fastenings system	-	-	-	$K_p$ : 60	$C_p$ : 77,76
Ballast	-	-	$M_b$ : 270	$K_b$ : 158,11 $K_w$ : 78,40	$C_b$ : 41,30 $C_w$ : 80,00
Asphalt Sub-ballast	$m_{sb}$ : variable	$\zeta_{sb}$ : variable	-	$K_{sb}$ : variable	$C_{sb}$ : variable
Over-compacted layer	-	-	$M_{oc}$ : 2300	$K_{oc}$ : 2066,63	$C_{oc}$ : 21,830
Subgrade	-	-	-	$K_g$ : 78,40	$C_g$ : 80,00

An ALn 668 train in single configuration characterised by a speed of 90 km/h was used [28]. The load frequency in the sub-ballast, according to the data (in particular the speed), is equal around to 2 Hz.

The vertical irregularities of the track were calculated using the PSD proposed by the America Railway Standard for class 6 lines [27].

## 7. Results

Maximum deflections of every element of the structure during the transit of the train are shown in Figure 6. It can be noticed that HMA-RFI and DAR1.5 show similar deflection values for the chosen temperatures (10°C, 17°C and 25°C). Also, smaller values of stiffness in dry asphalt material do not increase the deflections values of the rail track due to the more viscous behaviour of the asphalt when it contains rubber.



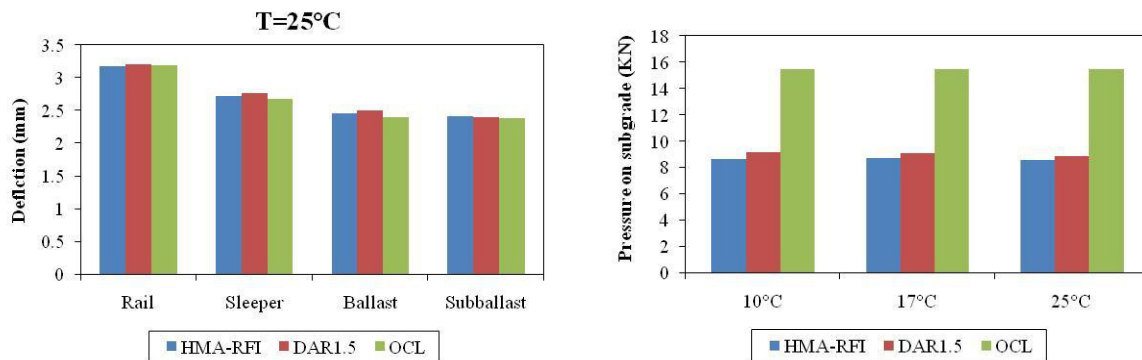


Fig. 6. Deflection of the track at 10°C (a), 17°C (b) and 25°C (c); (d) Dynamic force on the top of subgrade

Maximum dynamic pressures (i.e. forces on sleeper spacing) are shown in Figure 6d. These are the first vibration mode and thus they are very important in the control of vibrations in the environment due to the rail track. As said before, the deflection values are similar in both cases but maximum dynamic pressures are 40% less in the case of sub-ballast with asphalt material. That means a major capacity of asphaltic sub-ballast in spreading the loads in the sub-grade.

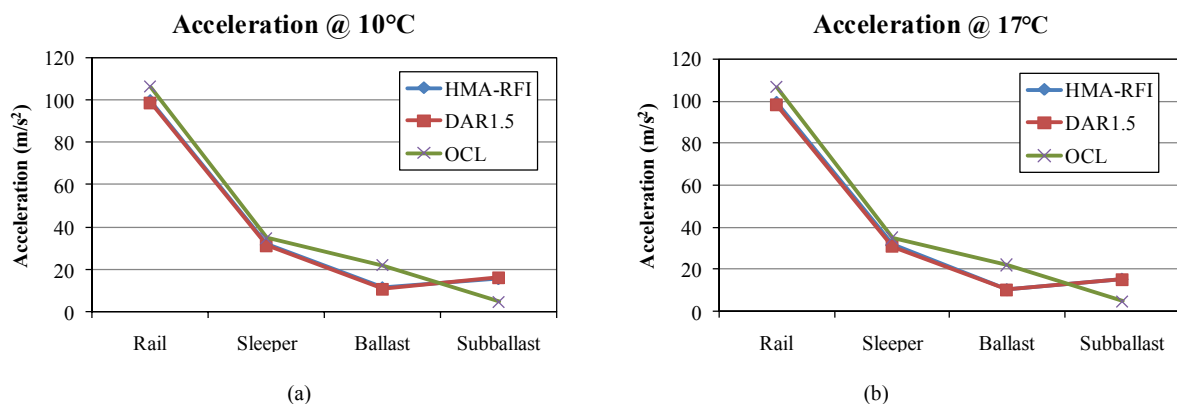
The inclusion of rubber in the asphalt material leads to a slightly bigger dynamic force on the top of sub-grade compared to the case of HMA-RFI.

Vertical acceleration peak values are reported in Figure 7. They are obtained for every kind of rail track considered in this paper.

Both rail tracks with asphalt material are characterised by similar vertical accelerations. The rail track with over compacted layer shows acceleration values slightly bigger.

Finally, constructing a sub-ballast of asphalt material involves a reduction of the acceleration peak values, leading to a reduction of tensions and thus a reduction of fatigue phenomena which characterises ballast during its life.

This implies a reduction of damage and of ground borne vibrations in rail track can be achieved.



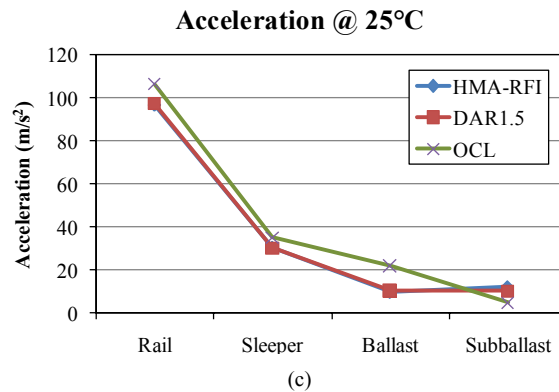


Fig. 7. Acceleration of the track at 10°C (a), 17°C (b) and 25°C (c)

## 8. Conclusion

The suggested model seems to be suitable to assess the real mechanical behavior of the rail track by means of Bernoulli viscoelastic continuous beam. The results show previous trends both of peak vertical deflection and peak vertical acceleration which confirm the performances of the rail track in terms of reduced stresses on both ballast layer and sub-grade due to the insertion of a sub-ballast compared with a ballasted rail track with an over compacted layer.

Finally, notwithstanding DAR was not precisely designed for tracks bed purposes, within the analysed frequency range, it highlights a better results because of its viscous nature than ones obtained from sub-ballast with traditional asphalt concrete.

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