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Weak action representability and categories of algebras

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Abstract.

It is well known that in the semi-abelian category **Grp** of groups, internal actions are represented by automorphisms. This means that the category **Grp** is *action representable* with the actor of a group being its group of automorphisms. The notion of action representability has proven to be quite restrictive: for instance, it was proved that the only non-abelian variety of non-associative algebras which is action representable is the variety of Lie algebras. More recently G. Janelidze introduced the concept of *weakly action representable* category, which includes a wider class of categories, such as the variety of associative algebras and the variety of Leibniz algebras.

The notion of weak action representability was studied in the context of varieties of algebras: it was shown that every object X of an *algebraically coherent* variety $\mathcal{V}$ admits an *external weak representation* $\mathrm{Act}_{\mathcal{V}}(-,X) \rightarrowtail \mathrm{Hom}_{\mathbf{PAlg}}(U(-),\mathcal{E}(X))$, where $\mathcal{E}(X)$ is a partial algebra, called *external weak actor*, and U denotes the forgetful functor from $\mathcal{V}$ to the category **PAlg** of partial algebras.

The aim of this talk is to investigate the relationship between action accessibility and weak action representability in the context of varieties of algebras. Using an argument of J. R. A. Gray in the setting of groups, we prove that the varieties of k-nilpotent ($k \geq 3$) and n-solvable ($n \geq 2$) Lie algebras are not weakly action representable. This establishes that a subvariety of a (weakly) action representable variety of non-associative algebras need not be weakly action representable.

We then aim to study the representability of actions in the context of categories of *unitary* non-associative algebras, which are *ideally exact* in the sense of G. Janelidze. After describing the monadic adjunction associated with any category of unitary algebras, we prove that the categories of unitary associative algebras, unitary alternative algebras and unitary Poisson algebras are action representable. This is joint work with X. García Martínez (*University of Vigo*) and F. Piazza (*University of Messina*).

References

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