

MECHANICS

‘Naughty cylinder’ mechanical paradox

To illustrate the laws of mechanics and at the same time engage the attention of students beginning their physics studies, scientific toys [1] and mechanical paradoxes [2, 3] can be utilized. These devices allow astonishing effects to be shown that, in the case of paradoxes, contradict common sense. However, close observation of the experiments reveals that the phenomena are perfectly consistent with the laws of physics, overcoming the initial sense of amazement. Today, research into scientific toys and paradoxes continues for didactic [1, 2] and historical [3] purposes, as well as for technological applications [4].

Here we investigate the properties of a ballast cylinder, known as the ‘naughty cylinder’. When this particular cylinder (or disc) is placed on an inclined plane, it stays in an equilibrium position. Because of this apparently anomalous behaviour, the device is described as a ‘mechanical paradox’. Actually, the cylinder conceals a fixed metal ballast that shifts the centre of mass (CM) of the system away from the cylinder’s axis. This mass thus creates a mechanical torque that allows the cylinder to move uphill on an inclined plane and come to a stop in a well-determined equilibrium position, in which all the external torques acting on the cylinder cancel.

To calculate the maximum angle of the inclined plane, for which the cylinder can remain in static equilibrium, let us consider a cylinder of radius R_c and density ρ_c having a cylindrical hole of radius R_h with its axis centred at a distance d from the

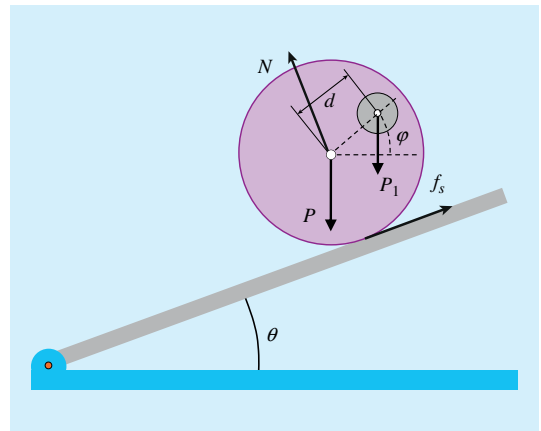


Figure 1. *The ballast cylinder on an inclined plane.*

cylinder’s axis, as shown in figure 1. The hole is filled with lead of density ρ_h . We indicate as P the weight of the full cylinder (without the hole), $P_1 = g(\rho_h - \rho_c)V_h$ the additional weight and f_s the friction force. We also assume that the static-friction coefficient is high enough to allow the cylinder to roll without slipping, therefore the static-friction force always equilibrates the component of the gravitational force parallel to the inclined plane.

The cylinder rolls uphill on the inclined plane if the momentum of P_1 with respect to the point of contact is larger than the momentum of P with respect to the same point (see figure 1). It reaches an equilibrium position when these two momenta are equal and opposite. Considering a generic position of the cylinder on the inclined plane, it is easy

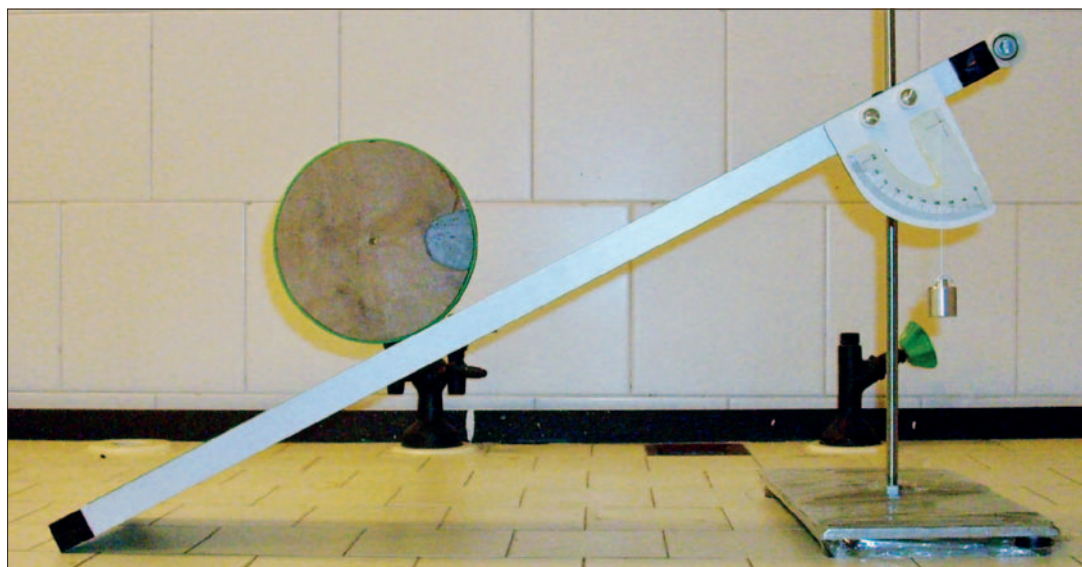


Figure 2. The ballasted wooden cylinder of the historic scientific-instrument collection displayed at the Department of Physics of the University of Palermo, Italy. The cylinder remains in equilibrium on the inclined plane at the maximum angle $\theta_{\max} = 28^\circ \pm 1^\circ$.

to show that the cylinder remains in equilibrium when $R_c \sin \theta = r_{cm} \cos \varphi$, where $r_{cm} = P_1 d / (P + P_1)$ is the radius of the CM of the system. For a fixed angle θ of the inclined plane, there are two equilibrium positions at the angles $+\varphi$ and $-\varphi$, which correspond to the position of maximum and minimum potential gravitational energy, respectively. On increasing the angle θ , we obtain the maximum angle, θ_{\max} , when $\varphi = 0$, that is when the cylinder's axis and the hole's axis lie on a horizontal line.

Figure 2 shows the wooden cylinder with a lateral lead disc as ballast, in equilibrium on the inclined plane at the maximum angle $\theta_{\max} = 28^\circ \pm 1^\circ$. The cylinder has radius $R_c = (66 \pm 1)$ mm, thickness $s = (19.4 \pm 0.2)$ mm and mass $M = (320 \pm 1)$ g. The ballast has an average radius $R_h = (17.7 \pm 0.7)$ mm, with its central axis distance $d = (48 \pm 2)$ mm from the cylinder's axis. Since the ballast is made of lead of a known density ($\rho_h = 11\,340 \text{ kg m}^{-3}$), we have calculated its weight and used it to find the density of the wood from which the cylinder is made, obtaining $\rho_c = (420 \pm 50) \text{ kg m}^{-3}$. By using these values, we obtain $\theta_{\max} = 29^\circ \pm 2^\circ$, which is consistent with experimental measurements.

On increasing the angle of the inclined plane, if the friction coefficient is relatively high the cylinder can remain in equilibrium until the maximum angle θ_{\max} , above which it starts rolling downwards

without sliding. The limit position corresponds to the condition in which the cylinder's and hole's axes lie on a horizontal line, while the cylinder's CM and the point of contact lie on the same vertical line. This finding allows the CM of the system to be determined experimentally.

Since the CM of the cylinder is r_{cm} apart from the cylinder's axis, when the cylinder is placed on the plane inclined by an angle $\theta < \theta_{\max}$, in a proper position, it can roll upwards. It stops in the position at which its potential gravitational energy has minimum value. By rolling up on the inclined plane, the position of the CM decreases. The same behaviour is observed in the well-known 'never fall doll', which has a weight in the bottom so it will never tip over—when gently knocked, it rocks back and forth and never falls.

In conclusion, we have discussed the physical properties of a ballast cylinder, known as 'the naughty cylinder'. This mechanical paradox can be exploited to increase the attention of students during lectures, as well as to build toys. Students beginning their study of mechanics are attracted to toys and paradoxes and can be helped by their teachers to explain the phenomena using their knowledge of the laws of mechanics.

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References

- [1] Güémez J, Fiolhais C and Fiolhais M 2009 Toys in physics lectures and demonstrations—a brief review *Phys. Educ.* **44** 53–64
- [2] Agliolo Gallitto A and Fiordilino E 2011 The double cone: a mechanical paradox or a geometrical constraint? *Phys. Educ.* **46** 682–4
- [3] De Luca R and Ganci S 2011 The uphill roller experiment and a variation on theme *Eur. J. Phys.* **32** 101–6
- [4] Leine R I, Van Campen D H and Glocker C H 2003 Nonlinear dynamics and modeling of various wooden toys with impact and friction *J. Vib. Control* **9** 25–78

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