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Measurement method of rainfall energetic characteristics using the Weibull drop size distribution

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ABSTRACT

Rainfall kinetic energy is a critical variable in assessing rainfall erosivity, that is the capability of rainfall to erode soil. Accurate measurements of rainfall erosivity are essential for predicting soil loss and refining soil erosion models. Currently, rainfall energy is derived from measurements of raindrop size distribution (DSD) and raindrop terminal velocity. However, traditional disdrometers, which measure DSD, face limitations related to both the produced large datasets and the high costs associated with their deployment, especially over large areas, limiting their use to experimental installations for scientific research. Recent advancements have introduced a patented method to measure rainfall energy, relying on the simultaneous detection of rainfall intensity and the number of raindrops impacting a surface within a specific time interval. This paper presents advances in the theoretical measurement method of rainfall kinetic power and momentum by using Weibull distribution and rainfall momentum distribution. The developed theoretical analysis aimed at enhancing the reliability of this innovative approach for measuring rainfall kinetic power and momentum, by the detection of the rainfall intensity, number of the raindrops and the mean raindrop diameter calculated by the raindrop momentum distribution. This analysis developed by using measured DSDs demonstrates that the proposed approach provides accurate estimates of rainfall kinetic power and momentum, but also of the shape and the scale parameters of the DSD. Furthermore, the results suggest that the momentum distribution-based estimates of rainfall momentum are more accurate than those of kinetic power, due its reduced sensitivity to uncertainties in terminal velocity calculations. Implementing direct measurements of rainfall erosivity, this method could significantly improve the accuracy of erosion models and contribute to more effective large-scale monitoring of soil erosion risk.

1. Introduction

One of the most important resources for ensuring human survival is soil. However, much of it around the world is severely degraded, mainly due to soil erosion phenomena (Alewell et al., 2019; Wuepper et al., 2020; Wang et al., 2024). Soil erosion is primarily caused by soil particle detachment, and consequent transport, by rainfall. For hydrological purposes, rainfall intensity and depth are the most relevant and commonly used variables for characterizing precipitation by its erosivity, since devices that can directly measure rainfall erosivity are not widely available. This circumstance favored the application of empirical relationships, relating rainfall intensity I (mm h^{-1}) with rainfall kinetic power P_n (J m^{-2}h^{-1}), i.e. the kinetic energy per unit time and area, useful for predicting soil loss by empirical soil erosion models (Salles et al., 2002; van Dijk et al., 2002; Wilken et al., 2018; Serio et al., 2019b; Wang et al., 2024). Salles et al. (2002), carrying out an overview of many P_n-I empirical relationships, showed that, for a fixed rainfall intensity, these relationships give very different values of kinetic power. According to Parsons and Gadian (2000) and Salles et al. (2002), a global parameter such as rainfall intensity is not sufficient to determine the rainfall erosivity since P_n measurements are also dependent on other variables, such as rain type, climate and measurement method. Using drop size distribution (DSD) measurements carried out with different techniques and in different geographical sites, Carollo et al. (2016b) demonstrated that the rainfall intensity is not sufficient to determine the rainfall kinetic power, and the most used empirical relationships between P_n and I (Salles et al., 2002) are site-specific.

Some authors (Waldvogel, 1974; Uijlenhoet and Stricker, 1999; Cevasco et al., 2015; Meshesha et al., 2019; Carollo et al., 2023) stated that the *DSD* is more suitable for characterizing the precipitation when

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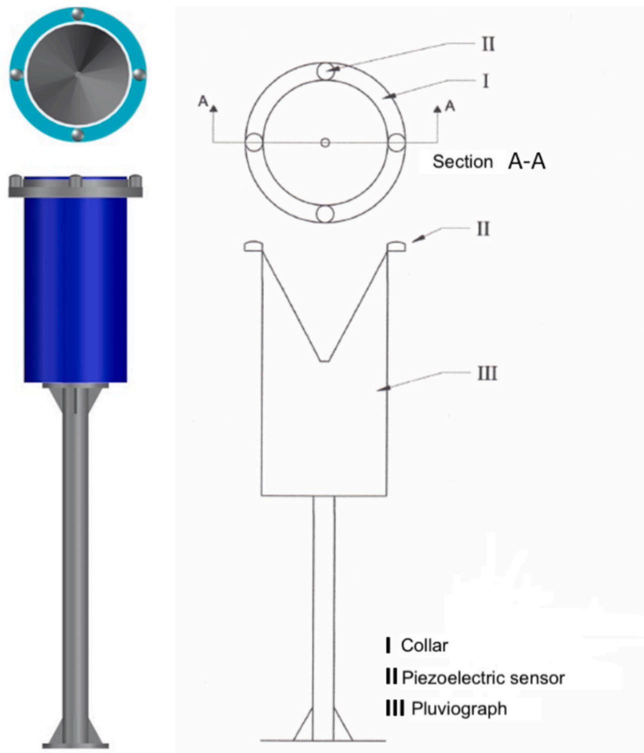

Fig. 1. Example of the patented device.

the soil erosion effects have to be studied. The knowledge of DSD enables to (i) understand how the rainfall is made up and (ii) compute, in addition to rainfall intensity and depth, the rainfall kinetic power, P_n (J $m^{-2}h^{-1}$), and momentum, M (N m^{-2}). These variables allow the energy characterization of precipitation, which is crucial for both soil sealing and soil erosion. In other words, the accuracy of the rainfall erosivity computation directly affects the accuracy of soil erosion models (Wang et al., 2024). Therefore, a direct measure or estimate of the rainfall erosivity is needed.

Over the years, several analytical forms of raindrop size parameterization have been proposed, such as lognormal (Feingold and Levin, 1986), Weibull (Sekine and Lind, 1982), exponential (Marshall and Palmer, 1948), and gamma (Ulbrich, 1983) distributions. However, to the best of our knowledge, for meteorological and soil erosion purposes, the most widely applied analytical form is the gamma distribution (Khrgian et al., 1952; Levin, 1963; Sulakvelidze, 1969; Sulakvelidze and Dadali, 1971; Ulbrich, 1983; Uijlenhoet and Stricker, 1999, Tokay et al., 2001; Salles et al., 2002; Bringi et al., 2003; Zhang et al., 2003; Carollo and Ferro, 2015; Carollo et al., 2016a, b; Seela et al., 2018; Serio et al. 2019a; Carollo et al., 2023). The Weibull distribution was proposed for the first time by the Swedish physicist Weibull (Weibull, 1951) and has been applied in many different scientific fields (like material science, engineering, physics, meteorology, medicine, economics, and quality control) over the years. It was applied only a few times for describing the raindrop size distribution.

For meteorological purposes, Sekine and Lind (1982) calculated attenuation due to rain by using the Mie scattering theory and the Weibull distribution, and proposed the Weibull distribution in the following form:

$$N(D)dD = N_0\frac{\eta}{\sigma}\left(\frac{D}{\sigma}\right)^{\eta-1} exp\left[-\left(\frac{D}{\sigma}\right)^{\eta}\right]dD \quad (1)$$

in which D is the drop diameter and $N(D)$ is the number of drops within the range (D, $D + dD$), N_0 is equal to 1000 m^{-3}, η is the dimensionless shape parameter, and σ (cm) is the scale parameter.

To date, the Weibull distribution has mainly been used and tested for meteorological purposes and rarely for hydrological ones. The representation of measured DSDs by the Weibull distribution and its application to estimate rainfall kinetic energy was presented in Mualem and Assouline (1986), Assouline and Mualem (1989), and Assouline (2009; 2020). Assouline and Mualem (1989) suggested a new approach for studying the mechanism of the forces that shape the raindrop size distribution through coalescence and breakup processes along their pathway from the cloud to the soil surface. The Authors stated that the hypothesis of uniform random raindrop fragmentation leads to considering the measured DSD identical to the Weibull distribution since the probability for a drop break up is proportional to its relative volume. Moreover, they suggested a new probability distribution function, named "*universal distribution*", which is independent of both the rainfall intensity and site where precipitation occurs (Assouline and Mualem, 1989). Assouline and Mualem (1989) and Assouline (2009) positively tested this approach using DSD measurements carried out in different experimental areas (Rhodesia by Hudson (1965); Baton Rouge and Holly Springs by Carter et al. (1974); Princeton (New Jersey) by Smith et al. (2009)).

Recently, Carollo et al. (2024a) positively tested the reliability of the Weibull distribution to reproduce DSDs measured at the El Teularet experimental site, located in Spain, using an optical disdrometer (model ODM 470 made by Eigenbrodt). Carollo et al. (2024a) also proposed a theoretical approach to determine rainfall kinetic power by the Weibull distribution. The kinetic power, P_n (J $m^{-2}h^{-1}$), is calculated by adding the contribution of each drop that composes precipitation, once their terminal velocity and DSD are known, as:

$$P_n = 10^{-6}\frac{\rho\pi}{12}\int_0^{\infty}[V(D)]^2D^3N(D)dD \quad (2)$$

in which ρ is the water density, equal to 1000 kg m^{-3}, and $V(D)$ is the raindrop terminal velocity (m s^{-1}) of the droplet having a diameter D (cm). Considering Eq. (1) and that proposed by Atlas and Ulbrich (1977) to estimate $V(D)$

$$V(D) = 17.67D^{0.67} \quad (3)$$

Eq. (2) can be rewritten as:

$$P_n = 10^{-6}\frac{\rho\pi}{12}17.67^2N_0\sigma^{4.34}\Gamma\left(\frac{4.34}{\eta}+1\right) \quad (4)$$

where Γ is the gamma function. According to Carollo et al. (2024a), N_0 parameter is a function of I and the other two Weibull parameters:

$$N_0 = \frac{I}{3.6\frac{\pi}{6}\sigma^3\Gamma\left(\frac{3}{\eta}+1\right)} \quad (5)$$

Therefore, introducing Eq. (5) into Eq. (4), one obtains:

$$P_n = 6.273510^{-5}\rho\sigma^{1.34}\frac{\Gamma\left(\frac{4.34}{\eta}+1\right)}{\Gamma\left(\frac{3}{\eta}\right)}I \quad (6)$$

According to Eq. (6), the kinetic power per unit volume of rainfall, P_n/I, is only a function of the two parameters of the Weibull distribution. Carollo et al. (2024a) verified the reliability of Eq. (6) at the El Teularet experimental site, using both the maximum likelihood and the momentum methods to estimate the Weibull parameters. In the latter method, the equality between the measured and theoretical values of both the median diameter, D_{50}, and the median volume diameter of the distribution, D_0 (that is the diameter value that divides the DSD into two parts of equal volume), was imposed.

To date, the most commonly used instruments to detect the DSD are

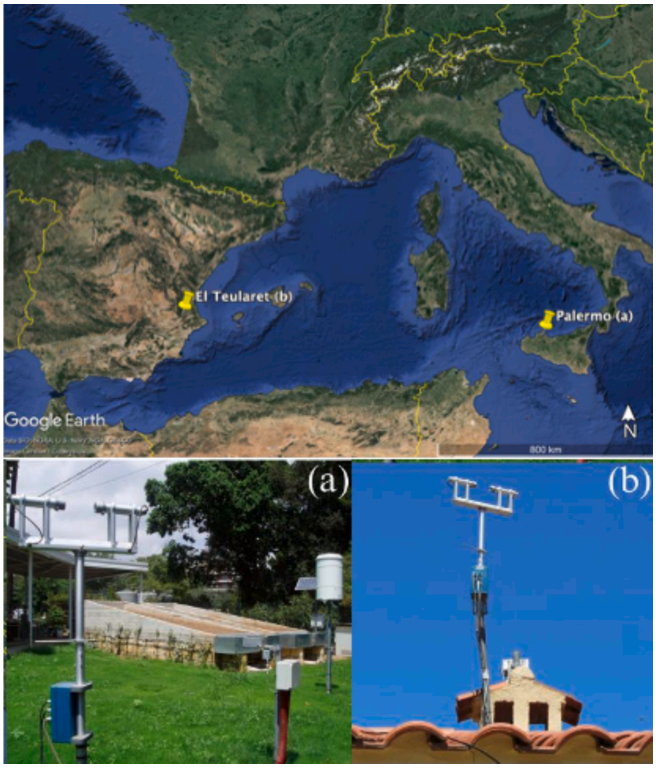

Fig. 2. View of Palermo (a) and El Teularet (b) experimental sites.

the disdrometers, which detect a high amount of numerical data that are difficult to elaborate and manage remotely. Moreover, considering the noticeable installation, management cost, and uncertainties in rainfall erosivity estimates, these instruments are not suitable for detecting rainfall energetic characteristics at a large spatial scale.

Currently, the rainfall impact energy is directly measured through instruments named impactometers (force transducers), which are often piezoelectric sensors. The rainfall impact is converted by a pressure transducer to an electric signal whose magnitude allows for determining, using a specific empirical calibration curve, the energy or momentum associated with the impact of raindrops.

Recently, an innovative device and method to measure rainfall energy, subject to a patent (https://www.uibm.gov.it/bancadati/Number_search/type_url?type=wpn, file nr. 102018000010691 of 29/10/2020) was proposed. This method is based on the simultaneous detection, in a given time interval, of the rainfall intensity, I, and the number of raindrops, N, that hits a specific surface. These variables can be measured by a simple and cheap device described in the same filed patent.

The device (Fig. 1) consists of a pluviograph, which measures I, equipped with a collar fixed to its upper part that houses four sensors that count the number, N, of pulses originating from the collisions of the raindrops on each sensor. The patented method and device have the following benefits: (i) the measurement technique is simple; (ii) the collar to be manufactured is inexpensive and can be applied to existing pluviographs; and (iii) the new device can be integrated into the pluviographs' production and marketing process.

To improve the patented device's measurement technique and the reliability of the rainfall energy measurement, Carollo et al. (2024b) proposed a new theoretical analysis, based on the hypothesis that the rainfall momentum distribution, $M(D)dD$, is the same as the distribution of the electrical signal detected by a specific (i.e., manufacturing characteristics, shape and size) piezoelectric sensor where raindrops impact. Therefore, the detection of I, N and the mean, $m(D)$, or standard deviation, $s(D)$, of the diameter deduced by the $M(D)dD$ allow for estimating the μ and Λ parameters of the Ulbrich's distribution (Ulbrich, 1983). The knowledge of both the shape (μ) and the scale (Λ) parameters permitted to reproduce the DSDs and, thus, calculate the rainfall kinetic power and momentum. Carollo et al. (2024b) tested the reliability of the proposed approach using DSD measurements carried out by an optical disdrometer placed, in different periods, at three experimental areas located in the Mediterranean Basin, Palermo (Italy), Sparacia (Italy), and El Teularet (Spain). For all the experimental sites, the measurements suggested that the knowledge of I, N, and $m(D)$ allows us not only to estimate M and P_n accurately but also to well reproduce the entire DSD (Carollo et al., 2024b). The authors stated that the construction of the prototype of the device (Fig. 1) will permit to verify the hypothesis that the rainfall momentum distribution, $M(D)dD$, is the same as the distribution of the electrical signal detected by a specific piezoelectric sensor on which the raindrops impact. This would imply that the mean of the diameters deduced from $M(D)dD$ coincides with the mean value of the impulse generated by the raindrops that will impact on the piezoelectric sensor.

In this paper, advances in the theoretical measurement method of rainfall kinetic power and momentum by using both the Weibull distribution and the rainfall momentum distribution are presented. Specifically, this investigation is aimed to present a new theoretical procedure to estimate the Weibull parameters, the kinetic power P_n, and the rainfall momentum M using the measurement of N, I, and the mean diameter deduced by rainfall momentum distribution $M(D)$.

Considering that the mean drop diameter deduced by $M(D)$ was successfully used for estimating the μ and Λ parameters of the Ulbrich's distribution, P_n and M in a previous investigation (Carollo et al., 2024b), the present analysis allows us for testing if the reliability of the proposed approach, based on the rainfall momentum distribution, can be assumed independent of the applied theoretical DSD.

2. Materials and methods

Drop size distributions were detected using the same optical disdrometer (model ODM 470 made by Eigenbrodt) placed, in different periods, at Palermo and El Teularet experimental areas (Fig. 2).

The Palermo experimental station is located at the Department of Agricultural, Food and Forest Sciences of the University of Palermo, at 40 m a.s.l. (Fig. 2a). The climate is Mediterranean temperate (Köppen's Csa type), characterized by dry and hot summer and mild and rainy winter. The average annual temperatures range between 15 °C and 22 °C, while rainfalls, mainly concentrated in autumn and winter, show mean annual value of 654 mm.

The El Teularet experimental site is located at Sierra de Enguera, 100 km southwest of Valencia, at 760 m a.s.l. (Fig. 2b). Climate is Typical Mediterranean with 3–5 months of summer drought, usually from late June to September. Mean annual rainfall at the study area range from 479 mm at the Enguera—Las Arenas meteorological station to 590 mm at the Enguera Confederación Hidrográfica del Jucar (CHJ) meteorological station. Rainfall is distributed homogenously among spring, autumn and winter, while the summer is extremely dry due to high temperatures and lack of rainfall (García-Orenes et al., 2009). Mean annual temperature ranges from 12.7 °C at the Enguera—Las Arenas meteorological station to 14.2 °C at the Enguera Confederación Hidrográfica del Jucar (CHJ) meteorological station (García-Orenes et al., 2009).

A rainfall detector (model IRSS88 made by Eigenbrodt), placed close to the disdrometer, signals rainfall occurrence (at least five drops in 90 s) and so switches on the disdrometer. After 60 s without rain, the disdrometer switches off. For each raining minute, the optical disdrometer measured drop diameters in the range of 0.05–0.6 cm. Each drop was separately measured and registered into classes of about 0.005 cm in width. The disdrometer divides the diameter range into 128 classes and gives for each raining minute the number of drops belonging to each class. Drop diameter is measured by registering light damping due to the passage of the drop in the control volume between two diodes (Fig. 3).

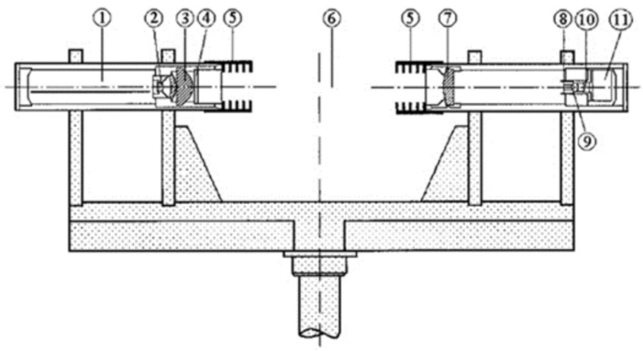

Fig. 3. Sketch of the optical disdrometer ODM470 by Eigenbrodt.

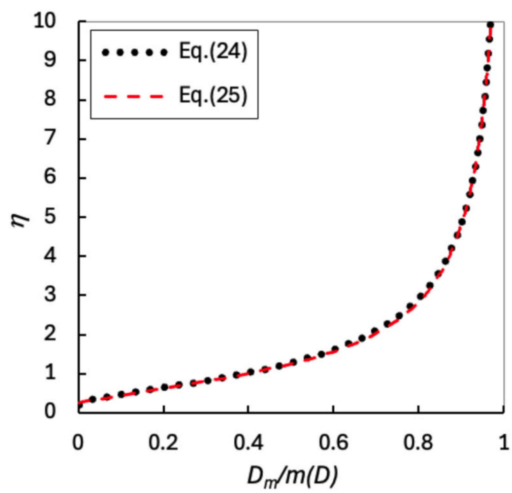

Fig. 4. Comparison between Eq. (24) and Eq. (25).

This volume has a cylindrical shape with a length of 12 cm and a diameter of 2.2 cm. Moreover, the disdrometer simultaneously measures the diameter and the crossing time of raindrops through the control volume. The number of drops in each class and the corresponding rainfall intensity *I* are detected at 1 min time intervals. Further details on the DSD measuring technique are reported in previous papers (Carollo et al., 2016a; Carollo et al., 2018b; Serio et al., 2019a, b, Carollo et al., 2024b).

The disdrometer registered 544 rainfall events (rainy days) in the period June 2006–April 2014 at Palermo experimental site, and 79 events at El Teularet experimental site in the period July 2015–May 2016. For neglecting both rainfalls having a low erosive power and DSDs having a small sample size, only the DSDs with at least 20 measured diameter classes and rainfall intensity greater than or equal to 0.5 mm h^{-1} were considered (Carollo et al., 2016a; Carollo et al., 2018a, b; Serio et al., 2019a, b). This procedure resulted in 48,439 DSDs for Palermo and 5585 DSDs for El Teularet.

The absolute error *AE* (%), the mean absolute error (*MAE*) (%), and the mean relative error (*MRE*) (%) were considered as error indices in the present investigation and calculated as follows:

$$AE = 100\left|\frac{x_{calculated} - x_{measured}}{x_{measured}}\right| \tag{7}$$

$$MAE = \frac{100}{n}\sum_{i=0}^{n}\left|\frac{x_{calculated} - x_{measured}}{x_{measured}}\right| \tag{8}$$

$$MRE = \frac{100}{n}\sum_{i=0}^{n}\left(\frac{x_{calculated} - x_{measured}}{x_{measured}}\right) \tag{9}$$

in which $x_{calculated}$ and $x_{measured}$ are the calculated and measured variable, and n is the number of experimental data.

3. Results and discussion

3.1. A theoretical approach to determine rainfall momentum by Weibull distribution

The momentum, M (N m^{-2}), is expressed by the following relationship (Carollo et al., 2018a):

$$M = 10^{-6}\frac{\rho\pi}{6}\int_0^\infty V(D)D^3N(D)dD \tag{10}$$

where the $V(D)$ (m s^{-1}) is the falling velocity of the raindrop having a diameter D (cm). Considering the Weibull distribution of Eq. (1), which is equal to the drop size distribution that reaches a unit horizontal area during a unit time, and using the relationship proposed by Atlas and Ulbrich (1977) to calculate the falling velocity, Eq. (10) can be rewritten as:

$$M = 10^{-6}\frac{\rho}{3.6}17.67\sigma^{0.67}\frac{\Gamma\left(\frac{3.67}{\eta}+1\right)}{\Gamma\left(\frac{3}{\eta}+1\right)}I \tag{11}$$

According to Eq. (11), the rainfall momentum depends on the rainfall intensity, and the shape, η, and scale, σ, parameters of the Weibull distribution.

3.2. Estimation of the Weibull distribution parameters by D_m and $m(D)$ from momentum distribution

Considering that the integral function, *F(D)*, of the Weibull distribution (Eq. (1)) is given by

$$F(D) = N_0\left(1 - exp\left[-\left(\frac{D}{\sigma}\right)^\eta\right]\right) \tag{12}$$

the total number of droplets, *N*, can be calculated as follows

$$N = \int_0^\infty N(D)dD = N_0\int_0^\infty \frac{\eta}{\sigma}\left(\frac{D}{\sigma}\right)^{\eta-1} exp\left[-\left(\frac{D}{\sigma}\right)^\eta\right]dD = N_0 \tag{13}$$

In disagreement with the results by Sekine and Lind (1982), Eq. (13) demonstrates that N_0 is not a constant parameter but is equal to the number of the raindrops that reach the unit soil surface in the unit time.

The mean volume diameter, D_m (cm), corresponding to the mean value of the raindrops' volumes, can be calculated as a function of *I* and *N*, by the following equation (Carollo et al., 2024b):

$$D_m = \sqrt[3]{\frac{I}{N\frac{\pi}{6}}} \tag{14}$$

Considering that the rainfall intensity *I* (mm h^{-1}) can be calculated from the following expression (Salles et al., 2002; Carollo and Ferro, 2015)

$$I = 3.6\frac{\pi}{6}\int_0^\infty D^3N(D)dD \tag{15}$$

where *D* is expressed in cm and *N(D)dD* in m^{-2} s^{-1}, introducing Eq. (1)

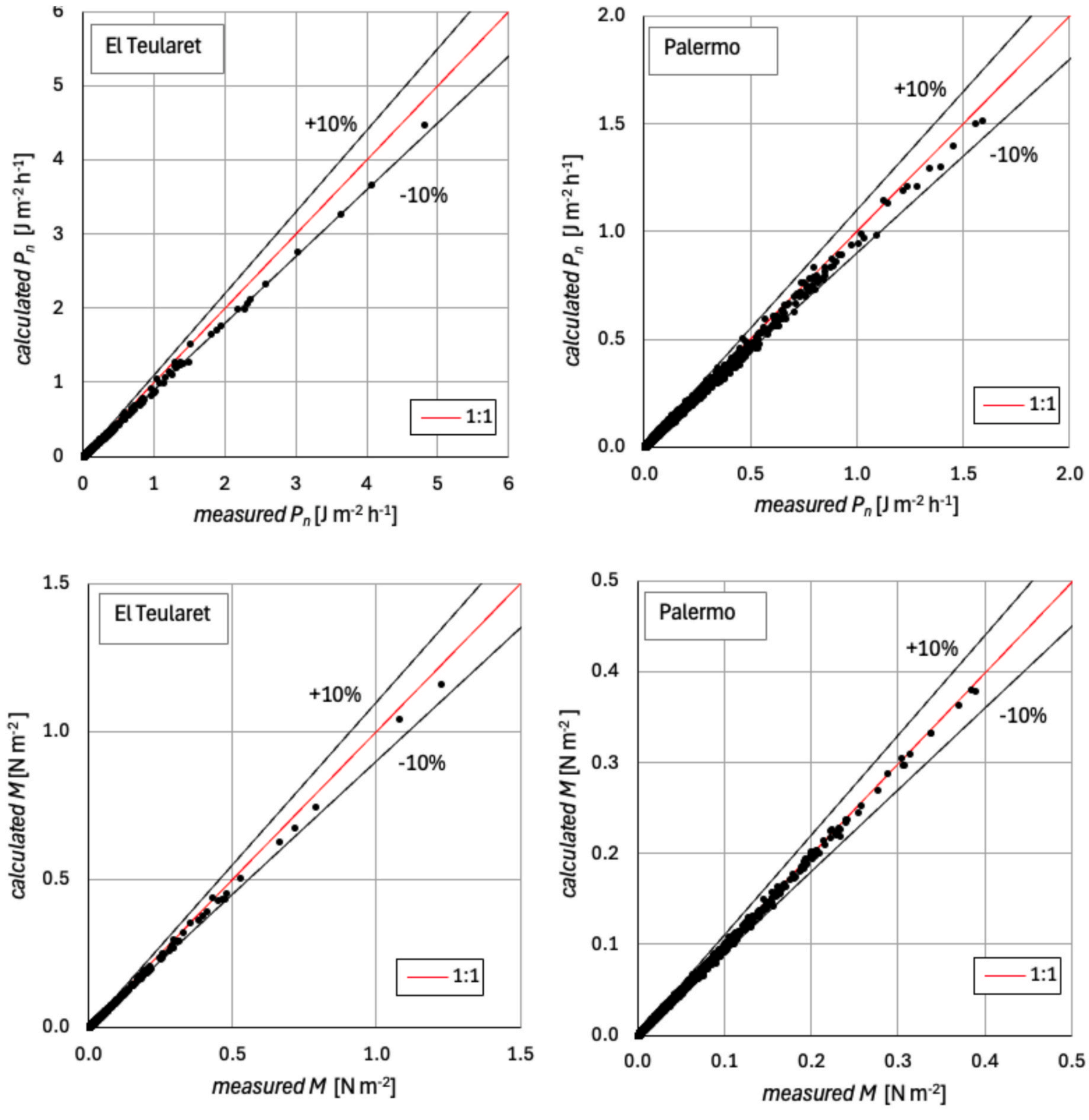

Fig. 5. Comparison between the measured values of P_n and M and those calculated by Eqs. (6) and (11) for Palermo and El Teularet datasets.

Table 1
Error analysis for Eqs. (6) and (11) with parameters estimated by the proposed approach.

	Palermo		El Teularet	
	M (Eq. (11))	P_n (Eq. (6))	M (Eq. (11))	P_n (Eq. (6))
MRE [%]	3.57	7.17	3.53	6.16
MAE [%]	3.77	7.48	4.24	7.45
% estimates with AE < 10 %	95.07	88.98	96.45	77.37

into Eq. (15) and solving the integral, one obtains:

$$I = 3.6\frac{\pi}{6}N\sigma^3\Gamma\left(\frac{3}{\eta}+1\right) \tag{16}$$

where Γ is the gamma function. Introducing Eq. (16) into Eq. (14), the latter can be rewritten as:

$$D_m = \sigma\sqrt[3]{\Gamma\left(\frac{3}{\eta}+1\right)} \tag{17}$$

expressing that D_m is theoretically related to η and σ parameters of the Weibull distribution.

To improve the measurement method of rainfall energy characteristics proposed by Carollo et al. (2018b), the distribution of rainfall momentum, *M(D)dD,* corresponding to all droplets having diameter falling in the range (D, $D + dD$), is considered here, i.e., the distribution of impacts. Since the terminal velocity is calculated by Eq. (3), the distribution *M(D)dD* can be described as:

$$M(D)dD = N(D)\rho\frac{\pi}{6\,10^{-6}}D^3 17.67 D^{0.67}dD \tag{18}$$

where *N(D)* is the number of drops within the range (D, $D + dD$).

For the Weibull distribution, Eq. (18) can be rewritten as:

Table 2
Values of the mean absolute error, *MAE*, for Eqs. (6) and (11) with parameters estimated by the proposed approach, and each rainfall intensity *I* class.

Palermo				El Teularet			
		M (Eq. (11))	P_n (Eq. (6))			*M* (Eq. (11))	P_n (Eq. (6))
I_{mean} [mm h^{-1}]	Number of DSDs falling in the class	*MAE*	*MAE*	I_{mean} [mm h^{-1}]	Number of DSDs falling in the class	*MAE*	*MAE*
0.75	7289	4 %	9 %	0.75	885	4 %	8 %
1.5	13,840	4 %	8 %	1.5	1200	4 %	8 %
2.5	8134	4 %	8 %	2.5	701	4 %	8 %
3.5	5263	4 %	7 %	3.5	478	5 %	8 %
4.5	3421	4 %	7 %	4.5	332	4 %	7 %
5.5	2484	3 %	6 %	5.5	273	5 %	8 %
6.5	1691	3 %	6 %	6.5	247	4 %	7 %
7.5	1197	3 %	6 %	7.5	202	5 %	7 %
8.5	898	3 %	6 %	8.5	177	5 %	8 %
9.5	654	3 %	6 %	9.5	166	4 %	7 %
12.5	1784	3 %	5 %	12.5	441	4 %	6 %
17.5	708	2 %	5 %	17.5	170	4 %	6 %
22.5	365	2 %	4 %	22.5	95	3 %	5 %
27.5	202	2 %	4 %	27.5	54	3 %	5 %
32.5	125	2 %	3 %	32.5	32	3 %	5 %
37.5	94	2 %	3 %	37.5	27	3 %	6 %
42.5	68	2 %	4 %	42.5	19	4 %	7 %
47.5	49	2 %	3 %	47.5	7	3 %	5 %
52.5	25	3 %	5 %	52.5	9	4 %	6 %
57.5	28	2 %	4 %	57.5	10	3 %	5 %
62.5	16	2 %	3 %	62.5	7	3 %	6 %
67.5	18	2 %	4 %	67.5	3	5 %	9 %
72.5	8	2 %	4 %	77.5	6	4 %	8 %
77.5	11	2 %	4 %	82.5	3	7 %	12 %
82.5	12	2 %	3 %	87.5	3	5 %	9 %
87.5	7	3 %	6 %	92.5	3	5 %	8 %
92.5	4	2 %	4 %	97.5	1	2 %	2 %
97.5	9	2 %	3 %	105	5	3 %	6 %
105	8	2 %	4 %	115	5	3 %	7 %
115	9	1 %	2 %	125	2	4 %	8 %
125	5	1 %	2 %	135	3	3 %	6 %
135	2	2 %	3 %	145	2	3 %	7 %
145	4	1 %	3 %	155	1	4 %	8 %
155	2	1 %	1 %	165	4	3 %	5 %
165	2	1 %	4 %	180	5	5 %	8 %
180	2	1 %	3 %	210	1	4 %	8 %
200	1	2 %	3 %				

$$M(D)dD = N\rho\frac{\pi}{6\,10^{-6}}D^3 17.67D^{0.67}\frac{\eta}{\sigma}\left(\frac{D}{\sigma}\right)^{\eta-1} exp\left[-\left(\frac{D}{\sigma}\right)^{\eta}\right]dD \tag{19}$$

According to Eq. (19), the distribution of the rainfall momentum is obtained by the knowledge of the raindrop diameter and the η and σ parameters of the Weibull distribution.

The momentum *M(D)* of a single raindrop, which is the mass of the raindrop having diameter *D* multiplied by its falling velocity obtained by Eq. (3), results in:

$$M(D) = \rho\frac{\pi}{6\,10^{-6}}17.67D^{3.67} \tag{20}$$

from which the raindrop diameter follows as

$$D = \left(\frac{6\,10^{-6}}{17.67\pi\rho}M(D)\right)^{\frac{1}{3.67}} \tag{21}$$

If a device measuring the momentum *M(D)* is available, the sample mean value of the drop diameters *m'(D)*, referred to the detected distribution of raindrop momentum *M(D)*, is obtained by the following expression:

$$m'(D) = \frac{\sum M(D)\left(\frac{6\,10^{-6}}{17.67\pi\rho}M(D)\right)^{\frac{1}{3.67}}}{M} \tag{22}$$

in which *M* is the sum of the momentum of all the drops recorded in the sampling time interval.

Considering the distribution of Eq. (19), the theoretical mean value of the raindrop diameter, *m(D)*, is calculated by the following expression:

$$m(D) = \frac{\int_0^\infty DM(D)dD}{\int_0^\infty M(D)dD} = \frac{\int_0^\infty D^{4.67}\frac{\eta}{\sigma}\left(\frac{D}{\sigma}\right)^{\eta-1} exp\left[-\left(\frac{D}{\sigma}\right)^{\eta}\right]dD}{\int_0^\infty D^{3.67}\frac{\eta}{\sigma}\left(\frac{D}{\sigma}\right)^{\eta-1} exp\left[-\left(\frac{D}{\sigma}\right)^{\eta}\right]dD} = \sigma\frac{\Gamma\left(\frac{4.67}{\eta}+1\right)}{\Gamma\left(\frac{3.67}{\eta}+1\right)} \tag{23}$$

The estimation of the η and σ parameters can be performed by solving the system constituted of Eq. (14) and Eq. (23). Specifically, by dividing each member of the two equations, one obtains:

$$\frac{D_m}{m(D)} = \frac{\sqrt[3]{\Gamma\left(\frac{3}{\eta}+1\right)}}{\frac{\Gamma\left(\frac{4.67}{\eta}+1\right)}{\Gamma\left(\frac{3.67}{\eta}+1\right)}} = \frac{\sqrt[3]{\Gamma\left(\frac{3}{\eta}+1\right)}}{\Gamma\left(\frac{4.67}{\eta}+1\right)}\Gamma\left(\frac{3.67}{\eta}+1\right) \tag{24}$$

Eq. (24) highlights that $D_m/m(D)$ is a function of the only η parameter. Fig. 4 shows that, in the η range of 0.25 to 10, $D_m/m(D)$ is a monotonic function and, therefore, it is an invertible function. Fig. 4 also shows the curve of Eq. (25):

$$\eta = 0.53 + \tan\frac{\frac{D_m}{m(D)} - 0.15}{0.5593} \tag{25}$$

that is an empirical relationship enabling to reproduce adequately the inverse function of Eq. (24), as the mean absolute error is equal to 1.8 %.

Therefore, considering the momentum method (MM), η is estimated by Eq. (25) setting $D_m/m(D)$ equal to $\sqrt[3]{\frac{I}{N\frac{\pi}{6}}}/\ m'(D)$, and then σ is estimated by Eq. (23) setting $m(D) = m'(D)$.

3.3. Testing the new approach in the Mediterranean area

The reliability of the theoretically deduced relationships to calculate P_n (Eq. (6)) and M (Eq. (11)), using the Weibull distribution and the new approach to estimate its parameters, was tested by more than 54,000 DSD measurements carried out at the El Teularet and Palermo experimental sites.

At first, for both sites and each DSD, the measured values of I, N, and $m'(D)$ obtained by the rainfall momentum distribution allowed us to calculate both parameters of the Weibull distribution by applying the MM (Eqs. (23) and (25)). Then, the measured values of kinetic power and rainfall momentum, named *measured* P_n and *measured* M were determined by associating each raindrop diameter with the corresponding terminal velocity calculated by Eq. (3).

Fig. 5 shows the comparison between the measured values of P_n and M and those calculated by Eqs. (6) and (11) using η and σ values estimated by MM. In particular, for both sites, the analysis highlighted that the suggested approach allows for obtaining reliable estimates of P_n even if with a slight overestimation (equal to 7.17 % for Palermo and 6.16 % for El Teularet). Moreover, 77.37 % and 88.98 % of the estimates for the Palermo and El Teularet experimental sites, respectively, have an absolute error (AE) less than 10 % (Table 1). Concerning the rainfall momentum, the analysis highlighted that the use of Eq. (11) with parameters estimated by MM also resulted in slight M overestimations. The best estimates for both sites were obtained for M and are characterized by values of the Mean Relative Error (MRE) (Eq. (9)) approximately equal to 3.5 % (Table 1) and AE less than 10 % for more than 95 % of the cases.

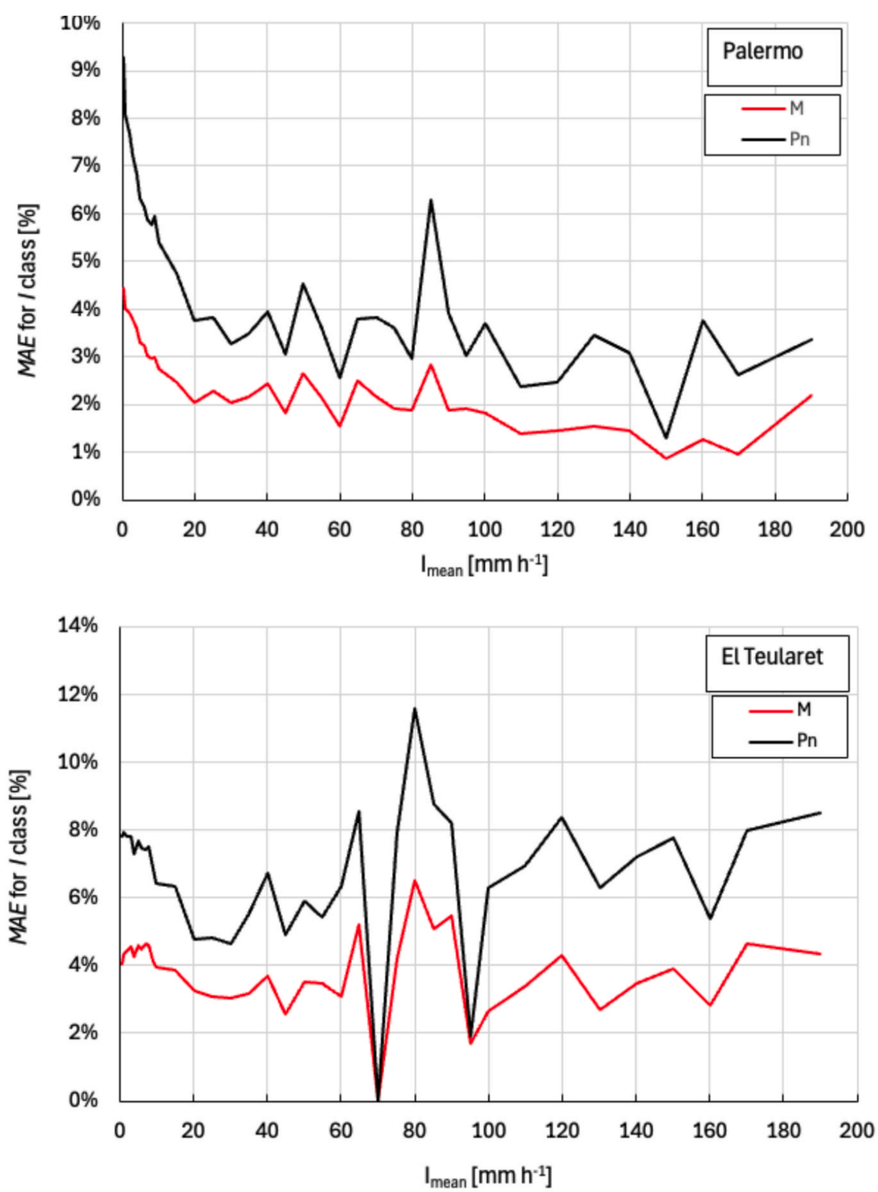

Fig. 6. Values of the mean absolute error (for P_n and M) MAE for rainfall intensity class versus I_{mean} at Palermo and El Teularet experimental sites.

These results at first highlight the reliability of Eqs. (6) and (11), theoretically deduced by Weibull distribution, in estimating P_n and M. Moreover, they also suggest the accuracy of the new proposed method to estimate the η and σ parameters of the Weibull distribution.

The comparison between the present results, based on the Weibull distribution, and those obtained by Carollo et al. (2024b), based on the Ulbrich's law (1983) and the same database (El Teularet and Palermo), support the hypothesis that the approach based on the rainfall momentum distribution can be considered independent of the applied theoretical distribution. In other words, whether we consider the theoretical Weibull distribution (present investigation) or the Ulbrich distribution (Carollo et al., 2024b), the knowledge of *I, N,* and *m(D)* enables accurate P_n and M estimates, as the shape and the scale parameters of the DSDs are well reproduced.

To identify the estimates affected by the highest absolute errors, the DSDs were separated into rainfall intensity classes differing in width (1 mm h^{-1} for $I \leq 10$ mm h^{-1}; 5 mm h^{-1} for 10 mm h^{-1} $< I \leq 100$ mm h^{-1}; 10 mm h^{-1} for 100 mm h^{-1} $< I \leq 170$ mm h^{-1}; and 20 mm h^{-1} for 170 mm h^{-1} $< I \leq 210$ mm h^{-1}). For each class, the mean rainfall intensity, I_{mean} (mm h^{-1}), the number of DSDs, and the *MAE* (Eq. (8) were calculated (Table 2).

Fig. 6shows the pattern of *MAE*, against I_{mean}. For Palermo experimental site, the analysis highlighted that the highest absolute errors occur for the DSDs characterized by $I_{mean} < 20$ mm h^{-1} with *MAE* referred to P_n and M, equal to, on average, 6.9 % and 3.5 %. For $I_{mean} >$ 20 mm h^{-1} *MAE* reduces (on average, *MAE* = 3.5 % for P_n and 1.9 % for M). For El Teularet experimental site, a clear rainfall intensity threshold was not observed, probably due to the lower DSD sample size compared to the Palermo site.

These results suggest that the use of the proposed approach, based on the measurement of the momentum distribution, provides reliable estimations of the energetic characteristics of precipitation, particularly for intensities greater than 10 mm h^{-1}, which are the most relevant from an erosive point of view. The analysis also demonstrated that the new approach gives M estimates more accurate than P_n ones for both experimental sites. This is probably due to the circumstances that the new approach is based on the the rainfall momentum distribution and the uncertainty in calculating raindrop terminal velocity has less impact on the calculation of M than P_n.

The three variables (*I, N,* and *m'(D)*) to detect according to the proposed approach, will be easily measurable once the patented device is built. This method can be practically applied to reproduce the shape and scale parameters of the DSDs, independently of the theoretical distribution used to describe the measured DSDs. As a result, it can also be utilized for the energy characterization of precipitation (P_n and M).

The findings presented in this paper encourage the authors to perform specific experiments to verify the hypothesis that the mean diameter deduced by the momentum distribution is the same as that deduced by the distribution of the electrical signal. If this hypothesis will be confirmed by future measurements that will be carried out after the realization of the patented device, the new instrument will be able to reproduce the DSDs, as a disdrometer but with the advantage of directly measuring the energetic characteristics of precipitation. This can overcome the limitations of traditional disdrometers (e.g., uncertainties in rainfall erosivity estimation, expensive, and not suitable for a widespread use) (Wilken et al., 2018; Shen et al., 2022; Johannsen et al., 2020) and may have a relevant impact on soil erosion research, allowing for monitoring rainfall erosivity at large scale.

4. Conclusions

This paper presented advancements in the theoretical measurement method of rainfall kinetic power and momentum that apply the Weibull distribution and raindrop momentum distribution. Specifically, the new theoretical procedure to estimate the Weibull parameters, used for calculating P_n and M, was based on the measurement of *N, I,* and the mean of the diameters, *m'(D),* deduced by rainfall momentum distribution *M(D)*.

The reliability of the proposed approach was positively tested using DSD measurements carried out with the same disdrometer located in two different experimental sites. The proposed approach gives accurate estimates of both rainfall kinetic power and rainfall momentum and the best performance for the latter. This result is probably influenced by less uncertainty in rainfall terminal velocity for rainfall momentum and the use of the distribution of the measured rainfall momentum rather than kinetic power.

The three variables required to apply the measurement method (*I, N,* and *m'(D)*) will be easily measurable once the patented device is built. This method can be practically applied to reproduce the shape and scale parameters of the DSDs, independently of the theoretical distribution used to describe the measured DSDs and can be used for the energy characterization of precipitation.

Future challenges concern the construction of the prototype of the patented device, which will enable to verify the underlying hypothesis of the proposed approach, that is, the mean diameter deduced from the momentum distribution is the same as that obtained from the electrical signal distribution. This new instrument, coupled with the approach proposed in this paper, will work as a disdrometer and, in addition, have the advantage of directly measuring the rainfall energy characteristics at a large spatial scale. This aspect may have a great impact on the field of soil erosion research as the measurement of the rainfall erosivity may also improve the predictive capability of soil erosion models.

CRediT authorship contribution statement

F.G. Carollo: Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Data curation, Conceptualization. **R. Caruso:** Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Data curation, Conceptualization. **C. Di Stefano:** Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Data curation, Conceptualization. **V. Ferro:** Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Data curation, Conceptualization. **V. Pampalone:** Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Data curation, Conceptualization. **M.A. Serio:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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All authors set up the research, analyzed and interpreted the results, and contributed to write the paper.

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Data availability

Data will be made available on request.

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