Nonequilibrium QFT of axion-like particles and condensates and cosmological constraints on freeze-in and domain wall scenarios

Amedeo M. Favitta[1,2] Lucia Rizzuto[1,2] Roberto Passante [1,2]

[1]Dipartimento di Fisica e Chimica—Emilio Segrè, Università degli Studi di Palermo, Via Archirafi 36, I-90123 Palermo, Italy
[2]INFN, Laboratori Nazionali del Sud, Via S. Sofia 62, I-95123 Catania, Italy

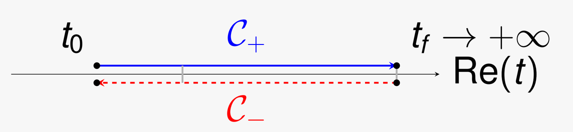
Abstract

We consider the effect of the interaction between axions and Standard Model particles, specifically their evolution during the earliest stages of the Universe, for three different axion models: the photophilic, photophobic, and GUT heavy axion cases. We analyse their scenarios of freeze-in and the case of axion domain walls created after the QCD phase transition with a domain wall number $N_{DW} > 1$ and subjected to thermal plasma friction.
To handle such effects involving both thermal and non-thermal processes, we develop a theoretical formalism with fully relativistic Boltzmann and modified Gross-Pitaevskii equations for the coupled particle and condensate phase-space distributions, including self-interactions, in an expanding universe (or general background metric) derived from the 2PI effective action.
We solve the coupled equations considering the numerically simplest irreducible scenario of ALPs produced via freeze-in with a low reheating temperature ($T \sim 5 - 10 MeV$) and respecting the limits by the Big Bang Nucleosynthesis (BBN) and the effective number of ultrarelativistic degrees of freedom $\Delta N_{\rm eff}$, keeping care of the evolution of the condensate misalignment angle and the contribution of muons.
We derive new bounds for all three models and utilise our formalism to enhance the existing results and relative bounds on the thermal plasma friction acting on the axion domain walls. In the two different scenarios, we solve the equations numerically with MicrOmegas 6.1.15
For the last aspect, we improve upon some known results in the literature by considering the complete quantum equations of motion for the condensate domain wall and the muon contribution relevant for high-mass ALPS, thereby determining the parameter space where the domain wall problem can be solved through the thermal friction mechanism.
In both cases, we get interesting bounds and results in the region of axion mass $m_a > 1 MeV$.

Nonequilibrium QFT and quantum equations of motion

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[\frac{1}{2} \partial_\mu \Phi \, \partial^\mu \Phi - V(\Phi) + K_\chi(x) + g_{\phi\chi} \Phi \, \mathcal{O}_\chi(x) \right],$$

- We work with axion field Φ interacting with a generic SM field χ
- We work with the Keildysh-Schwinger formalism of nonequilibrium QFT in a curved spacetime with metric $g_{\mu\nu}$
- $\Delta_\phi^{ab}(x_1, x_2) = \langle \Phi(x_1)\Phi(x_2) \rangle - \varphi(x_1)\varphi(x_2)$
- $\varphi(x) = \langle \Phi(x) \rangle$ and $a, b = +, -$.
- We start from the EoMs obtained by varying the 2PI effective action [1,2]



Graphical visualization of the Keyldish closed time contour (CTP) for our initial-value problem and the extension at $t_f \to +\infty$.

$$\frac{\delta \Gamma_{2PI}}{\delta \varphi} = 0 \quad , \quad \frac{\delta \Gamma_{2PI}}{\delta \Delta_\phi} = 0.$$

$$\Gamma_{2PI}[\varphi, \Delta_\phi, \Delta_\chi] = S[\phi] + \frac{i}{2} \mathrm{Tr} \ln \Delta_\phi^{-1} + \frac{i}{2} \mathrm{Tr}[G_\phi^{-1} \Delta_\phi] + i \sum_\chi a_\chi \left[\mathrm{Tr} \ln \Delta_\chi^{-1} + i \, \mathrm{Tr}[G_\phi^{-1} \Delta_\phi] \right] + \Gamma_2.$$

Quantum equations of motion (EoMs)

$$(D^\mu \partial_\mu + \tilde{m}_\varphi^2)\varphi + \frac{\lambda_\phi}{2} \Delta_\phi^{++}(x, x)\varphi - \frac{\delta \Gamma_2}{\delta \phi^+}\Big|_{\phi^+ = \phi^- = \varphi} + g_{\phi\chi} \Delta_{\mathcal{O}_\chi} = 0$$

$$-(D^\mu \partial_\mu + \tilde{m}_\phi^2)\Delta^{ab}(x_1, x_2) - c \int d^4x_3 \, \Pi_\phi^{ac}(x_1, x_3) \, \Delta^{cb}(x_3, x_2) = i c^{ab} \delta(x_1 - x_2)$$

with $\tilde{m}_\varphi$ and $\tilde{m}_\phi$ the effective masses of the two components, keeping care of self-interactions

Boltzmann and modified Gross-Pitaevskij equations

We obtain from Quantum EoMs the following equations

$$\frac{1}{k_0} \left[k_\mu \partial_\mu^{(x)} - \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta \partial_\mu^{(k)} + \frac{1}{2} \left(\partial_\mu^{(x)} M_\phi^2 \right) \partial^\mu_{(k)} \right] f_\phi(k, x) = \mathcal{C}[f_\phi, f_i]$$

$$\dot{\rho}_\varphi + 3H\rho_\varphi + H \sum_i p_{i\,\varphi} = 2\gamma\rho_\varphi + \frac{2\sigma}{M_\phi^2} \rho_\varphi^2$$

with [1,2,3]

$$-2\gamma = \sum_{\rm proc} \left[\frac{1}{2M_\phi} \int \Pi_i \frac{d^3\vec{k}_i}{(2\pi^3) 2k_i^0} (2\pi)^4 \delta^{(4)}(\bar{k} + k_{A1} + ... - k_{B1} - ...) \right.$$
$$\left. \times \{ (1 \pm f_{A1})...f_{B1}... - f_{A1}...(1 \pm f_{B1})... \} \, |\mathcal{M}_{\varphi A_1 ... \to B_1 ...}|^2 \right] ,$$

and

$$-2\frac{\sigma}{M_\phi} = \sum_{\rm proc} \left[\frac{1}{(2M_\phi)^2} \int \Pi_i \frac{d^3\vec{k}_i}{(2\pi^3) 2k_i^0} (2\pi)^4 \delta^{(4)}(2\bar{k} + k_{A1} + ... - k_{B1} - ...) \right.$$
$$\left. \times \{ (1 \pm f_{A1})...f_{B1}... - f_{A1}...(1 \pm f_{B1})... \} \, |\mathcal{M}_{(2\varphi) A_1 ... \to B_1 ...}|^2 \right] .$$

Some important generalisations from our work

- We generalise to QFT for a generic background spacetime
- For the FRLW metric case of our interest, we go beyond the slow-rate approximation and the k_0^i energies are to be thought to correspond to $a(t)k_0^i = \Omega_k(t)$ where $\Omega_k(t)$ are the time-dependent frequencies satisfying

$$\Omega_k^2 = \frac{3\dot{\Omega}_k^2}{4\Omega_k^2} - \frac{\ddot{\Omega}_k}{2\Omega_k} + \frac{k^2}{a^2} + \tilde{m}_\phi^2 - \frac{3}{4}H^2 - \frac{3}{2}\frac{\ddot{a}}{a},$$

- Relevance of Ω_k for axions when at least $H \sim m_\phi \to$ ultralight DM, DWs

Application 1: Axion freeze-in

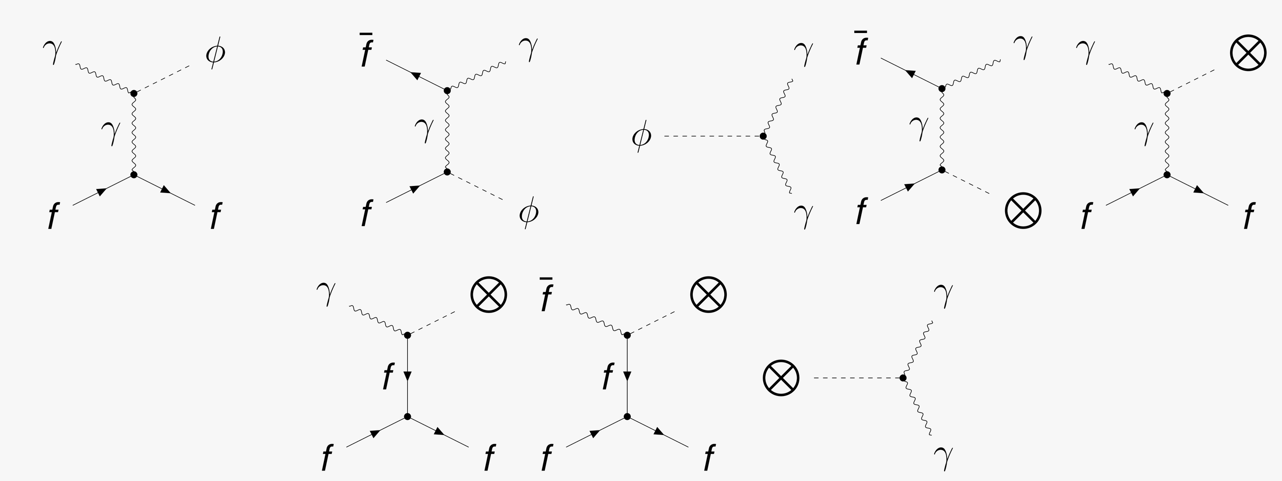
At leading order in the on-shell limit and adiabatic expansion, and with non-relativistic approximation, we obtain the following equations for number densities

$$\dot{n}_\phi + 3Hn_\phi = \mathcal{C}[f_\phi, f_\chi]$$
$$\dot{n}_\varphi + 3Hn_\varphi = \mathcal{C}_1[f_\chi] n_\varphi + \mathcal{C}_2[f_\chi] n_\varphi^2$$

Relevant aspects

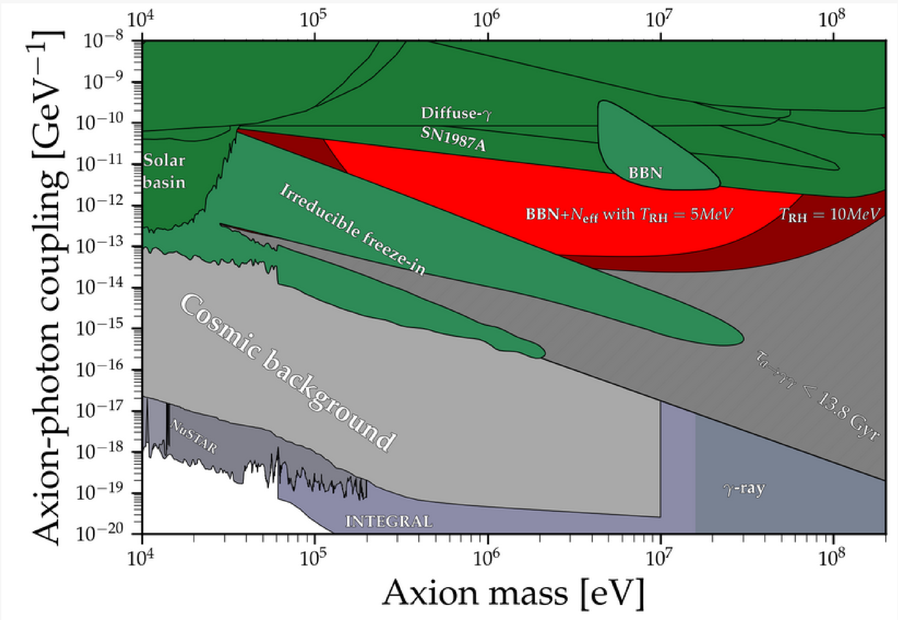
- Condensate production is analogous to particle production
- Relevant for irreducible freeze-in and BBN constraints

Relevant Feynman diagrams for our work



BBN+$N_{\rm eff}$ constraints: Photophilic ALP

- We solve Boltzmann equations numerically for an axion freeze-in scenario at low reheating temperature
- For photophilic and photophobic models, we take care of Primakoff and annihilation processes with electrons and muons
- We obtain constraints from excluding axions in parameter space exceeding limits from $N_{\rm eff}$ and DM abundance



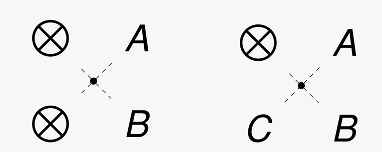
Application 2: Domain wall (DW) problem for $N_{\rm DW} > 1$

- We obtain from the quantum EoMs, assuming the DW network to be the condensate, the VOS model

$$\dot{\rho} - H\rho(1 + 3v^2) = 2\gamma(T, v)\,\rho + \frac{2\sigma(T, v)}{M_\phi^2} \rho^2,$$

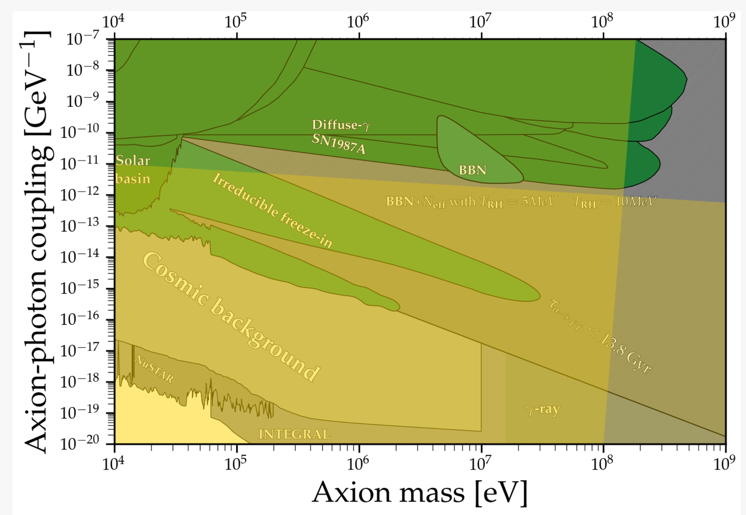
$$\dot{v} = (1 - v^2) \left[-3Hv - 2\gamma(T, v)\,v - \frac{2\sigma(T, v)}{M_\phi^2} \frac{\sigma_{\rm dw}}{L} v \right],$$

where $L = \sigma_{\rm DW}/\rho$ is the correlation lenght and $\sigma_{\rm DW}$ the DW surface energy density.
- Correspondence between microscopic terms $\leftrightarrow$ macroscopic terms obtained formerly in the literature in a heuristic way or by numerically fitting:
  - $2\gamma(T, v) \to l_f^{-1} \to$ inverse friction lenght
  - $\frac{2\sigma(T,v)}{M_\phi^2} \sigma_{\rm dw} \to -c_w v \frac{\rho}{L} - d' \left[k_0 - k(v) \right]^r \Rightarrow$
  - No need or heuristic pressure calculation, and go beyond particle scattering approximation



Constraints on DWs for photophilic ALP (VOS model)

- Friction terms influence the DW network evolution $\leftrightarrow$ network annihilation before or after the DW domination $\to$ relevant for cosmological observables [4,5]
- We require annihilation temperature $T_{\rm ann} > T_{\rm dom}$ $\to$influence on the SGWB contribution from DWs [1,4]
- Influence on CMB birefringence [1,4,5]
- Preliminary plot with constrained ALP models. Yellow-shaded areas are constrained by $N_{\rm eff}$ and DW domination constraints



References

[1] A. M. Favitta, L. Rizzuto, and R. Passante, "Nonequilibrium qft of axion-like particles and condensates and cosmological constraints on freeze-in and domain wall scenarios, in preparation,2025."
[2] W.-Y. Ai, A. Beniwal, A. Maggi, and D. J. Marsh, "From qft to boltzmann: freeze-in in the presence of oscillating condensates," *Journal of High Energy Physics*, vol. 2024, no. 2, pp. 1–37, 2024.
[3] N. P. Proukakis, G. Rigopoulos, and A. Soto, "Unified description of corpuscular and fuzzy bosonic dark matter," *Phys. Rev. D*, vol. 108, p. 083513, Oct 2023.
[4] S. Blasi, A. Mariotti, A. Rase, A. Sevrin, and K. Turbang, "Friction on alp domain walls and gravitational waves," *Journal of Cosmology and Astroparticle Physics*, vol. 2023, p. 008, apr 2023.
[5] A. M. Favitta, I. H. Brevik, and M. M. Chaichian, "Axion electrodynamics: Green's functions, zero-point energy and optical activity," *Annals of Physics*, vol. 455, p. 169396, 2023.