CHARACTERIZING THE SPONTANEOUS COLLAPSE OF A WAVEFUNCTION THROUGH ENTROPY PRODUCTION

S. Artini*, M. Paternostro*, *New Journal of Physics* 25.12 (2023): 123047.

In collaboration with S. Donadi**, G. Lo Monaco*.

*Università degli Studi di Palermo **Queen's University Belfast

Goals of the project:

- Develop a new theoretical framework to study collapse dynamics;
- Verify if this framework is useful to attest **physical consistency**;
- Study the emergence of **thermodynamic equilibrium**.

Content of this presentation:

- The CSL and DP models and their dissipative extensions;
- Thermodynamics of out-of-equilibrium systems: entropy production;
- Case study: 1D harmonic oscillator;

SPONTANEOUS COLLAPSE MODELS

The measurement problem and the UD approach

Inconsistency between the Schrödinger equation and the wavefunction collapse postulate (with the Born rule).

Unified Dynamics theories try to reconcile the microscopic and the macroscopic world under a **single dynamical principle**.

In our case the idea is to add to the Schrödinger equation an additional **non-linear** and **stochastic** term.

Collapse models: modified SE and ME

$$\mathrm{d}\psi_t = \Big[-\frac{\mathrm{i}}{\hbar}\hat{H}\,\mathrm{d}t + \sqrt{\lambda}\int \mathrm{d}^3x\,(\hat{m}(\mathbf{x}) - \langle \hat{m}(\mathbf{x})\rangle_t)\,\mathrm{d}W_t(\mathbf{x})$$
$$-\frac{\lambda}{2}\int \mathrm{d}^3x \int \mathrm{d}^3y\,\mathcal{G}(\mathbf{x}-\mathbf{y})(\hat{m}(\mathbf{x}) - \langle \hat{m}(\mathbf{x})\rangle_t)(\hat{m}(\mathbf{y}) - \langle \hat{m}(\mathbf{y})\rangle_t)\,\mathrm{d}t\Big]\psi_t$$

(ME for one particle)

$$\partial_t \hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(t)] + \mathcal{L}[\hat{\rho}(t)]$$

$$\mathcal{L}[\hat{\rho}(t)] = \int d^3P\,\Gamma(P)\left(e^{\frac{i}{\hbar}P\cdot\hat{r}}\hat{\rho}(t)e^{-\frac{i}{\hbar}P\cdot\hat{r}} - \hat{\rho}(t)\right)$$

Bassi, Groβardt, Ulbricht, *Classical and Quantum Gravity* 34.19 (2017): 193002.
Bahrami, Smirne, Bassi, *Physical Review A* 90.6 (2014): 062105.

The CSL and the DP models

$$\mathcal{G}_{CSL}(\mathbf{x}-\mathbf{y}) = e^{-\frac{(\mathbf{x}-\mathbf{y})^2}{4r_c^2}} \longrightarrow \Gamma_{\mathrm{CSL}}(\boldsymbol{P}) = \frac{\gamma}{(2\pi)^3}\frac{m^2}{m_0^2}e^{-P^2 r_c^2}$$

$$\mathcal{G}_{DP}(\boldsymbol{x}-\boldsymbol{y}) = \frac{G}{\hbar}\frac{1}{|\boldsymbol{x}-\boldsymbol{y}|} \longrightarrow \Gamma_{\mathrm{DP}}(\boldsymbol{P}) = \frac{Gm^2}{2\pi^2\hbar^2}\frac{1}{P^2}e^{-\frac{P^2 R_0^2}{\hbar^2}}$$

Dissipative extension of the models

$$\mathcal{L}[\hat{\rho}] = \frac{1}{\hbar^2} \int \frac{d^3k}{(2\pi)^3} \Gamma(k) \left(\hat{L}_k \hat{\rho} \hat{L}_k^\dagger - \frac{1}{2} \left\{ \hat{L}_k^\dagger \hat{L}_k , \hat{\rho} \right\} \right)$$

With Lindblad operators that depend on both the position and the momentum:

$$\hat{L}_k = e^{ik\cdot\hat{x}} - \frac{\hbar\beta}{8} \left\{ k \cdot \hat{p}, e^{ik\cdot\hat{x}} \right\}$$

Di Bartolomeo, G., Carlesso, M., Piscicchia, K., Curceanu, C., Derakhshani, M., & Diósi, L. (2023). *Physical Review A*, *108*(1), 012202.

Asymptotic behaviour of the energy

For the frictionless models:

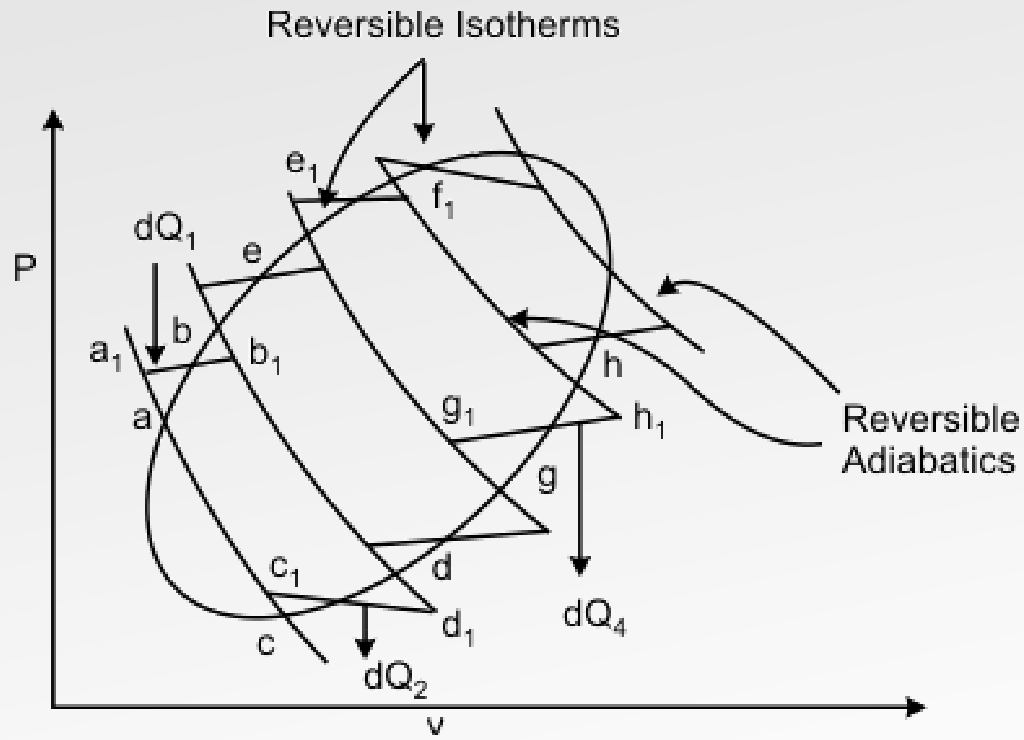
$$\frac{d < \hat{H} >}{dt} = P \quad \Longrightarrow \quad < \hat{H} >_\infty = +\infty$$

For the dissipative extensions:

$$\frac{d < \hat{H} >}{dt} = P - \Gamma < \hat{H} > \quad \Longrightarrow \quad < \hat{H} >_\infty = \frac{P}{\Gamma}$$

ENTROPY PRODUCTION

Entropy production and irreversibility

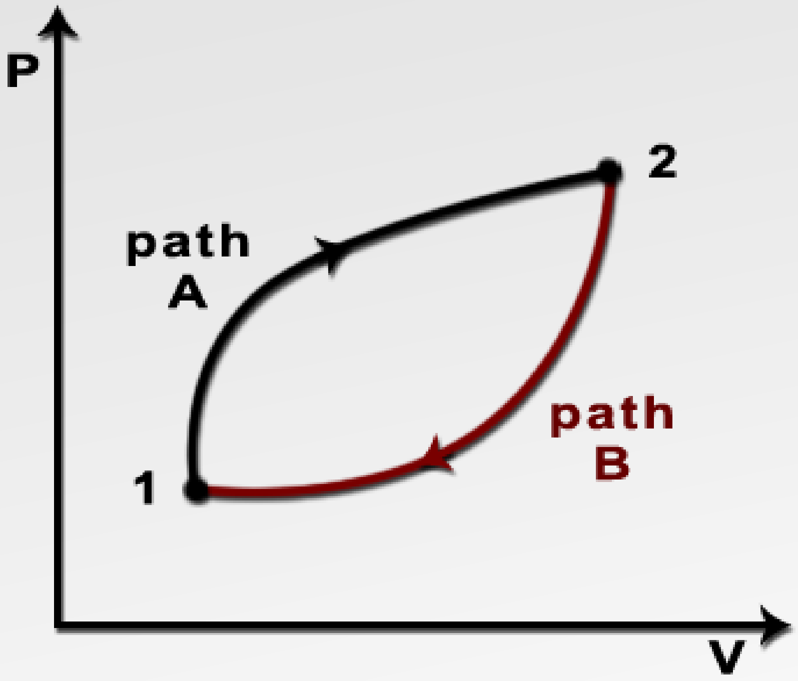


$$\Delta S = \int_\gamma \phi(t)dt = 0$$

Quasi-static ↔ equilibrium

https://home.iitk.ac.in/~gtm/thermodynamics/lecture21/21_3.htm

Entropy production and irreversibility



$$\int_\gamma dS = \Sigma - \int_\gamma \phi(t)dt$$

$$\frac{dS}{dt} = \Pi(t) - \phi(t)$$

Σ is not a state function!

Out-of equilibrium → we reintroduce time as a variable

https://www.learnthermo.com/T1-tutorial/ch07/lesson-C/pg01.php

Entropy production and irreversibility

- $\boldsymbol{\Sigma > 0}$ **for irreveresible processes** and $\Sigma = 0$ at equilibrium (Clausius inequality);

- Its rate $\boldsymbol{\Pi(t)}$ is used to study whether a system approaches thermal equilibrium during its dynamics;

- Not directly observable, needs a theoretical framework to be consistently defined.

VN Entropy vs Wigner entropy

$S_{VN} = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$ → $\Pi_{VN} = -\partial_t K_{VN}(\hat{\rho}||\hat{\rho}^*)$ → ULTRACOLD CATASTROPHE

VN Entropy vs Wigner entropy

$$S_{VN} = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$$

$$S_\alpha = (1-\alpha)^{-1}\ln(\mathrm{Tr}\hat{\rho}^\alpha)$$

($\alpha = 2$ and gaussian states)

$$S = -\int dq dp W(q,p) \ln\big(W(q,p)\big)$$

$$\Pi(t) = -\partial_t K(W(t)||W_{eq})$$

Santos, Jader P., Gabriel T. Landi, and Mauro Paternostro. *Physical review letters* 118.22 (2017): 220601 and references therein.

THERMODYNAMIC ANALYSIS OF AN HARMONIC OSCILLATOR SUBJECT TO SPONTANEOUS COLLAPSE

Collapse dynamics in phase-space - frictionless

Starting from the CSL\DP ME one can get a quantum Fokker-Planck equation via a Wigner-Weyl transform.

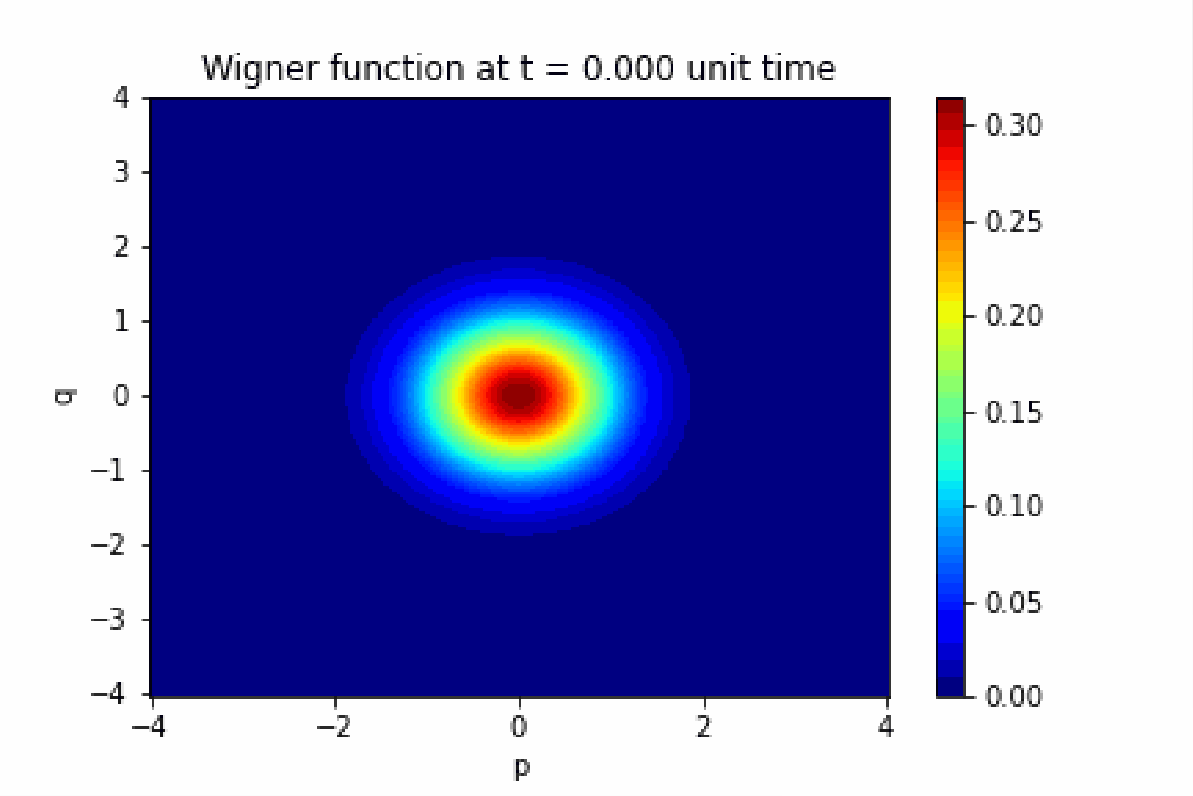
Considering states well concentrated around the origin of the phase-space (e.g. **gaussian states**), the equation in terms of the Wigner function of a 3D system is well approximated by:

$$\partial_t W_{\hat{\rho}(t)} = \{W_{\hat{H}}, W_{\hat{\rho}(t)}\}_\star + D\,\Delta_{(\boldsymbol{p})} W_{\hat{\rho}}(\boldsymbol{q}, \boldsymbol{p})$$

Diffusion in the momentum direction!

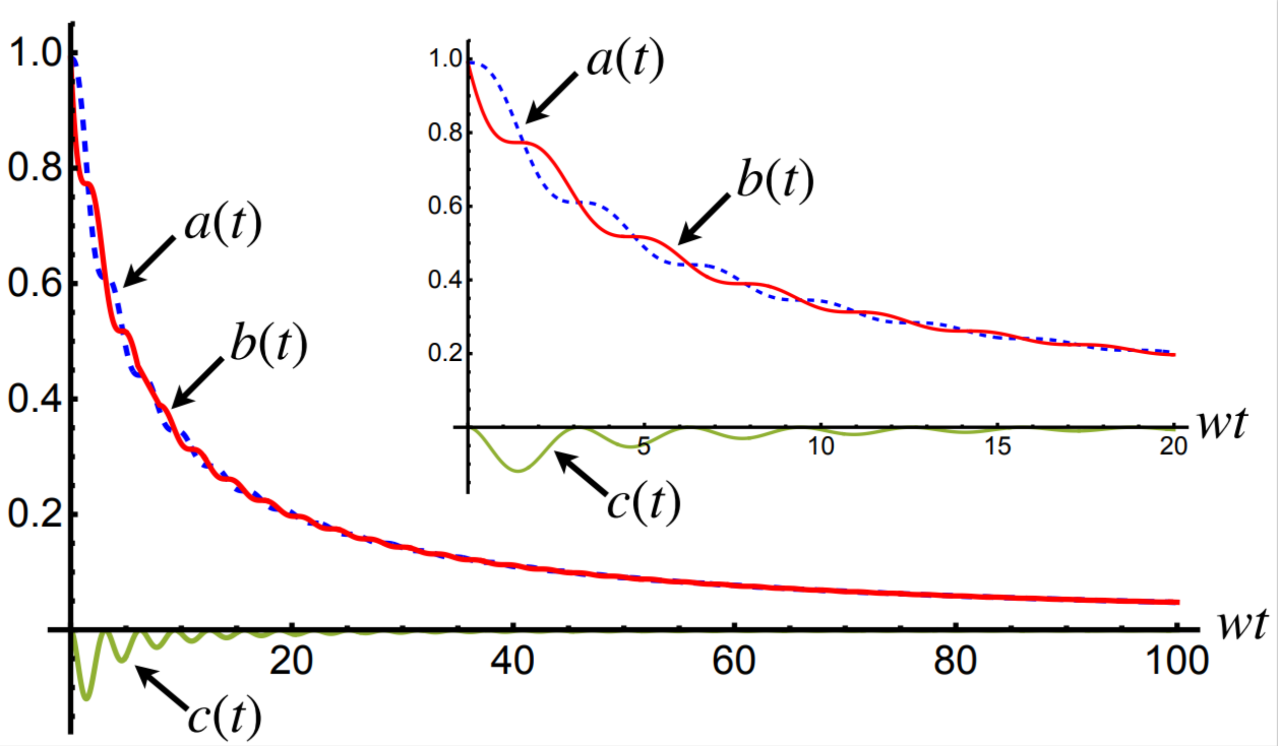
Collapse dynamics in phase-space (1D)

$W(q,p) = N(t)\exp\left\{-\left(a(t)mwq^2 + \frac{b(t)p^2}{mw} + 2c(t)pq\right)\right\}$ with thermal initial datum (isotropic)



(Time is in Planck's units)

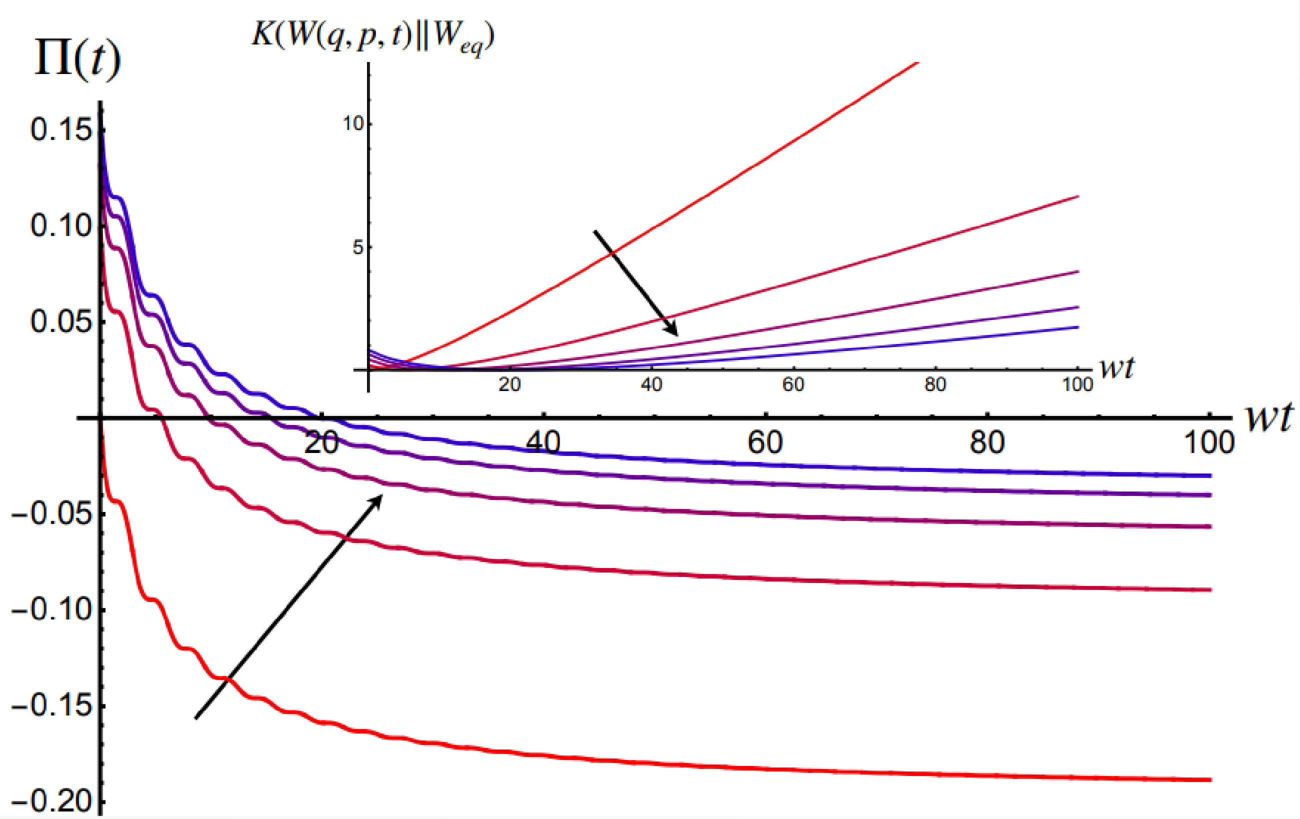
Collapse dynamics in phase-space (1D)



No (physical) asymptotic state!!!

(Time is in Planck's units)

Collapse entropic analysis (1D)



(Target temperature increases along the arrow)

(Time is in Planck's units)

Comments

- **Entropy production** always takes **negative** values at some point;
- The target state should have an **infinite temperature** to guarantee a thermodynamical consistent dynamics;
- This reflects the absence of a physical asymptotic state.

Collapse dynamics in phase-space – linear friction

$$\partial_t W_{\hat{\rho}(t)} = \{W_{\hat{H}}, W_{\hat{\rho}(t)}\}_\star + \nabla_p \cdot (f p W_{\hat{\rho}}) + \\ + \partial_{p_i}\partial_{p_j}(D_{ij} W_{\hat{\rho}}) + \partial_{q_i}\partial_{q_j}(\tilde{D}_{ij} W_{\hat{\rho}}) + \\ + R^{ijkl}\partial_{p_i}\partial_{p_j}\partial_{q_j}\partial_{q_k} W$$

- **Non-Gaussian terms** that could lead to negativity in the Wigner function;
- Very complicated to solve numerically even in 1D;

Collapse dynamics in phase-space – linear friction

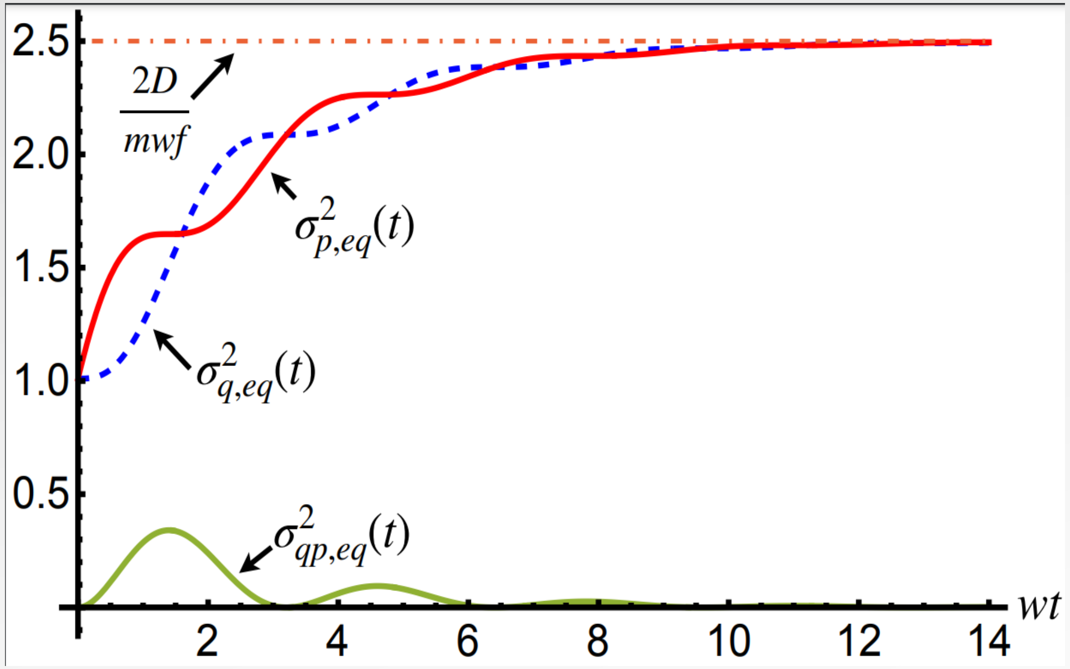
However, keeping only terms at most linear in β (**high temperature** approximation):

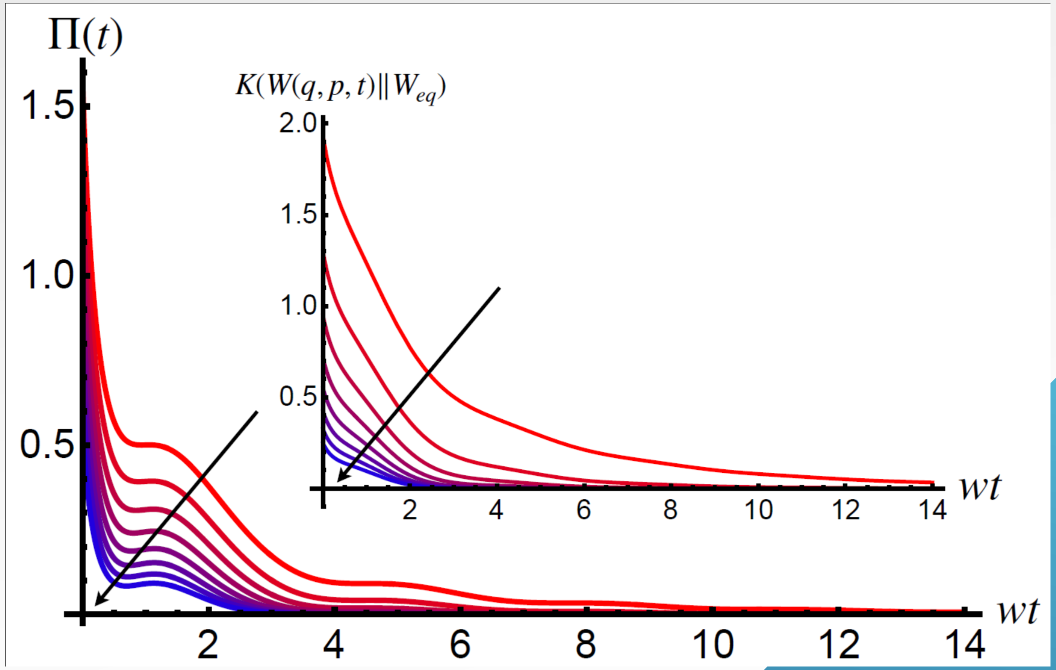
$$\partial_t W_{\hat\rho(t)} = \left\{W_{\hat H}, W_{\hat\rho(t)}\right\}_\star + \nabla_p \cdot (f p W_{\hat\rho}) + D\Delta_p W_{\hat\rho}$$

$$D = \frac{2m^2}{3}\int \frac{d^3k}{(2\pi)^3} k^2 \Gamma(k)\left(1 + \frac{\hbar^2\beta}{2m}k^2\right)$$

$$f = \frac{m\beta}{3}\int \frac{d^3k}{(2\pi)^3} k^2 \Gamma(k)$$

Collapse dynamics in phase-space and entropy production (1D)





(Time is in Planck's units)

Comments

- **Thermal equilibrium** is reached in a physical consistent way:

$$\Pi(t) \rightarrow 0 \; as \; t \; \rightarrow 0 \; with \; \Pi(t) \geq 0 \; \forall t$$

- The **uncertainty principle** yields an upper bound on the noise inverse temperature:

$$det(V_{eq}) \geq \hbar^2 \implies \beta \leq \frac{1}{\hbar m w - \frac{3\hbar\sqrt{\pi}}{8R_0^2 m}}$$

Outlook

- How can we take into account the **non-Gaussian terms** in the full QFP equation? (Q-function, Wehrl entropy, linear response regime…);

- Even in the small β approximation is still possible to get negative entropy production using **non-isotropic initial states** (e.g. squeezed states).

And after that:

- Characterize the entropy dynamics of other gravity-induced collapse or gravitational decoherence models.

THANK YOU FOR THE ATTENTION.

UNIVERSITÀ DEGLI STUDI DI PALERMO
Department for the Economy
THE ROYAL SOCIETY
MSCA COFUND
Horizon Europe
EPSRC
The Leverhulme Trust
Science Foundation Ireland sfi For what's next