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INVITED REVIEW

Review of conceptual and empirical approaches to characterize infiltration

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Abstract

Infiltration regulates the movement and storage of water at the soil–atmosphere interface and is, therefore, a key component of many related physical and biogeochemical processes. Numerous studies have examined infiltration over the past two centuries. These efforts have resulted in the development of numerous models that capture the effects of specific soil properties and initial and boundary conditions. This proliferation of models has advanced our collective ability to understand infiltration processes but has also made it challenging for researchers to select appropriate approaches for analyzing experimental infiltration data or for conducting basic research on soil parameters like saturated hydraulic conductivity or sorptivity. Here, we aimed to reduce this uncertainty by developing a comprehensive literature review of published infiltration models, including their underlying philosophies and evolution over the years. Through this effort, we compiled and examined 138 unique infiltration models. We grouped models into two major categories, empirical and conceptual, noting that boundaries between those two categories are at times debatable. After classifying and providing a full historical retrospective of these models, we examined specific model parameters and how their usage has changed

Karim Majdi Abou Najm inspired his father, his family and friends to ask the right questions and to be critical thinkers. Karim's journey ended on April 29, 2023, but his legacy continues. A first-friend to many, Karim found happiness in connecting people and in helping many friends realize their full potentials. As a brilliant computer science student, he enjoyed working on solutions for the betterment of humanity. The authors dedicate this work to his memory.

with time. We also reviewed different methods applied to estimate infiltration parameters, as well as the challenges that arise when using such methods. Finally, we proposed a framework for identifying suitable models depending on field conditions, experimental plan, and data availability.

Plain Language Summary

Infiltration is the process of water movement in soils. Understanding infiltration is critical to our understanding of how nature functions, but infiltration is controlled by very complex phenomena. So to understand infiltration, we need to mathematically create models that simplify its complicated structure. Thinking of one soil aggregate as one large city with a complex road network can demonstrate how sophisticated the soil pore network is. As researchers tried to understand this complex process, wide range of concepts were developed over the past 200 years. Some are complex and try to use much information from the soil, while others try to simplify it and generalize things so we can estimate infiltration with simple formulas. This has led to the development of 138 different models, which we tried to categorize and explain in this paper. Our objective was to aid researchers in finding the right models for their application from this wide selection of models.

1 | INTRODUCTION

Water infiltration is one of the most important processes of the hydrologic cycle for assessing runoff, soil moisture, groundwater recharge, nutrients leaching, plant growth, and many other soil-related processes (Fodor et al., 2011; Singh, 2010; Zadeh et al., 2007). Because of its importance, infiltration has been widely studied for over two centuries (Ghorbani et al., 2009). This attention led to the development of numerous infiltration models, classified into two categories: conceptual and empirical models. Conceptual models are derived from fundamental and physical theories and typically account for soil properties and specific boundary and initial conditions, while empirical models are based on fitting experimentally measured data using defined mathematical functions (Assouline, 2013; Jacka et al., 2016).

Among the conceptual models, most equations were derived from Darcy's law and substantially influenced by approximating solutions and assumptions of Buckingham and Richardson–Richards. Although some researchers have questioned the validity of Buckingham's assumptions and Richardson–Richards' equations (Abou Najm et al., 2019; Beven, 2018; Germann, 2021; Hunt et al., 2013), several extensions and new applications of the Darcy-Buckingham–Richardson-Richards paradigm for saturated and unsaturated flow (hereafter referred to as Darcy-BRR and explained in depth in the following section) were developed as an attempt to estimate infiltration. For example, flow in infiltration models was mostly derived from an abstracted and often overly simplified soil pore structure of a bundle of capillary tubes. Comparatively rare are advancements reporting progress in percolation theory, network models, and Stokes viscous approach (Beven, 2018; Germann, 2021; Hunt et al., 2013).

The abundance of developed equations motivated researchers to present reviews that investigate infiltration models. Some of these reviews were comprehensive summaries of conceptual and empirical equations (Barry et al., 2009; Clothier, 2001; Delleur, 2006; Kutilek and Nielsen, 1994; Raats et al., 2002). Others provided insight of the historical evolution of infiltration theory, including the classic solutions of the Richardson–Richards' equation (Assouline, 2013; Youngs, 1995). Building on these reviews (Assouline, 2013; Youngs, 1995), others have worked to identify models satisfactory for field applications. Typically, such models were fitted to measure infiltration data (e.g., cumulative infiltration vs. time of infiltration) for estimation of the parameters of fitted infiltration models. The ease and accuracy of estimated parameters were further assessed through evaluation techniques that identify the best fit infiltration models for different datasets (Fodor et al., 2011; Jacka et al., 2016; Bayabil et al., 2019; Nie et al., 2017b).

Most reviews on infiltration theory provided a comprehensive evaluation of the commonly used infiltration models developed over the past century, highlighting the seminal works of Green and Ampt (1911), Philip (1957a, 1957b), Parlange et al. (1982), Swartzendruber (1987b), and Haverkamp et al. (1990, 1994), along with key empirical equations by Kostiakov (1932), Horton (1940), Mezencev (1948), and Holtan (1961). These models are still extensively studied and applied. However, in the last two decades, substantial efforts have been made on the abovementioned models, leading to theoretical improvements and a wider body of research on modeling infiltration. These advancements have ultimately led to a larger and more diverse set of infiltration models and to more open discussions on the validity and application of some of the dominant theoretical frameworks. To this end, it became challenging for researchers to select appropriate approach(es) for analyzing experimental infiltration data or for conducting basic research on soil parameters like saturated hydraulic conductivity or sorptivity. Therefore, a comprehensive and current review covering both past and recent model development is still needed.

Here, we present a comprehensive critical review that traces the development of conceptual and empirical models over the past two centuries through both historical and theoretical lenses. With this review, our objectives are to (1) build an inclusive historical retrospective of the evolution of the infiltration problem and its solutions, (2) highlight the diversity in form, origin, and theory among reviewed infiltration models, and (3) explore methods applied to estimate the basic model parameters, including saturated hydraulic conductivity and sorptivity. To this end, we aimed for models that provided theoretical advancements, novel frameworks, and unique empirical fits, and excluded papers that focused on iterative or incremental advancements of existing models, as well as papers that adapted these models to other uses (e.g., hydraulic characterization).

2 | UNDERLYING PHYSICS OF INFILTRATION PROCESSES

Water infiltrates because of two forces: capillarity and gravity (Buckingham, 1907). Absorption of water due to capillarity, also in some contexts referred to as sorptivity S (L T$^{-0.5}$), controls water movement at the early state of infiltration into dry soils; while the hydraulic conductivity K (L T^{-1}) dominates the infiltration process at steady state, which is mainly driven by gravity (Philip, 1957b). The hydraulic conductivity, K (L T^{-1}), varies rapidly and nonlinearly with matric head, ψ (L) (Richards, 1931). Integration of K (L T^{-1}) over a range of ψ (L) is defined as matric flux potential, φ (L^2 T^{-1}), which provides a useful approach to the infiltration problem in unsaturated soils (Raats, 1971). The relative importance of capillarity versus gravity during water infiltration in unsaturated soils is expressed by a scaling factor defined as the capillary length, λ_c (L) (Philip, 1985a). λ_c is equal to the inverse of a parameter α (L^{-1}) introduced by Gardner (1958) as a general description of soil textural and structural characteristics. α tends to be small in fine-textured soils with capillarity dominant, while large in coarse-textured soils where gravity is important. Typically, α is about 1 m^{-1}, and the range 0.1–5 m^{-1} can be representative of the full moisture range in real soils (Philip, 1984a). Elrick and Reynolds (1992) suggested four values, that is, $\alpha = 36,\ 12,\ 4,$ and 1 m^{-1} for practical use of permeameters and infiltrometers in soils varying from coarse sands to compacted clays. The reader should note that the larger values (i.e., 12 and 36 m^{-1}) are for "sand fractions" and/or apply over small capillary ranges.

Core Ideas

- A total of 138 unique infiltration models were identified, spanning a wide range of theoretical and empirical boundaries.
- The proliferation of infiltration models created challenges for researchers in identifying which models to use.
- Extracting Ksat and sorptivity from infiltration data becomes a challenge with the increased number of models.
- Most of the developed infiltration models clustered theoretically around few early major milestones

To determine water infiltration into soils, it is necessary to impose appropriate boundary conditions. Such boundaries may be classified as Dirichlet and Neumann boundary conditions.

1. The Dirichlet boundary condition may be expressed in terms of pressure head or water content. Pressure head is typically controlled and can be constantly negative, equal to zero, constantly positive, or changing (typically falling from some positive value to some lesser positive value or zero) in a furrow, lake, or river.
2. The Neumann boundary condition may be expressed in terms of water flux. This boundary condition specifies the rate at which water is entering or leaving the system, rather than the water content or pressure directly. It is typically found in sprinkling irrigation or rainfall-based measurements.

The boundary conditions may change from prescribed flux to prescribed head type conditions (and vice versa). For instance, when precipitation or irrigation rate q_0 exceeds infiltration capacity of the soil, ponding will occur. In this case, infiltration rate is no longer controlled by precipitation rate but instead by infiltration capacity of soil.

Furthermore, soil characteristics greatly affect water infiltration. Some soil characteristics of importance include:

1. Rigid versus deformable or shrink-swell soils
2. Isotropic versus anisotropic soils

3. The degree of heterogeneity of soils
4. Hydrophobicity
5. Presence of multi-porosity domains
6. Soil layering
7. Slope

Infiltration models are mostly designed to characterize infiltration into idealized soils, that is, rigid, homogeneous, isotropic, and non-hydrophobic. The cumulative infiltration curves, obtained by plotting the cumulative infiltration as function of time, reflect a characteristic concave shape demonstrating higher infiltration capacity at early stages that reaches a constant slope at steady state. However, real soils can have cumulative infiltration curves that exhibit convex, mixed, or non-standard shapes (e.g., Chen et al., 2020; Abou Najm et al., 2021; Pachepsky and Karahan, 2022).

Altogether, the physics underlying infiltration processes are highly complex. To select and apply appropriate model(s), it is important to understand how variables such as flow direction and dimensionality, driving forces, non-uniformities in drivers and media, and boundary conditions all interact. To guide the model selection process, we emphasized, in the first section of our review, the underlying physics of infiltration processes that encompass dimensions and directions, forces, processes, homogeneity versus heterogeneity and nonuniform flow drivers, as well as a brief discussion of common boundary conditions and dominant soil processes as could be inferred from field conditions and the shape of the cumulative infiltration curve. In the following sections, we provided a narrative of the history and evolution of major infiltration models, divided into (1) empirical models and (2) conceptual models divided as macro-scale depictions of flow in rigid versus deformable soils. Recently, modeling infiltration shifted from use of analytical to numerical methods, with the emergence of advanced computers.

3 | EVOLUTION OF THE BASIC FLOW MODELS FOR RIGID SOILS

We summarized basic and fundamental physical flow models used as the foundations for modeling infiltration in rigid (non-swelling) soils (Figure 1). Below, we present the equations that underpin most of the infiltration models reviewed in this paper.

The Navier–Stokes equation (Navier, 1823; Stokes, 1850) described laminar motion of incompressible fluids and could be used to describe fluids flowing through homogeneous and smooth surfaces in both horizontal and vertical directions:

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}.\nabla\right)\vec{v}\right) = -\nabla P + \rho\vec{g} + \mu\nabla^2\vec{v} \qquad (1)$$

where v (L T^{-1}) is the velocity, $\frac{\partial \vec{v}}{\partial t}$ (L T^{-2}) is the change of velocity with respect to time t (T), ∇P (M L^{-2} T^{-2}) is the pressure gradient, ρ (M L^{-3}) is the fluid density, μ (M L^{-1} T^{-1}) is the dynamic viscosity of the fluid, and g (L T^{-2}) is the acceleration of gravity, and ∇ stands for the nabla or del operator, $\nabla = \frac{\partial}{\partial x}\overrightarrow{e_x} + \frac{\partial}{\partial y}\overrightarrow{e_y} + \frac{\partial}{\partial z}\overrightarrow{e_z}$.

Based on Navier's work, Poiseuille (1844) derived a physical law, known as Hagen–Poiseuille equation, that describes the pressure drop, ΔP (M L^{-1} T^{-2}), in an incompressible and Newtonian fluid with laminar volumetric flow, Q (L^3 T^{-1}), flowing through a cylindrical pipe of length l (L) and radius r (L):

$$Q = \frac{\pi \Delta P r^4}{8\mu l} \qquad (2)$$

While the concept of infiltration was discussed and described in the literature before Darcy's work, Darcy (1856) formulated the first empirical quantitative description of flow through a saturated porous medium, known as **Darcy's law**, based on water infiltration experiments through sand beds, valid for both horizontal and vertical flow:

$$q = -K_s \nabla h \qquad (3)$$

where q (L T^{-1}) is the flux, K_s (L T^{-1}) is the saturated hydraulic conductivity, and ∇h (L L^{-1}) is the hydraulic gradient describing the difference in total hydraulic head h (L) between any two points within the porous medium.

A few decades after Darcy, Buckingham (1907) built on the earlier work of Lyman Briggs, his senior colleague at the U.S. Bureau of Soil, who introduced capillary flow due to capillary gradient. Buckingham's work resulted from deviations of the equilibrium between gravity and capillarity in unsaturated soils (Briggs, 1897). More about this development can be found in Germann (2021). Buckingham, who was unaware of the work of Darcy (Beven, 2018), extended Briggs' concepts and drew an analogy with Ohm's law and Fourier's law to define unsaturated water flow as a function of a gradient of attraction and a constant of proportionality, namely "capillary conductivity," based on two main assumptions:

1. The driving force for water flow in isothermal, rigid, unsaturated soil containing no solute membranes, and zero air pressure potential is the gradient between two points of the combination of matric ψ (L) and gravitational z (L) potentials, that is, $h = \psi + z$
2. The hydraulic conductivity of unsaturated soil is a function of water content θ (L^3 L^{-3}) or matric potential ψ (L).

Based on Buckingham's approach, vertical and 3D infiltration processes are generally considered to be in response to a combination of gravity and capillarity, and horizontal

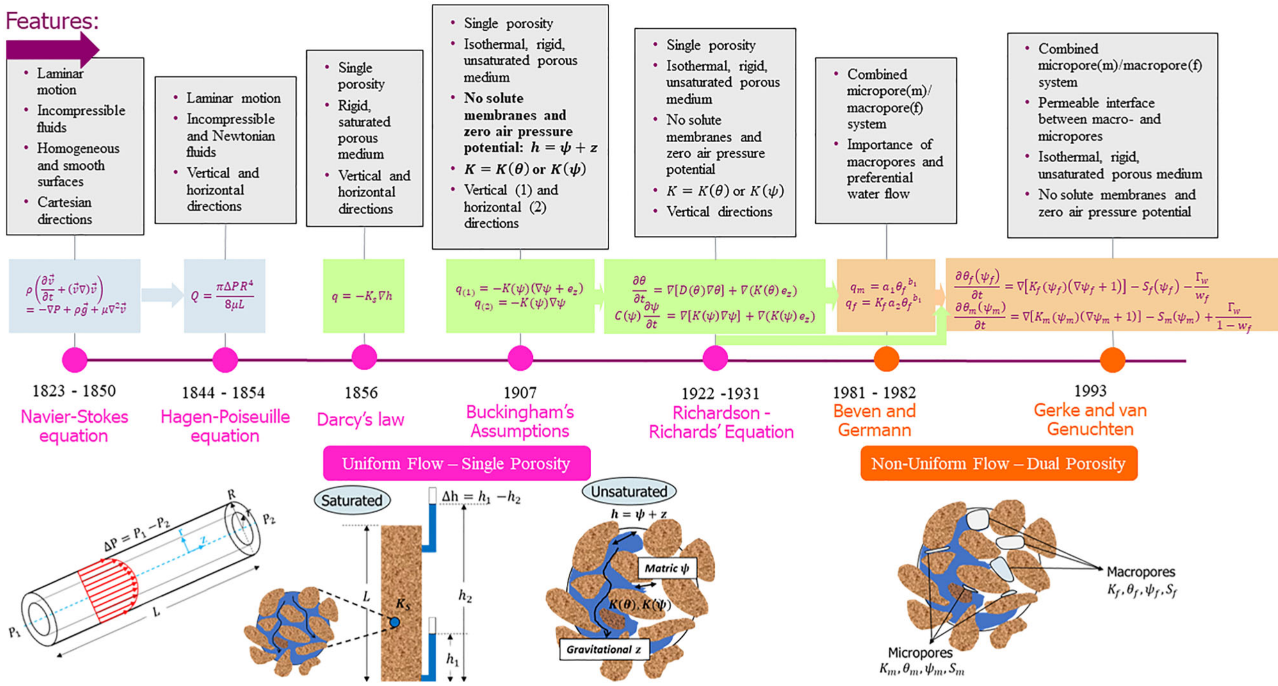

FIGURE 1 Evolution of the basic flow models for rigid (i.e., non-swelling) soils. It is worth noting that nonuniform preferential flows were recognized long before the equilibrium concepts of Buckingham and Richardson–Richards (Brewer, 1960; Kubiena, 1938; Lawes et al., 1882; Schumacher, 1864). However, the nonuniform flow was not modeled until early 1980s.

infiltration is considered to be driven exclusively by capillarity. To this end, Buckingham's assumptions have become the fundamental "depictions of the underlying physics".

$$q = -K(\psi)\left(\nabla\psi + e_z\right) = -K(\theta)\left(\nabla\psi + e_z\right) \text{ vertical flow} \tag{4}$$

$$q = -K(\psi)\ \nabla\psi = -K(\theta)\,\nabla\psi \text{ horizontal flow} \tag{5}$$

where $\nabla\psi$ (L L^{-1}) is the matric hydraulic gradient, and $K(\psi)$ and $K(\theta)$ (L T^{-1}) are the unsaturated hydraulic conductivities as respective functions of ψ (L) and water content θ (L^3 L^{-3}), and e_z stands for the vertical axis. Equations (4) and/or (5) can be viewed as a generalization of Darcy's Law and can be referred to as the Buckingham–Darcy law.

In 1911, Green and Ampt modeled the capillary force resulting from the matric potential of unsaturated soils as a single-valued wetting front potential. Green and Ampt (1911) assumed a wetting front that goes downward at constant water content and, thus, constant matric potential ψ (L), as well as constant hydraulic conductivity K (L T^{-1}), since, in their conceptualization, water always moves within a saturated soil. The Green–Ampt (GA) model works best when a relatively sharp wetting front exists throughout the infiltration process. Such a distinct wetting front was adopted later by several infiltration models, such as those developed by Beven (1984), Li et al. (1976), Mein and Larson (1973), Selker et al. (1999a), Swartzendruber (2000), Selker and Assouline (2017), and Stewart (2019). The GA model has proven to be a useful framework for understanding and predicting infiltration.

The next advancement, in theory, was to extend the sharp wetting front theory to nonsteady conditions, in which soil water content can change over time and space. Two different scientists (Richardson, 1922; Richards, 1931) independently developed similar solutions to this problem through a second-order convection-diffusion equation, conventionally known as Richards' equation (Equation 6 or 7). However, the resulting equation should rightly be called the **Richardson–Richards' equation**. Unfortunately, Richardson's work was barely noticed in the soil physics or hydrology literature, most likely because it was presented as part of a book on weather prediction (Raats & Knight, 2018).

As Richards (1931) attempted to extend Darcy–Buckingham work, he defined "the essential difference between flow through a porous medium which is saturated and flow through a medium which is unsaturated is that under the latter condition the pressure is determined by capillary forces and the conductivity depends on the moisture content of the medium" (Richards, 1931, p. 323). However, Richards constrained his derivation to low-pressure gradients and highlighted the need for different model approximations for high-pressure gradients: "For low pressure gradients it has been found by numerous investigators that this law (Darcy) is in exact agreement with experiment, and it is entirely analogous to the well-known law of Poiseuille for the flow of liquids through capillary tubes. However, both of these laws fail to hold for high pressure gradients. The limit within which they are true and the modifications which a second approximation requires can be determined only by exhaustive experiments on a wide range of materials" (Richards, 1931). The validity problem of Richards' assumption is related to the validity of Darcy's law. Very large hydraulic gradients will induce inertial forces that could dominate the flow, leading to larger Reynolds number (the ratio of inertial to viscous forces) beyond the range of validity of Darcy's law.

The Buckingham–Richarson-Richard's (BRR) equation can be derived by combining Darcy's equation and conservation of mass (along with Buckingham's paradigm under isothermal conditions in a homogeneous and rigid porous medium).

$$\frac{\partial \theta}{\partial t} = \nabla \left[D(\theta) \nabla \theta \right] + \nabla \left(K(\theta)\ e_z \right) \quad (6)$$

The equation is also written in terms of head:

$$C(\psi) \frac{\partial \psi}{\partial t} = \nabla \left[K(\psi) \nabla \psi \right] + \nabla \left(K(\psi)\ e_z \right) \quad (7)$$

The soil infiltration characteristics are hydraulic conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1}), hydraulic diffusivity $D(\theta) = K(\psi)\frac{\partial \psi}{\partial \theta}$ (L^2 T^{-1}), and water-holding capacity $C(\psi) = \frac{\partial \theta}{\partial \psi}(1\ L^{-1})$.

The concept of water movement as a diffusion phenomenon, implicit in Buckingham's approach, and later in that of Richards as well as Green and Ampt's model, was explicitly proposed by Childs (1936a,1936b) who studied the hypothesis of constant diffusivity. Later, Childs and Collis-Georges (1950) introduced the concept of concentration-dependent diffusivity characterized by the strong dependence of diffusivity on water content. Embracing this concept, Philip (1957a) developed, based on a Taylor Series expansion of Equation (6), the first specific quasi-analytical solution of the nonlinear Richards' equation, in the form of a power series in $t^{1/2}$ describing infiltration in a homogeneous, isotropic, and rigid porous medium. However, Philip's time expansion series converges only for finite t; thus, the solution becomes unreliable at infinite times. To this end, Philip (1957b) approximated his equation by the two first terms, describing one-dimensional, unsaturated infiltration for relatively short times. At this point, he defined the first term as "sorptivity" or "a term embracing both absorption and desorption". In his words, it is "a measure of the capillary uptake or removal of water." Later, in 1969, Philip showed that sorptivity can be determined from horizontal infiltration where water flow is mostly controlled by capillary absorption (Philip 1969a). The second term in Philip's two-term equation reflects gravity effects and can be assumed equal to the saturated hydraulic conductivity multiplied with a constant between 1/3 and 2/3 (Davidoff & Selim, 1986; Ghorbani et al., 2009; Swartzentruber & Young, 1974).

Following Philip's work, Parlange obtained, in a series of papers, a quasi-analytical solution for water infiltration, by first developing a one-dimensional absorption equation (Parlange, 1971a), which was then extended to infiltration to include gravity effects (Parlange 1971b), and further extended in subsequent papers to problems in two and three dimensions (Parlange 1971c, 1972a, 1972b).

The series of papers published by Philip and Parlange more than 50 years ago remains today the basis of our understanding of infiltration theory. However, a serious limitation of applying their quasi-analytical solutions emerges from the representativity of the initial and boundary conditions. Philip and Parlange's solutions often assume uniform initial water content throughout the soil profile. As for boundary conditions, these models assume a constant value of water pressure head at the soil surface. However, in natural settings, initial and boundary conditions can be highly variable with changing precipitation intensity, intermittent irrigation, and varying soil properties.

Later, the analysis of the highly nonlinear flow problem has been simplified and made tractable by applying the matric flux potential using Kirchhoff transformation. Raats (1971) defined the matric flux potential, φ (L^2 T^{-1}), by: $\varphi = \int_{\psi_i}^{\psi} K(\bar{\psi}) d\bar{\psi} = \int_{\theta_i}^{\theta} D(\bar{\theta}) d\bar{\theta}$; where subscript i denotes the initial state. Using this definition, Redinger et al. (1984) and Campbell (1985) were the first to apply the Kirchhoff transform (K transform) of the matric potential ψ (L) to linearize the second-order spatial term in Richards' equation (Equation 7). Their method introduced a simple but efficient method of solution for homogeneous unsaturated soils. However, K transform depends on the soil's hydraulic properties and will therefore vary spatially with any spatial change in the hydraulic properties. Therefore, to characterize scale-heterogeneous soils, Ross (1990) used an inverse hyperbolic sine transform of ψ (L) in preference to the K transform. Later, Ross and Bristow (1990) capitalized on the numerical advantages of using the K transform to solve Richards' equation for water flow in scale-heterogeneous soils when the appropriate correction to spatial changes in soil properties is made.

Clearly, the BRR equation became the mainstream approach for modeling flow in unsaturated soils. Researchers developed simplified analytical approximations following Richards' paper, and later on, thanks to advancements in numerical computation, developed robust numerical models, particularly the HYDRUS software packages, which simulate water, heat, and solute movement in 1-D, 2-D, or 3-D porous media and provide numerical solutions to Richards' equation (Šimůnek et al., 1998, 1999, 2005, 2006). HYDRUS uses linear finite elements to numerically solve Richardson–Richards' equation for saturated or unsaturated water flow under uniform or nonequilibrium conditions, as well as Fickian-based advection–dispersion equations for solute transport. In 2008, Simunek and his coworkers described the entire history of the development of the various HYDRUS programs with related models and tools (Šimůnek et al., 2008). Later, Šimůnek et al. (2016) reviewed recent developments and applications of the HYDRUS program implemented after 2008.

In fact, the ubiquity and ease-of-use of Richards-based model frameworks transitioned research in the area of infiltration and unsaturated flow almost exclusively from a theoretical search for a better infiltration theory into an expanded field of highly applied and practical research. Therefore, many researchers have been applying Richards' equation to understand processes or develop solutions addressing environmental, agricultural, and ecological issues like irrigation efficiency, pollutant and nutrient transport, heat transfer, water recharge, and stormwater management (Lassabatère et al., 2010; Kandelous & Simunek, 2010; Goutaland et al., 2013; Slimene et al., 2017; Stewart et al., 2017; Autovino, 2018; Coppola et al., 2019; Fields et al., 2020).

Despite its widespread adoption, Richards' equation fails to adequately describe infiltration in different flow situations and scales (e.g., Beven, 2018; Germann, 2021 Hunt et al., 2013). Fundamentally, the Richards' equation was developed using equilibrium-based measurements, such that equilibrium between soil pressure head and water content is instantaneous, opposite to nonequilibrium conditions that occur during rapid flow (Šimůnek et al., 2003). Based on the theoretical dominance of capillary forces in unsaturated flow, Richards proposed an elegant hierarchy of active–inactive pores in which larger pores empty, while smaller pores conduct water in partially saturated soils. He stated that if "there is a steady flow of liquid through a porous medium which is only partially saturated, then the larger pore spaces contain air and the effective cross-sectional area of the water conducting region is reduced" (Richards, 1931). This hierarchy completely deactivates those larger pores "If these air spaces could in some way be filled with solid, the condition of the flow would be unchanged" (Richards, 1931). On the other hand, Germann's last published work was critical of Richards experiment "which relied on the application of air pressure to ensure that larger pores were empty at each potential and flow rate, was simply the wrong experiment for flows under more natural conditions" (Germann, 2021). Germann's stated that "while capillary flow relies on the strongest force extended on water in an unsaturated soil, the weaker force of viscosity can dominate during infiltration as a result of formation of film flows at and near the soil surface" (Germann, 2021). Similar criticism was levied by Hunt et al. (2013) and Beven (2018).

The inadequacies of the Darcy-BRR framework were also becoming apparent as interest increased in depicting preferential flow processes through a wide range of theoretical advancements (Jarvis, 2007; Šimůnek et al., 2003; Šimůnek et al., 2008). In particular, studies were increasingly noting pollutant and nutrient transport processes that were happening at faster rates or over greater distances than seemed possible based on the depictions provided by BRR-based models (Beven, 2010; Reichenberger et al., 2002; Zehe and Fluhler, 2001; Zheng and Gorelick, 2003). Other work at hillslope and watershed scales also indicated that BRR-based models did not accurately capture soil wetting and streamflow responses to precipitation, even when treated with parameter values deemed as "effective" for the larger scales (James et al., 2010; Nimmo et al., 2021; van Schaik et al., 2008).

These issues were first raised by Beven and Germann (1982), who demonstrated the importance of macropores and preferential water flow in soils, as well as identified the need for future comprehensive characterization of flow that goes beyond Richards' equation. Beven and Germann's pioneering work (1982) led to mainstream recognition and acknowledgment of the limitations of current physical theory. Thirty years later, many of the same discrepancies persisted, as noted by the revised work in Beven and Germann (2013).

The first theoretical attempt at addressing the discrepancies of Darcy-BRR framework was held by Beven and Germann (1981), who described flow in high-permeability domains by the kinematic wave approach. To this end, Beven and Germann (1981) developed a one-dimensional model of bulk flow in a combined micropore/macropore system by approximating the micropore (subscript m) flow, q_m (L T^{-1}), for various arrangements of macropores (subscript f) at different water contents, θ_f (L^3 L^{-3}):

$$q_\mathrm{m} = a_1\, \theta_f{}^{b_1} \tag{8}$$

And the flow in macropores, q_f (L T^{-1}), by:

$$q_f = K_f\, a_2 \theta_f{}^{b_1} \tag{9}$$

where K_f (L T^{-1}) is the hydraulic conductivity of the macropores, a_1 (L T^{-1}), a_2, and b_1 (dimensionless) are the fitting parameters.

Recognizing the importance of preferential and nonequilibrium flow that is not captured by Darcy-BRR framework, Gerke and van Genuchten (1993) transformed Richards' equation into a dual permeability model that describes two single-permeability media, one associated with macropores (fracture and inter-porosity domain) of high permeability and the other associated with micropores (matrix and intra-porosity domain) of low permeability, with exchange possible through a permeable interface. This approach assumed constant fluid densities, no hysteresis in the hydraulic properties, and no effects of swelling, shrinking, temperature, air pressure, and solute concentration on water flow. Based on Richards' equation, Gerke and van Genuchten (1993) model uses the flow equations for the fast-flow region (subscript f) and the matrix (subscript m), respectively, as follows:

$$\frac{\partial\theta_f(\psi_f)}{\partial t} = \nabla\left[K_f(\psi_f)(\nabla\psi_f + \mathrm{e}_z)\right] - s_f(\psi_f) - \frac{\Gamma_w}{w_f} \tag{10}$$

And

$$\frac{\partial\theta_m(\psi_m)}{\partial t} = \nabla\left[K_m(\psi_m)(\nabla\psi_m + \mathrm{e}_z)\right] - s_m(\psi_m) + \frac{\Gamma_w}{1 - w_f} \tag{11}$$

where $s(\psi)(\mathrm{T}^{-1})$ is the sink–source term, w_f ($\mathrm{L}^3\ \mathrm{L}^{-3}$) is the ratio of the volume occupied by the fast-flow region and relative to the total volume, and Γ_w (T^{-1}) is a space- and time-dependent exchange term describing the transfer of water between the two pore systems.

Other further approaches for modeling preferential and nonequilibrium flow in the vadose zone have used the kinematic approach (Jarvis, 1994; Jarvis & Larsson, 1998; Larsbo & Jarvis, 2003), Darcy-BRR (Lewandowska et al., 2004), or Green and Ampt (Stewart, 2018; Weiler, 2005) by describing water movement in the soil matrix in combination with a depiction of water flow through macropores.

4 | EVOLUTION OF BASIC FLOW CHARACTERIZATION FOR DEFORMABLE SOILS

Many soils have clay minerals or other particles (e.g., organic substrates) that change volume when wetting and drying, which causes the soil matrix to deform and typically leads to excessive error when infiltration models developed for rigid soils are applied. Therefore, substantial research efforts have been devoted to characterizing water movement through deformable soils with time-variable pore structures. Here, we list the basic conceptual models developed to characterize water flow into deformable soils (Figure 2).

Using Darcy's law, Biot (1955) first described the flow of fluids in deformable anisotropic soils by studying the problem of soil consolidation through the mixture theory in the absence of gravity effects:

$$v_{\mathrm{liquid}} - v_{\mathrm{solid}} = -\frac{k}{\theta\mu}\nabla P \tag{12}$$

where v_{liquid} and v_{solid} are the vertical velocity components of the liquid and solid phases, respectively, and k (L^2) is the solid permeability.

Then, mass conservation of the solid (Equation 13) and liquid (Equation 14) phases, yielded (Bowen, 1980):

$$\frac{\partial\theta}{\partial t} - \nabla\left[(1-\theta)\,v_{\mathrm{solid}}\right] = 0 \tag{13}$$

$$\frac{\partial\theta}{\partial t} + \nabla\left(\theta v_{\mathrm{liquid}}\right) = 0 \tag{14}$$

Later, researchers' attention was skewed toward characterizing soils that shrink and swell in volume when drying and wetting, namely, shrink-swell soils. To understand the behavior of shrink-swell soils, the soil shrinkage curve (SSC) was developed describing the change in soil volume as function of the change in water content from a fully saturated state to a completely dry state. The SSC is generally sigmoidal in shape and is represented by numerous models (McGarry and Malafant, 1987; Tariq and Durnford, 1993; Braudeau et al., 1999; Crescimanno and Provenzano, 1999; Chertkov 2000, 2003; Peng and Horn, 2005; Cornelis et al., 2006a, 2006b; Lu and Dong, 2017; Chen and Lu, 2018; Gupt et al., 2022).

Different modeling analogs of Richardson–Richards' equation were developed to study and solve the problem of infiltration into shrink-swell soils. For instance, Philip (1969b) modified Richardson–Richards' equation for rigid soil and developed the standard Fokker–Planck equation (FPE; Equation 15) describing flow in unsaturated swelling media:

$$\frac{\partial\theta}{\partial t} = \nabla\left[D(\theta)\nabla\theta\right] + \nabla\left[\left(1-\gamma_w\right)K(\theta)\ e_z\right] \tag{15}$$

where γ_w (dimensionless) is the wet specific gravity of the swelling soil.

Smiles and Raats (2005) reformulated the standard (FPE) by including the dimensionless constant, here called α_{swell}, defined as a water content-dependent value and as an average over a defined pressure range, to deal with curvilinearity in the shrinkage curve:

$$\frac{\partial\theta}{\partial t} = \nabla\left[D(\theta)\nabla\theta\right] + \nabla\left[\left(1-\alpha_{\mathrm{swell}}\gamma_w\right)K(\theta)\ \mathrm{e}_z\right] \tag{16}$$

Furthermore, a special form of the (FPE) was investigated by Su (2009), who considered the inclusion of a dimensionless parameter β_{swell}, the order of fractional derivative, which enables a clear explanation of the anomalous infiltration into swelling porous media:

$$\frac{\partial^{\beta_{\mathrm{swell}}}\theta}{\partial t^{\beta_{\mathrm{swell}}}} = \nabla\left[D(\theta)\nabla\theta\right] + \nabla\left[\left(1-\alpha_{\mathrm{swell}}\gamma_w\right)K(\theta)\ \mathrm{e}_z\right] \tag{17}$$

Another governing equation for water movement in swelling soils was provided by Giraldez and Sposito (1985) who used a generalized version of the Richardson–Richards equation:

$$\frac{\rho_b}{\rho}\frac{\partial\theta}{\partial t} = \nabla\left[D(\theta)\nabla\theta\right] + \nabla H \tag{18}$$

where ρ_b ($\mathrm{M\ L}^{-3}$) is the soil bulk density and ρ ($\mathrm{M\ L}^{-3}$) is the fluid density. H ($\mathrm{L\ T}^{-1}$) is a gravity-envelope-pressure parameter given by:

$$H = K(\theta)\left(1-\rho_{b,w}\bar{V}\right) \tag{19}$$

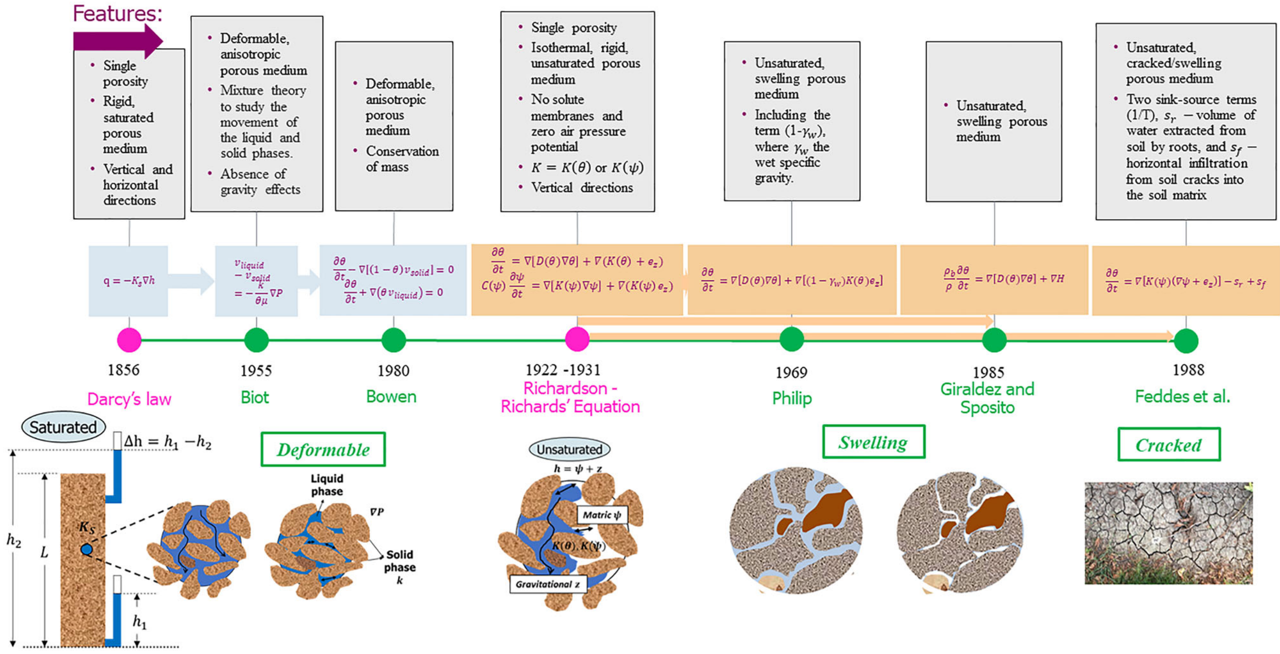

FIGURE 2 Evolution of the basic flow models for deformable (i.e., swelling) soils.

where $\rho_{b,w}$ (M L^{-3}) is the wet bulk density (subscript w) and $\bar{V}$ (L^3 M^{-1}) is the slope of the shrinkage curve.

Furthermore, Feddes et al. (1988) extended Richardson–Richards' equation to characterize water flow in swelling/cracked soils by adding two sink-source terms, s_r (T^{-1}) quantifying the volume of water extracted from soil by roots, and s_f (T^{-1}) quantifying the horizontal infiltration from soil cracks (subscript f) into the soil matrix:

$$\frac{\partial\theta}{\partial t} = \nabla\left[K(\psi)\left(\nabla\psi + \mathrm{e}_z\right)\right] - s_r + s_f \tag{20}$$

Shifting from Richardson–Richards' equation, Davidson (1984), and later Weiler (2005), built on the classic GA model to characterize infiltration into macroporous soils. Neither approach, however, describes the dynamic shrinkage and swelling processes. In response, Stewart (2018) combines the GA model with a multidomain framework previously developed by Stewart et al. (2016), allowing for variations in properties of the different porosity domains, and therefore a better characterization of the dynamic properties of shrink-swell soils.

Another approach toward modeling water movement through shrink-swell soils was developed by Bennethum and Cushman (1996) who derived a new constitutive theory for multiphase, multicomponent, three-scale, and swelling systems with interfaces. This effort resulted in a generalized form of Darcy's law where flow is governed by gravity and the Gibbs free energy, temperature, and concentration gradients:

$$q = \kappa^L \left[-\phi^L\rho^L\nabla\left(\tilde{A}^L - \frac{1}{\rho^L}\sigma\right) + \left(\frac{1}{\rho^L}\sigma^L + \frac{1}{\rho^L}P^L\right)\nabla \left(\phi^L\rho^L\right) + \phi^L\rho^L g^L - \phi^L\rho^L \mathrm{E}^L \nabla T^0 + \sum_{j=1}^{N-1}\phi^L\rho^L\tilde{\mu}^{L_j}\nabla C^{L_j}\right] \tag{21}$$

where $L = lA$ refers to the fluid phase in the particle at the macroscale, B and C refer to the liquid and air phase at the mesoscale, respectively, κ (L^3 T M^{-1}) is the second-order positive semi-definite tensor, ϕ (L^3 L^{-3}) is the volume fraction, ρ (M L^{-3}) is the fluid density, E (L^2 T^{-2} T$^{0(-1)}$) is the entropy, $\tilde{A}$ (L^2 T^{-2}) is the specific Helmholtz free energy, ∇T^0 (T°) is the temperature gradient, $\tilde{\mu}$ (L^4 M T^{-2}) is the chemical potential, and ∇C (M L^{-4}) is the concentration gradient.

5 | PORE-SCALE MODELS

The infiltration process in soils through which air is replaced by water is typically investigated using Richards' equation at the continuum scale, which requires the determination of soil hydraulic properties, such as water retention and unsaturated hydraulic conductivity curves. During infiltration within the complex structure of soils, one observes phenomena such as irregular wetting front and hydraulic non-equilibrium that are controlled by pore-scale characteristics of soils. Therefore, to better understand infiltration at the continuum scale, one should first investigate it at the pore scale—the scale of processes taking place within porous media. Different visualization tools were used to investigate the pore scale such as X-ray microcomputed tomography, confocal microscopy, and optical microscopy (Geistlinger & Ataei-Dadavi, 2015; Krummel et al., 2013; Liu and Song, 2015; Song et al., 2020). Magnetic resonance imaging has also been used to detect tracers and characterize complex flows in soils including heterogeneous systems (e.g., Lehoux et al., 2016; Raimbault et al., 2021).

Several pore-scale approaches emerged to address fluid flow and transport in the complex geometry and topology of porous media, as well as to solve physics within the given domain at the pore scale (Liu et al., 2006; Martys & Hagedorn, 2002; Valvatne & Blunt, 2004; Wilkinson, 1984; Prodanovic & Bryant, 2006; Blunt et al., 2013). The most widely used approaches for pore-scale modeling are lattice Boltzmann methods, smoothed particle hydrodynamics approach, computational fluid dynamics-based techniques, and pore-network models (Blunt, 2017).

Here, we present a brief summary of the emergence of pore-scale models and their co-evolution to broaden our review aspects, as well as our understanding of the modeling and application of flow into porous media. Mapping the pore space of porous media into a network of connected pores was first developed by Fatt (1956a, 1956b, 1956c) who stressed the inadequacy of parallel tube models to describe the complex structure of soils and rocks. Fatt (1956a) proposed a regular 2D network of tubes with various pore size distributions, which evolved to a ball-and-stick network proposed by Chandler et al. (1982) and Koplik (1982), and then to a biconical pore network proposed by Toledo et al. (1994). Recognizing that real porous media are three-dimensional, Rose (1957) developed the first computer-based network characterized by 3D lattices, which were further used to study hysteresis (Nicholson, 1968), dispersion (Torelli and Scheidegger, 1971), and waterfloods (Simon & Kelsey, 1971, 1972).

Since the 1980s up to now, research in pore-scale modeling evolved from the computation of simple two-phase flow processes and relative permeability (Blunt & King, 1991; Blunt et al., 1992; Heiba et al., 1984; Koplik & Lasseter, 1985; Larson et al., 1981; Mogensen & Stenby, 1998) to a huge range of pore-scale events, for example, wettability, three-phase flow, hysteresis, mass transfer between phases, and to a more accurate abstraction of porous media (Blunt, 2001; Blunt et al., 2002; Blunt & Scher, 1995; Bultreys et al., 2015; Dong & Blunt, 2009; Joekar-Niasar et al., 2008, 2009; Raoof & Hassanizadeh, 2010; Wilkinson & Willemsen, 1983).

For instance, Pomchaitaward et al. (2003) applied a lattice Boltzmann method to study capillary infiltration in porous spheres and cubes. They further validated their approach by comparing the results of their lattice-Boltzmann simulations with those obtained from a continuum model based on the kinetic of capillary infiltration. Later, Tzavaras et al. (2017) used pore-network modeling to simulate infiltration at the pore scale. In their study, the structure of pores and their topology were adapted from a loess soil sample. Using the same pore size distribution, they generated two types of pore networks: structured and random. The former was based on the measured topology, while the latter on the basis of random pore connection. Their pore networks, however, were only composed of $16 \times 16 \times 32$ nodes. The authors compared their pore-scale simulation from pore-network modeling with the continuum-scale simulations obtained by solving the Richards' equation. More specifically, the horizontally averaged dynamics of water content and water potential at the two scales were compared. Although reasonable agreements were found, the authors stated that assuming fluid phases were immiscible and incompressible led to unrealistic air trapping in the studied pore networks.

6 | PERCOLATION THEORY

Concepts from the percolation theory were widely applied in conjunction with network modeling to study flow and transport in complex porous media (Hunt et al., 2014; Hunt & Sahimi, 2017; Sahimi, 2011). Percolation theory addresses the effect of geometrical and topological properties of the pore space, particularly pore size distribution and connectivity at both small and large scales. An important feature within percolation theory is the presence of a percolation threshold (critical fraction) below which the network is not connected and, therefore, there is no macroscopic flow or transport. The basic concept of percolation threshold has also been broadly referred to in soil physics using other terms such as residual water saturation or critical air-filled porosity.

Early percolation models were based on networks (or lattices) composed of bonds and sites. Bonds act like links (or pore throats) connecting sites (or pore bodies). The early advancement in percolation theory for vertical downward infiltration was described by Glass and Yarrington (1996) as "gravity fingering in porous media" emphasizing that water mainly percolates due to gravity. More specifically, Glass and Yarrington (1996) proposed a modified invasion percolation approach for the immiscible displacement of a nonwetting fluid (e.g., air) by a wetting one (e.g., water). They further found that their modified invasion percolation model yielded substantially different structures in the wetting front compared to the standard invasion percolation. Within their framework, gravity and capillary fingering, as well as capillary facilitation, contributed to the determination of wetting front and its structure. In their own words, they stated that, "results suggest capillary forces to stabilize downward infiltration events either in very narrow or very wide pore-size distribution media. While this stabilization is intuitive for wide pore-size distributions, gravity fingering has generally been considered to dominate as the pore-size distribution narrows."

Later, Glass et al. (2000) expanded these efforts and developed a macroscale growth "structural" model to model invasion percolation at the near pore scale. Under non-negligible viscous forces conditions, Glass et al. (2000) redefined (1) the total pore filling pressure to incorporate viscous losses within the invading phase (e.g., water) from one hand and (2) the viscous effect to reduce randomness caused by capillary forces at the front. By comparing their simulations with CO_2-water experiments where viscous forces were negligible, Glass et al. (2000, 2001) found a fair agreement between simulations and experimental data. However, for trichloroethylene-water experiments, discrepancies between simulations and measured data were considerable. It is worth mentioning that the viscosity of trichloroethylene is nearly 40 times greater than that of CO_2.

Another approach toward modeling infiltration was developed by Hunt (1998) who presented random percolation theory and accordingly applied statistics of clusters showing that cumulative infiltration was analogous to electrical polarization. Using such an analogy, Hunt (1998) derived a general expression for cumulative infiltration including the two-term Philip (1957b) model as a special case (see his eq. 42).

More recently, Hunt et al. (2017) developed a theoretic model for vertical infiltration using concepts from random percolation theory. They proposed a two-term relationship for transient and steady-state infiltration. Hunt et al. (2017) assumed that the transient term describes solute transport under saturating conditions and the steady-state term represents advection fluid.

$$I\ (t) = \frac{x_0}{t_{x0}}\,t + \frac{x_0}{t_{x0}^{1/D_b}}\,t^{\frac{1}{D_b}} \tag{22}$$

where $D_b = 1.861$ represents the backbone fractal dimension in three dimensions (Hunt et al., 2014), x_0 denotes the characteristic length scale (e.g., typical pore diameter), and t_{x0} is the time required for fluid to traverse the distance x_0. By setting $D_b = 1.861$, the exponent in the transient term is found to be approximately 0.54 (= 1/1.861), which is close to the 0.5 exponent in Philip's equation (1957b). Using experimental data from Sharma et al. (1980), Hunt et al. (2017) scaled infiltration data and demonstrated that a much better agreement with actual data was obtained when infiltration data were scaled using the percolation theory exponent (e.g., 0.54). They concluded that the exponent 0.54 might be a better approximation than 0.5.

7 | BUNDLE OF CAPILLARY TUBES APPROACH

Most of the fundamental flow concepts (described in Sections 2 and 3) modeled water flow into porous media while characterizing the pore space with bundles of capillary tubes of either similar or varying radii. By simplifying the pore space topology, such models do not capture the fundamental randomness of porous media characterized by a wide distribution of interconnected channels and pore sizes. However, the different models based on bundles of capillary tubes developed later can still provide an improved representation of complex pore structures, while their formulation remains an open theoretical and experimental challenge.

One of the early infiltration models based on the bundle of capillary tubes approach was developed by Chu (1993). In his study, first soil water retention data were used to determine pore size distribution. Then, the Hagen–Poiseuille equation was applied to calculate hydraulic conductivity of each individual tube. Chu (1993) considered a scale factor to match theoretical and experimental saturated hydraulic conductivities. He generalized the GA model to describe water flow in all tubes determined from water retention data and then applied the van Genuchten model to derive a special case. Chu (1993) showed that his model captured spatial variation in the wetting front—an improvement over the traditional GA model, which assumes a constant depth front. In another study, Chu (1994) used the same approach but replaced the van Genuchten model with the Brooks–Corey model to represent water retention data. Using a Yolo light clay soil sample, he compared his theoretical results with those by the Philip model and found reasonable agreement between the two models.

Recently, a more advanced approach based on the bundle of capillary tubes was proposed to predict pore size distribution using non-Newtonian fluids offering new possibilities for improving porous media characterization (Abou Najm & Atallah, 2016; Atallah & Abou Najm, 2019; Basset et al., 2019; Hauswirth et al., 2019). Experimental evidence was presented validating the ability of a non-Newtonian fluid at different concentrations to infer the pore structure of simple and synthetic porous media using the Abou Najm Atallah (ANA) model (Atallah and Abou Najm, 2019; Hauswirth et al., 2019) and that of dual-porosity media using the dual-permeability ANA-2 model (Basset et al., 2019).

8 | SUMMARY OF THE INFILTRATION MODELS

Building on the fundamental physical flow models for rigid and deformable/swelling soils (Sections 2 and 3), several concepts were derived across the years to estimate infiltration for a specific porous media under specific boundaries and initial conditions. Empirical models were also developed describing infiltration with fitted data derived from either field or laboratory experiments. Here, we present a comprehensive summary of the evolution of 138 infiltration models. Our summary is by no means an exhaustive list of all the attempts of infiltration modeling in literature.

We illustrated the historical evolution of infiltration models as listed in Table 1, in an ascending historical order based on their year of publication. Table 1 summarizes those models based on their category, origin, and assumptions, as well as the type and behavior of infiltration. For additional information and formulation, Table 1 is further expanded into Tables A1 and A2 to describe the model parameters, equations, and concepts.

From a historic timeline demonstrating the evolution of knowledge from earlier theories into the most recent conceptual models based on citations (Figure 3), we identified areas that witnessed the most developments versus areas that have been understudied. Studies based on Darcy's law, the Richardson–Richards–Buckingham paradigm, and the Green–Ampt models are very common, while models based on Stokes' work and other earlier theories are areas for possible investigation and advancements.

One additional finding from our inclusive survey is that almost all infiltration models for early- and transient-time behavior, except for Abou Najm et al. (2021) and Di Prima et al. (2021), were designed to reflect typical infiltration results from infiltration experiments revealing a concave cumulative infiltration curve. The concave curve demonstrates higher infiltration capacity at early stages that reaches a constant slope at a steady state. This concave shape mimics what we call "perfect" infiltration conditions with no water-repellency, preferential flow, or any other factors present. However, different soils and field conditions can alter this behavior leading to cumulative infiltration curves that exhibit convex, mixed, or non-standard shapes. More recently, Pachepsky and Karahan (2022) analyzed the soil water infiltration global database called SWIG developed by Rahmati et al. (2018). By analyzing 5023 infiltration curves, they found 12 types of cumulative infiltration curve shapes. Nearly one-third of the SWIG database showed a non-classic shape. They applied a classification tree approach to divide those data into non-classic shapes. Their results showed that measurement method, clay content, and organic matter were among the most influential predictors of the shape type. In fact, non-classic shapes were previously reported and associated with soil structure and hydrophobicity (Angulo-Jaramillo et al., 2019). Given different land use and field conditions, different structural interactions come into play such as soil water-repellency, hydrophobicity, and preferential flow paths. As an attempt to model infiltration behavior covering the full range of shapes, Abou Najm et al. (2021) introduced a soil water-repellency parameter, α_{WR} (T^{-1}) that can be used with any infiltration model. α_{WR} accounts for water repellency using an exponential scaling factor that mimics the attenuation of infiltration rates observed at the start of infiltration (Figure 4). The α_{WR} family of models originated by Abou Najm et al. (2021) presented a macroscopic approach addressing water repellency (Di Prima et al., 2021; Yilmaz et al., 2022); other more microscopic and process-based approaches can be found in Shillito et al. (2020) and Hammecker et al. (2022).

Figure 5, which illustrates arc diagrams tracing the evolution of model advancements using the citations within the article in Table 1, further shows how conceptual models are interrelated and how newer models built on the foundations of previous models, particularly Darcy, Buckingham, Richardson–Richards, and GA. However, we noticed that starting around 1990, the citations of these classic works began to decrease. Instead, new models started to build on one another, citing only earlier works that were one or two steps away, but not many of those models compiled the entire chain of references to the original or fundamental source. This is evidenced by the fact that the blue arc/links should have been denser on the left side; instead, Figure 5 demonstrates the relatively small number of citations that the original flow papers received (e.g., Darcy or Buckingham), with the interesting exception of the GA model.

Therefore, the literature has tended to treat earlier classic models like Darcy and Richardson–Richards as accepted laws rather than fundamental studies to be cited. Another way to interpret this result is that the origins of many infiltration models became obscured and overlooked as model variations proliferated. To this end, Figure 5 clearly provides alarming evidence, not only for the observation that papers Darcy and Richardson–Richards are no longer cited regularly (of course we believe they should), but for what this can imply, which is a general belief, that flow theory in porous media is resolved and related macroscopic laws are utterly determined, and that the challenges are more on the application and mathematical interpretation of current theories.

To demonstrate this issue further, we present Table 2 that gathers some data from a literature search carried out in Google Scholar and Scopus. The first two columns in Table 2 represent the actual number of citations to the papers that we considered (column 1), whereas the last two columns refer to the number of mentions of those references in papers (with or without citations). Table 2 shows how the literature refers to Darcy and Richards' work using key terms such as law, model, equation, or theory and in many instances not citing them. For example, Darcy (1856) was cited 8067 times, but the terms Darcy law or Darcy model or Darcy equation were mentioned a total of 21,381 times in the literature. We noticed that the keyword "law" is used more commonly to represent Darcy,

TABLE 1 Summary of 138 infiltration models.

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Driving forces for infiltration	Cited (mentioned in text) origins
Green and Ampt (1911)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Darcy (1856); Buckingham (1907)
Kostiakov (1932)	Empirical	Arbitrary	Arbitrary	1D, local and field-scale, surface	Early and transient state						Experimental infiltration data
Horton (1940)	Empirical	Arbitrary	Arbitrary	1D, local and field-scale, surface	Early and steady-state						Experimental infiltration data
Mezencev (1948)	Empirical	Arbitrary	Arbitrary	1D, local and field-scale, Surface	Early, transient, and steady-state						Experimental infiltration data —Kostiakov (1932)
Hansen (1955)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 > 0$) [constant head]	No	K is constant within the transmission zone, then K within the wetting zone decreases toward the wetting front	Capillarity and gravity	[Darcy's law]
Philip (1957a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and transient (implicit)		x ($h_0 \leq 0$)	No	D and K are non-linear, strongly varying functions of θ	Capillarity and gravity	Buckingham (1907); Richards (1931)
Philip (1957b)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state (explicit)		x ($h_0 \leq 0$)	No	D approaches a delta function	Capillarity and gravity	Philip (1957a)

(Continues)

TABLE 1 (Continued)

						Boundary conditions		Assumptions			
Model	**Category**	**Soil type**	**Porosity domain**	**Type of infiltration**	**Infiltration behavior**	**Neumann (prescribed flux)**	**Dirichlet (prescribed water content or pressure)**	**Step-function moisture profile**	**Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})**	**Driving forces for infiltration**	**Cited (mentioned in text) origins**
Holtan (1961)	Empirical	Arbitrary	Arbitrary	1D, local and field-scale, surface	Early, transient, and steady-state						Experimental infiltration data
Overton (1964)	Semi-empirical	Arbitrary	Arbitrary	1D, local and field-scale, surface	Early, transient, and steady-state						Holtan (1961)
Huggins and Monke (1966)	Semi-empirical	Arbitrary	Arbitrary	1D, local and field-scale, surface	Early, transient, and steady-state						Holtan (1961)
Fok and Hansen (1966)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface (furrow)	Early, transient, and steady-state		x ($h_0 > 0$) [constant head]	No	K is the constant within the transmission zone, then within the wetting zone, K decreases toward the wetting front	Capillarity and gravity	Hansen (1955) [Darcy's law]
Philip (1967)	Conceptual	Rigid, scale-heterogeneous, anisotropic, non-hydrophobic, flat surface	Single	1D, field-scale, surface	Early and transient		x ($h_0 \leq 0$)	No	K varies spatially with ψ in a mutually consistent way	Capillarity and gravity	Richards (1931); Philip (1957a, 1957b) [Darcy's law]
Philip (1968)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, subsurface (point sources and spherical cavities)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Richards (1931) [Darcy's law]

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ ($L^2 T^{-1}$) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Wooding (1968)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface (shallow pond)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Richards (1931); Philip (1968) [Darcy's law]
Philip (1969a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, 2D, and 3D, local scale, surface and subsurface (cylindrical and spherical cavities)	Early, transient, and steady-state		x ($h_0 \leq 0$)	No	(A) D and K are non-linear, strongly varying functions of θ (B) D is constant, K represented by an exponential function of ψ (Gardner, 1958) (C) D approaches delta function, very sharp peak in K then K varies very slowly near saturation	Capillarity and gravity	Darcy (1856); Buckingham (1907); Richards (1931); Philip (1957a, 1957b, 1967, 1968)
Parlange (1971a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and transient		x ($h_0 \leq 0$)	No	D vary markedly with θ and rapidly near saturation	Capillarity	Philip (1957a, 1957b, 1969a)
Parlange (1971b)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$)	No	D vary markedly with θ and rapidly near saturation, very sharp peak in K then K varies very slowly near saturation	Capillarity and gravity	Philip (1957a, 1969a); Parlange (1971a)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ ($L^2 T^{-1}$) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Parlange (1971c)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D and 3D, local scale, surface	Early and transient		x ($h_0 \leq 0$)	No	Arbitrary	Capillarity	Philip (1969a); Parlange (1971a, 1971b)
Philip (1971)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D and 3D, local scale, surface and Subsurface (point and line sources)	Steady-state	x [constant rainfall rate]		No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Philip (1968, 1969a)
Parlange (1972a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D and 3D, local scale, Subsurface (cavity)	Steady-state		x ($h_0 \leq 0$)	No	D and K strongly depend on θ	Capillarity and gravity	Philip (1968, 1969a); Parlange (1971a, 1971b, 1971c)
Parlange (1972b)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D and 3D, local scale, subsurface (cavity)	Early and transient		x ($h_0 \leq 0$)	No	D and K strongly depend on θ	Capillarity and gravity	Philip (1969a); Parlange (1971a, 1971b, 1971c, 1972a)
Smith (1972)	Semi-Empirical	Arbitrary	Arbitrary	1D, local and field-scale, surface	Steady-state						Extensive numerical solutions of Richards (1931)
Philip (1972)	Conceptual	Rigid, scale-heterogeneous, anisotropic, non-hydrophobic, flat surface	Single	2D and 3D, field-scale, surface and subsurface (point and line sources)	Steady-state	x [constant rainfall rate]		No	D is constant, K depends exponentially on ψ and z	Capillarity and gravity	Philip (1968, 1969a, 1971)

(Continues)

TABLE 1 (Continued)

						Boundary conditions		Assumptions			
Model	**Category**	**Soil type**	**Porosity domain**	**Type of infiltration**	**Infiltration behavior**	**Neumann (prescribed flux)**	**Dirichlet (prescribed water content or pressure)**	**Step-function moisture profile**	**Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)**	**Driving forces for infiltration**	**Cited (mentioned in text) origins**
Talsma and Parlange (1972)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$)	No	(1) D approaches a delta function, very sharp peak in K then K varies very slowly near saturation (2) $dK/d\theta$ and D are proportional (3) D is constant	Capillarity and gravity	Philip (1957a, 1957b, 1969a); Parlange (1971a, 1971b)
Parlange (1972c)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x [constant rainfall rate]		No	D and K strongly depend on θ	Capillarity and gravity	Philip (1969a); Parlange (1971a, 1971b)
Mein and Larson (1973)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911) [Darcy's law]
Smiles (1974)	Conceptual	Swelling, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Transient state		x ($h_0 \leq 0$)	No	Arbitrary	Capillarity and gravity	Philip (1969a, 1969b) [Darcy's law]
Turner and Parlange (1974)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early state		x ($h_0 \leq 0$)	No	Arbitrary	Capillarity and gravity	Parlange (1971a); Talsma and Parlange (1972)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Swartzendruber (1974)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Mein and Larson (1973)
Morel-Seytoux and Khanji (1974)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, gravity, and air resistance	Green and Ampt (1911); Mein and Larson (1973) [Darcy's law]
Morel-Seytoux (1976)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x Before ponding [Arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	No	Arbitrary	Hydrostatic pressure, capillarity, gravity, and air resistance	Mein and Larson (1973); Morel-Seytoux and Khanji (1974) [Darcian sense]
Li et al. (1976)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)
Brutsaert (1977)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 \leq 0$)	Yes	D and K are represented by Averjanov–Irmay formula (1964)	Capillarity and gravity	Green and Ampt (1911); Richards (1931); Philip (1957a, 1969a); Parlange (1971a)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ ($L^2 T^{-1}$) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Collis-George (1977)	Empirical	Arbitrary	Arbitrary	1D, local and field-scale, surface	Early, transient, and steady-state						Experimental infiltration data
Hachum and Alfaro (1977)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state	x Before ponding [variable rainfall rate]	x After ponding [variable head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Mein and Larson (1973) [Darcy's equation]
Smith and Parlange (1978)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x Before ponding [arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	No	(1) D approaches a delta function, very sharp peak in K then K varies very slowly near saturation (2) $dK/d\theta$ and D increase rapidly and in similar fashion, K varies rapidly near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Philip (1957b, 1969a); Parlange (1971b); Talsma and Parlange (1972)
Batu (1978)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, surface (single and periodic strip sources)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Philip (1968, 1969a, 1971)
Kutilek (1980)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x Before ponding [constant rainfall rate]	x After saturation ($h_0 = 0$)	No	D approaches a delta function	Capillarity and gravity	Philip (1957a)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Assumptions: Driving forces for infiltration	Cited (mentioned in text) origins
Parlange (1980)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	(1) D approaches a delta function, very sharp peak in K then K varies very slowly near saturation (2) $dK/d\theta$ and D increase rapidly and in similar fashion	Hydrostatic pressure, capillarity, and gravity	[Darcy's law]
Parlange et al. (1982)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 \leq 0$)	No	D approaches a delta function, very sharp peak in K then K varies very slowly near saturation	Capillarity and gravity	Philip (1957a, 1969a); Parlange (1980)
Scotter et al. (1982)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Steady-state		x ($h_0 > 0$) [constant head]	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Hydrostatic pressure, capillarity, and gravity	Wooding (1968)
Fok et al. (1982)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local, surface (furrow)	Early, transient, and steady-state		x ($h_0 > 0$) [constant head]	No	K is constant within the transmission zone, then within the wetting zone K decreases toward the wetting front	Hydrostatic pressure, capillarity, and gravity	Hansen (1955); Fok and Hansen (1966) [Darcy's law]
Brakensiek and Rawls (1983)	Conceptual	Rigid, heterogeneous, two-layer (surface crust + subsoil), isotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early and Steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Assumptions: Driving forces for infiltration	Cited (mentioned in text) origins
Reynolds et al. (1983)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, subsurface (Well)	Steady-state		x ($h_0 > 0$) [constant head]	No	Constant saturated K	Hydrostatic pressure and gravity	[Darcy's law]
Novak and Soltesz (1984)	Empirical	Swelling/ cracked, heterogeneous, anisotropic, non-hydrophobic, flat surface	Multi	1D, Local and field-scale, surface	Early, transient, and steady-state						Experimental infiltration data
Fok and Chiang (1984)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, surface (furrow)	Early, transient, and steady-state		x ($h_0 > 0$) [constant head]	No	K is constant within the transmission zone, then within the wetting zone K decreases toward the wetting front	Hydrostatic pressure, capillarity, and gravity	Fok and Hansen (1966); Fok et al. (1982)
Philip (1984a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, subsurface (cylindrical cavities)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Philip (1957b, 1969a)
Philip (1984b)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, subsurface (spherical cavities)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Philip (1968, 1969a); Philip (1984a)
Beven (1984)	Conceptual	Rigid, Scale-Heterogeneous, Anisotropic, Non-hydrophobic, Sloping surface	Single	1D, field-scale, surface	Early and Steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, saturated K decreases as an exponential function of depth	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Mein and Larson (1973) [Darcy's law]

(Continues)

TABLE 1 (Continued)

						Boundary conditions		Assumptions			
Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Neumann (prescribed flux)	Dirichlet (prescribed water content or pressure)	Step-function moisture profile	Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Germann (1985)	Conceptual	Rigid, Heterogeneous, Isotropic, Non-hydrophobic, Flat surface	Multi	1D, local scale, surface	Steady-state	x [constant rainfall rate]		No	Kinematic K	Gravity	Beven and Germann (1981, 1982)
Warrick et al. (1985)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$)	No	Arbitrary	Capillarity and gravity	Richards (1931); Philip (1957a)
Reynolds et al. (1985)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, subsurface (well)	Steady-state		x ($h_0 > 0$) [constant head]	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Hydrostatic pressure, capillarity, and gravity	Scotter et al. (1982); Reynolds et al. (1983)
Parlange et al. (1985)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	$dK/d\theta$ and D increase rapidly and in similar fashion	Hydrostatic pressure, capillarity, and gravity	Parlange (1980)
Chu (1985)	Conceptual	Rigid, heterogeneous, three-layer (surface crust + till layer + subsoil), isotropic, non-hydrophobic, flat surface	Triple	1D, local scale, surface	Early and steady-state	x Before ponding [arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Brakensiek and Rawls (1983)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Assumptions: Driving forces for infiltration	Cited (mentioned in text) origins
Waechter and Philip (1985)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D and 3D, local scale, Subsurface (cylindrical and spherical cavities)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity (weak) and gravity	Philip (1957b, 1984a, 1984b)
Philip (1985b)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D and 3D, local scale, Subsurface (Cavities)	Steady-state	Any		No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Van de Hulst (1949); Philip (1957b, 1984a, 1984b); Waechter and Philip (1985)
Philip (1986a)	Conceptual	Rigid, homogeneous, isotropic and anisotropic, non-hydrophobic, flat surface	Single	3D, local scale, subsurface (spheroidal cavities)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Philip (1957b, 1984a, 1984b, 1985b)
Philip (1986b)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D and 3D, local scale, subsurface (discs, cylinders, spheres)	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Van de Hulst (1949); Philip (1957b, 1984a, 1984b, 1985b, 1986a)
Kutilek and Krejca (1987)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and transient state		x ($h_0 \leq 0$)	No	Arbitrary	Capillarity and gravity	Philip (1957a)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Driving forces for infiltration	Cited (mentioned in text) origins
Swartzendruber (1987a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and Steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Darcy (1856); Green and Ampt (1911)
Swartzendruber (1987b)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	Arbitrary	Hydrostatic pressure, capillarity, and gravity	Richards (1931); Philip (1957a)
Broadbridge and White (1988)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	No	D and K depend on a single free parameter C and readily measured soil hydraulic properties	Hydrostatic pressure, capillarity, and gravity	[Darcy–Buckingham approach]
Yeh (1989)	Conceptual	Rigid, heterogeneous, isotropic, non-hydrophobic, flat surface	Multi	1D, local scale, surface	Early and steady-state		x ($h_0 \leq 0$)	No	K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	[Buckingham]
Reynolds and Elrick (1990)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Steady-state		x ($h_0 > 0$) [constant head]	No	K represented by an exponential function of ψ (Gardner, 1958)	Hydrostatic pressure, capillarity, and gravity	[Darcy, Buckingham relationships]
Haverkamp et al. (1990)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	D approaches a delta function, $dK/d\theta$ and D increase rapidly and in similar fashion, K varies rapidly near saturation	Hydrostatic pressure, capillarity, and gravity	Parlange et al. (1985)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Smith (1990)	Conceptual	Rigid, heterogeneous, two-layer (surface crust + subsoil), isotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early, transient, and steady-state	x Before ponding [variable rainfall rate]	x After ponding [variable head $h_0 > 0$]	No	K represented as function of θ using Brooks and Corey (1964) and van Genuchten (1980)	Hydrostatic pressure, capillarity, and gravity	Philip (1957b); Parlange (1971b); Talsma and Parlange (1972); Smith and Parlange (1978) [Darcy's law], [Richards' equation]
Schmid (1990)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and Steady-state	x Before ponding [variable rainfall rate]	x After ponding [variable head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Mein and Larson (1973)
Ankeny et al. (1991)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Steady-state		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Wooding (1968)
Swartzendruber and Hogarth (1991)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	No	Arbitrary	Hydrostatic pressure, capillarity, and gravity	Richards (1931); Swartzendruber (1987b)
Philip (1992)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [Falling head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ ($L^2 T^{-1}$) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
White et al. (1992)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Steady flow		x ($h_0 \leq 0$)	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Wooding (1968) [Darcy's equation]
Barry et al. (1993)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	D represented as function of θ using Bruce and Klute (1956), K depends strongly on the soil moisture characteristic curve	Hydrostatic pressure, capillarity, and gravity	Richards (1931) [Darcy's law]
Fonteh and Podmore (1993)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, surface (Furrow)	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Hansen (1955); Fok and Chiang (1984)
Smith et al. (1993)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x Before ponding/post hiatus [arbitrary rainfall rate]	x During rainfall hiatus ($h_0 \leq 0$) After ponding [arbitrary head $h_0 > 0$]	No	K represented as function of θ using Brooks and Corey (1964)	Hydrostatic pressure, capillarity, and gravity	Parlange et al. (1982) [Darcy flow] [Richards' equation]
Philip (1993)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state	x Before ponding [arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Stone et al. (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Philip (1957a)
Mandal and Waechter (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, subsurface (cylinders)	Steady-state		x ($h_0 \leq 0$)	No	Arbitrary	Capillarity and gravity	Philip (1968, 1984a, 1984b, 1985b); Waechter and Philip (1985)
Fallow et al. (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant and Falling head]	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity	Philip (1957a, 1969a)
Basha (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, 2D and 3D, local scale, surface and subsurface	Early and steady-state	x Before ponding [arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	No	D is constant, K represented by an exponential function of ψ (Gardner, 1958)	Capillarity and gravity	Buckingham (1907); Richards (1931); Greenberg (1971)
Salvucci and Entekhabi (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Philip (1957a)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)	Assumptions: Driving forces for infiltration	Cited (mentioned in text) origins
Corradini et al. (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x [arbitrary rainfall rate]		No	K represented as function of θ using Brooks and Corey (1964)	Capillarity and gravity	Parlange et al. (1980); Smith et al. (1993) [Richards' equation]
Smettem et al. (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Early State		x ($h_0 \leq 0$)	No	D increases sharply with increasing θ, K varies rapidly near saturation	Capillarity	Parlange (1971a); Turner and Parlange (1974)
Haverkamp et al. (1994)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D and 3D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	$dK/d\theta$ and D increase rapidly and in similar fashion, K varies rapidly near saturation	Hydrostatic pressure, capillarity, and gravity	Parlange et al. (1982); Smetten et al. (1994); Haverkamp et al. (1990)
Barry et al. (1995)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	No	$dK/d\theta$ and D increase rapidly and in similar fashion, K varies rapidly near saturation	Hydrostatic pressure, capillarity, and gravity	Richards (1931); Philip (1957a); Parlange et al. (1980); Haverkamp et al. (1990)
Elrick et al. (1995)	Semi-empirical	Arbitrary	Single	1D, local and field-scale, surface	Transient and steady-state		x ($h_0 > 0$) [Falling head]				Experimental infiltration data; Richards (1931)
Sommer and Mortensen (1996)	Conceptual	Deformable, homogeneous, anisotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Liquid pressure or hydrostatic pressure, and capillarity	Biot (1955) [Darcy's law]

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Srivastava et al. (1996)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Mein and Larson (1973)
Preziosi et al. (1996)	Conceptual	Deformable, homogeneous, anisotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Liquid pressure or hydrostatic pressure, and capillarity	Darcy (1856); Bowen (1980)
Corradini et al. (1997)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x Before ponding/ post hiatus [arbitrary rainfall rate]	x During rainfall hiatus ($h_0 \leq 0$) After ponding [arbitrary head $h_0 > 0$]	No	K represented as function of θ using Brooks and Corey (1964)	Hydrostatic pressure, capillarity, and gravity	Parlange et al. (1985); Smith et al. (1993); Corradini et al. (1994) [Darcy's law] [Richards' equation]
Parlange et al. (1997)	Conceptual	Arbitrary	Arbitrary	1D, local scale, surface	Early and steady-state	Any		No	D approaches a delta function, very sharp peak in K then K varies very slowly near saturation	Hydrostatic pressure, capillarity, and gravity	Parlange et al. (1980) [Richards' equation]
Wu and Pan (1997)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	K represented by an exponential function of ψ (Gardner, 1958)	Hydrostatic pressure, capillarity, and gravity	Richards (1931); Warrick et al. (1985); Reynolds and Elrick (1990)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Wang et al. (1997)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, gravity, and air entrapment effects	Green and Ampt (1911)
Enciso-Medina et al. (1998)	Conceptual	Rigid, heterogeneous, three-layer (surface seal + till layer + subsoil), isotropic, non-hydrophobic, flat surface	Triple	1D, local scale, surface (furrow)	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911) [Darcy's law]
Philip (1998)	Conceptual	Rigid, heterogeneous, two-layer (surface crust + subsoil), anisotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	D and K are nonlinear functions of θ in the crust and in the soil	Hydrostatic pressure, capillarity, and gravity	Philip (1957a, 1957b, 1967, 1969a) [Darcy's law]
Smith et al. (1999)	Conceptual	Rigid, heterogenous, two-layer (surface crust + subsoil), isotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early, transient, and steady-state	x Before ponding/ post hiatus [arbitrary rainfall rate]	x During rainfall hiatus ($h_0 \leq 0$) After ponding [arbitrary head $h_0 > 0$]	No	K represented as function of θ using Brooks and Corey (1964);- Upper layer has always the lowest K	Hydrostatic pressure, capillarity, and gravity	Smith et al. (1993); Corradini et al. (1994, 1997) [Darcy's law] [Richards' equation]

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Assumptions: Driving forces for infiltration	Cited (mentioned in text) origins
Selker et al. (1999a)	Conceptual	Rigid, Scale-heterogeneous, anisotropic, non-hydrophobic, flat surface	Single	1D, Field-scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, saturated K function of depth following linear, power law and exponential relationship	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Beven (1984)
Wu et al. (1999)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	K represented by an exponential function of ψ (Gardner, 1958)	Hydrostatic pressure, capillarity, and gravity	Philip (1957a); Reynolds and Elrick (1990); Wu and Pan (1997) [Richards' equation]
Novak et al. (2000)	Conceptual	Swelling/cracked, heterogeneous, isotropic, non-hydrophobic, flat surface	Multi	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	Arbitrary	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Feddes et al. (1988) [Darcy's law] [Richards' equation]
Corradini et al. (2000)	Conceptual	Rigid, heterogeneous, two-layer (surface crust + subsoil), isotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early, transient, and steady-state	x Before ponding/post hiatus [arbitrary rainfall rate]	x During rainfall hiatus ($h_0 \leq 0$) After ponding [arbitrary head $h_0 > 0$]	No	K represented as function of θ using Brooks and Corey (1964); either layer may be less permeable	Hydrostatic pressure, capillarity, and gravity	Smith et al. (1993, 1999); Corradini et al. (1994, 1997) [Darcy's law] [Richards' equation]

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Driving forces for infiltration	Cited (mentioned in text) origins
Swartzendruber (2000)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [Variable head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Darcy (1856); Green and Ampt (1911)
Govindaraju et al. (2001)	Semi-analytical	Rigid, scale-heterogeneous (at the field scale), anisotropic, non-hydrophobic, flat surface	Single	1D, local and field-scale, surface	Early, transient, and steady-state	x Before ponding [arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	(1) Yes (2) No	(1) At the local scale, rapidly varying D, constant K near saturation, (2) At the field-scale, saturated K varies spatially and represented by a correlated lognormal random field	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)
Serrano (2001)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and Steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)
Corradini et al. (2002)	Semi-analytical	Rigid, scale-heterogeneous, anisotropic, non-hydrophobic, sloping surface	Single	1D, field-scale, surface	Early, transient, and steady-state	x Before ponding [arbitrary rainfall rate]	x Downslope after ponding (no run-on) (1) After ponding [arbitrary head $h_0 > 0$] (2)	No	(1) Saturated K varies spatially and represented as a log-distributed random variable with PDF (2) Effective Saturated K represented empirically	Hydrostatic pressure, capillarity, and gravity	Govindaraju et al. (2001)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ ($L^2 T^{-1}$) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Elrick et al. (2002)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and Steady State		x ($h_0 > 0$) [constant and Falling head]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Philip (1957b); Elrick et al. (1995)
Parlange et al. (2002)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and Steady-state		x ($h_0 = 0$)	No	D and K lie between the assumption of sharp wetting front and the assumption of D and $dK/d\theta$ proportional	Capillarity and gravity	Green and Ampt (1911); Philip (1969a); Talsma and Parlange (1972); Parlange (1980); Parlange et al. (1982)
Warrick et al. (2005)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and Steady-state		x ($h_0 > 0$) [Variable head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)
Weiler (2005)	Conceptual	Rigid, heterogeneous, isotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911) [Buckingham-Darcy law]
Govindaraju et al. (2006)	Semi-analytical	Rigid, scale-heterogeneous, anisotropic, non-hydrophobic, sloping surface	Single	1D, field-scale, surface	Early, transient, and steady-state	x Before ponding [Variable rainfall rate]	x Downslope after ponding (no run-on) After ponding [variable head $h_0 > 0$]	No	Saturated K varies spatially and represented as a log-distributed random variable with PDF	Hydrostatic pressure, capillarity, and gravity	Govindaraju et al. (2001); Corradini et al. (2002)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Assumptions: Driving forces for infiltration	Cited (mentioned in text) origins
Morbidelli et al. (2006)	Semi-analytical	Rigid, scale-heterogeneous, anisotropic, non-hydrophobic, sloping surface	Single	1D, field-scale, surface	Early, transient, and steady-state	x Before ponding [variable rainfall rate]	x After ponding [variable head $h_0 > 0$]	No	Saturated K varies spatially and represented as a log-distributed random variable with PDF	Hydrostatic pressure, capillarity, and gravity	Govindaraju et al. (2001, 2006); Corradini et al. (2002)
Lassabatère et al. (2006)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Transient and Steady-state		x ($h_0 \leq 0$)	No	K represented as function of θ using Brooks and Corey (1964)	Capillarity and gravity	Philip (1969); Smetten et al. (1994); Haverkamp et al. (1994)
Chen and Young (2006)	Conceptual	Rigid, infinitely deep, homogeneous, isotropic, non-hydrophobic, sloping surface	Single	1D, local scale, surface	Early and Steady-state	x Before ponding [arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911) [Darcy's law]
Warrick and Lazarovitch (2007)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, surface (strip source)	Early, transient, and steady-state		x ($h_0 > 0$) [constant head]	No	Arbitrary	Hydrostatic pressure, capillarity and gravity	Turner and Parlange (1974); Smetten et al. (1994); Haverkamp et al. (1994) [Richards' equation]
Warrick et al. (2007)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, surface (furrow)	Early, transient, and steady-state		x ($h_0 > 0$) [constant head]	No	Arbitrary	Hydrostatic pressure, capillarity and gravity	Haverkamp et al. (1994); Warrick and Lazarovitch (2007) [Richards' equation]

(Continues)

TABLE 1 (Continued)

						Boundary conditions		Assumptions			
Model	**Category**	**Soil type**	**Porosity domain**	**Type of infiltration**	**Infiltration behavior**	**Neumann (prescribed flux)**	**Dirichlet (prescribed water content or pressure)**	**Step-function moisture profile**	**Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})**	**Driving forces for infiltration**	**Cited (mentioned in text) origins**
Assouline et al. (2007)	Conceptual	Arbitrary	Arbitrary	Arbitrary	Early, transient, and steady-state	x Before ponding [Variable rainfall rate]	x After ponding [variable head $h_0 > 0$]	No	Arbitrary	Hydrostatic pressure, capillarity, and gravity	Smith et al. (2002); Brutsaert (2005)
Germann et al. (2007)	Conceptual	Rigid, heterogeneous, isotropic, non-hydrophobic, flat surface	Multi	1D, local scale, surface	Steady-state	x [constant rainfall rate]		Yes	Saturated K is time-variant	Gravity	[Stokes flow]
Essig et al. (2009)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, sloping surface	Single	1D, local scale, surface	Early and Steady-state	x Before ponding [constant rainfall rate]	x Before ponding and after saturation ($h_0 = 0$) After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	[Darcy; Buckingham law] [Darcy's law]
Valiantzas (2010)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	D and K lie between the assumption of sharp wetting front and the assumption of D and $dK/d\theta$ proportional	Hydrostatic pressure, capillarity, and gravity	Philip (1957a, 1957b); Talsma and Parlange (1972) [Darcy's law] [Richards' equation]
Su (2010)	Conceptual	Swelling, heterogeneous, isotropic, non-hydrophobic, flat surface	Multi	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$)	No	constant D and linear K as function of θ (Fleming et al., 1984)	Capillarity and gravity	Philip (1969b); Smiles and Raats (2005);- Su (2009) [Richards' equation]

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Assumptions: Driving forces for infiltration	Cited (mentioned in text) origins
Corradini et al. (2011)	Conceptual	Rigid, heterogeneous, two-layer (surface crust + subsoil), isotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early, transient, and steady-state	x Before ponding/ post hiatus [arbitrary rainfall rate]	x During rainfall hiatus ($h_0 \leq 0$) After ponding [arbitrary head $h_0 > 0$]	No	K represented as function of θ using Brooks and Corey (1964);- Upper layer is more permeable	Hydrostatic pressure, capillarity, and gravity	Smith et al. (1993); Corradini et al. (1997, 2000) [Richards' equation]
Swamee et al. (2012)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and Steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	(1) Yes (2) No	(1) Rapidly varying D, nearly constant K (2) $dK/d\theta$ and D are proportional	Hydrostatic pressure, capillarity, and gravity	Richards (1931); Green and Ampt (1911); Taslma and Parlange (1972)
Govindaraju et al. (2012)	Conceptual	Rigid, scale-heterogeneous, anisotropic, non-hydrophobic, flat surface	Single	1D, local and field-scale, surface	Early, transient, and steady-state	x Before ponding [arbitrary rainfall rate]	x After ponding [arbitrary head $h_0 > 0$]	(1) Yes (2) No	(1) At the local scale, saturated K continuously decreasing with depth according to a power law (2) At the field-scale, saturated K represented spatially by a correlated lognormal random field	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)
Ali et al. (2013)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Driving forces for infiltration	Cited (mentioned in text) origins
Almedeij and Esen (2014)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Mein and Larson (1973)
Bautista et al. (2014)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, surface (furrow)	Early, transient, and steady-state		x ($h_0 > 0$) [Variable head]	No	Arbitrary	Hydrostatic pressure, Capillarity and gravity	Richards (1931); Haverkamp et al. (1994); Warrick and Lazarovitch (2007); Warrick et al. (2007)
Lassabatere et al. (2014)	Conceptual	Rigid, heterogeneous, isotropic, non-hydrophobic, flat surface	Dual	1D and 3D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$)	No	K represented as function of θ using van Genuchten; Mualem model (Mualem, 1976; van Genuchten, 1980)	Capillarity and gravity	Gerke and van Genuchten (1993); Haverkamp et al. (1994) [Buckingham; Darcy law] [Richards' equation]
Vatankhah (2015)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [arbitrary head]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Almedeij and Esen (2014)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ ($L^2 T^{-1}$) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L T^{-1}$)	Driving forces for infiltration	Cited (mentioned in text) origins
Bautista et al. (2016)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	2D, local scale, surface (furrow)	Early, transient, and steady-state		x ($h_0 > 0$) [Variable head]	No	Arbitrary	Hydrostatic pressure, capillarity and gravity	Richards (1931); Haverkamp et al. (1994); Warrick and Lazarovitch (2007); Warrick et al. (2007); Bautista et al. (2014, 2016)
Nie et al. (2017a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and steady-state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911), Valiantzas (2010) [Darcy's equation]
Selker and Assouline (2017)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early state		x ($h_0 > 0$) [constant head]	Yes	Rapidly varying D, nearly constant K	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911)
Stewart and Abou Najm (2018a)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	3D, local scale, surface	Transient and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	K represented as function of ψ using Brooks and Corey (1964)	Hydrostatic pressure, capillarity, and gravity	Philip (1957a); Philip (1969a); Reynolds and Elrick (1990); Wu and Pan (1997); Wu et al. (1999)
Stewart (2018)	Conceptual	Shrink-swell, Heterogeneous, Isotropic, Non-hydrophobic, Flat surface	Dual	1D, local scale, surface	Early and steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Selker and Assouline (2017)

(Continues)

TABLE 1 (Continued)

Model	Category	Soil type	Porosity domain	Type of infiltration	Infiltration behavior	Boundary conditions: Neumann (prescribed flux)	Boundary conditions: Dirichlet (prescribed water content or pressure)	Assumptions: Step-function moisture profile	Assumptions: Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ (L T^{-1})	Assumptions: Driving forces for infiltration	Assumptions: Cited (mentioned in text) origins
Rahmati et al. (2019)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early and transient State		x ($h_0 \leq 0$)	No	Arbitrary	Hydrostatic pressure, capillarity, and gravity	Haverkamp et al. (1994); Parlange et al. (1982); Richards (1931); Philip (1957a)
Stewart (2019)	Conceptual	Rigid, heterogeneous, isotropic, non-hydrophobic, flat surface	Dual	1D, local scale, surface	Early and Steady-state	x Before ponding [constant rainfall rate]	x After ponding [constant head $h_0 > 0$]	Yes	Rapidly varying D, very sharp peak in K to a constant K near saturation	Hydrostatic pressure, capillarity, and gravity	Green and Ampt (1911); Beven and Germann (1982); Gerke and Van Genuchten (1993); Selker and Assouline (2017) [Darcy's law]
Baiamonte (2020)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state	x [constant rainfall rate]		No	(1) K represented using Torricelli's law (2) K represented as function of ψ using Brooks and Corey (1964)	Gravity	Richards (1931)
Su et al. (2020)	Conceptual	Deformable, scale-heterogeneous, anisotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$) and ($h_0 > 0$) [constant head]	No	K represented as function of θ using van Genuchten model (1980)	Hydrostatic pressure, capillarity, and gravity	Richards (1931)

(Continues)

TABLE 1 (Continued)

						Boundary conditions		Assumptions			
Model	**Category**	**Soil type**	**Porosity domain**	**Type of infiltration**	**Infiltration behavior**	**Neumann (prescribed flux)**	**Dirichlet (prescribed water content or pressure)**	**Step-function moisture profile**	**Diffusivity $D(\theta)$ (L^2 T^{-1}) and/or Hydraulic Conductivity $K(\theta)$ or $K(\psi)$ ($L\ T^{-1}$)**	**Driving forces for infiltration**	**Cited (mentioned in text) origins**
Poulovassilis and Argyrokastritis (2020)	Conceptual	Rigid, homogeneous, isotropic, non-hydrophobic, flat surface	Single	1D, local scale, surface	Early, transient, and steady-state		x ($h_0 \leq 0$)	No	Arbitrary	Capillarity and gravity	Richards (1931); Philip (1957a, 1957b)
Abou Najm et al. (2021)[a]	Semi-conceptual	Arbitrary	Arbitrary	Arbitrary	Arbitrary	Any		No	Arbitrary	Arbitrary	-
Di Prima et al. (2021)[a]	Conceptual	Rigid, homogeneous, isotropic, hydrophobic, flat surface	Single	3D, local scale, surface	Transient and steady-state		x ($h_0 \leq 0$)	No	K represented as function of θ using Brooks and Corey (1964)	Capillarity and gravity	Haverkamp et al. (1994); Lassabatère et al. (2006); Abou Najm et al. 2021)

[a]All infiltration models were designed to reflect theoretical infiltration results revealing a concave cumulative infiltration curve, except for Abou Najm et al. (2021) and Di Prima et al. (2021) models that reflect cumulative infiltration curves exhibiting concave, convex, mixed, or non-standard shapes.

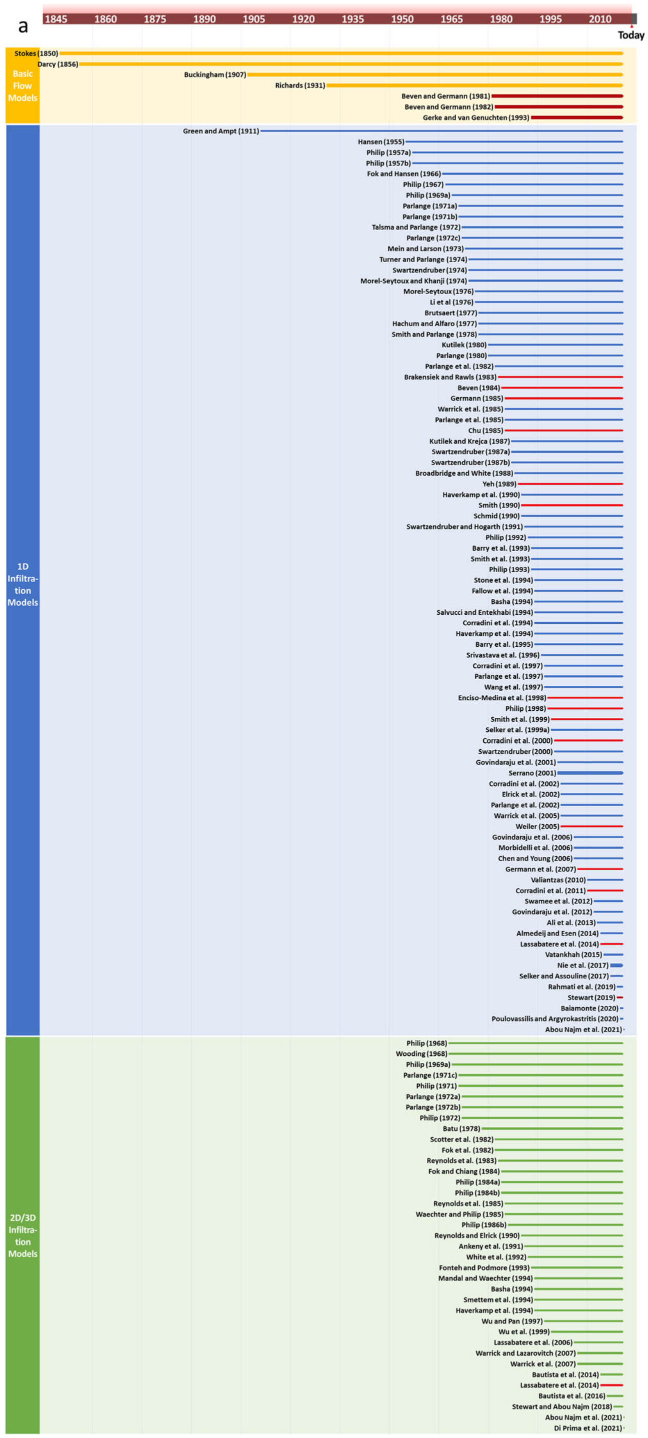

FIGURE 3 Schematic overview of (a) the historical evolution of infiltration theory from basic flow models to 1D and 3D infiltration models, and (b) the evolution of knowledge from earlier theories into the most recent infiltration models. Red symbols refer to infiltration models characterizing heterogenous pore domains.

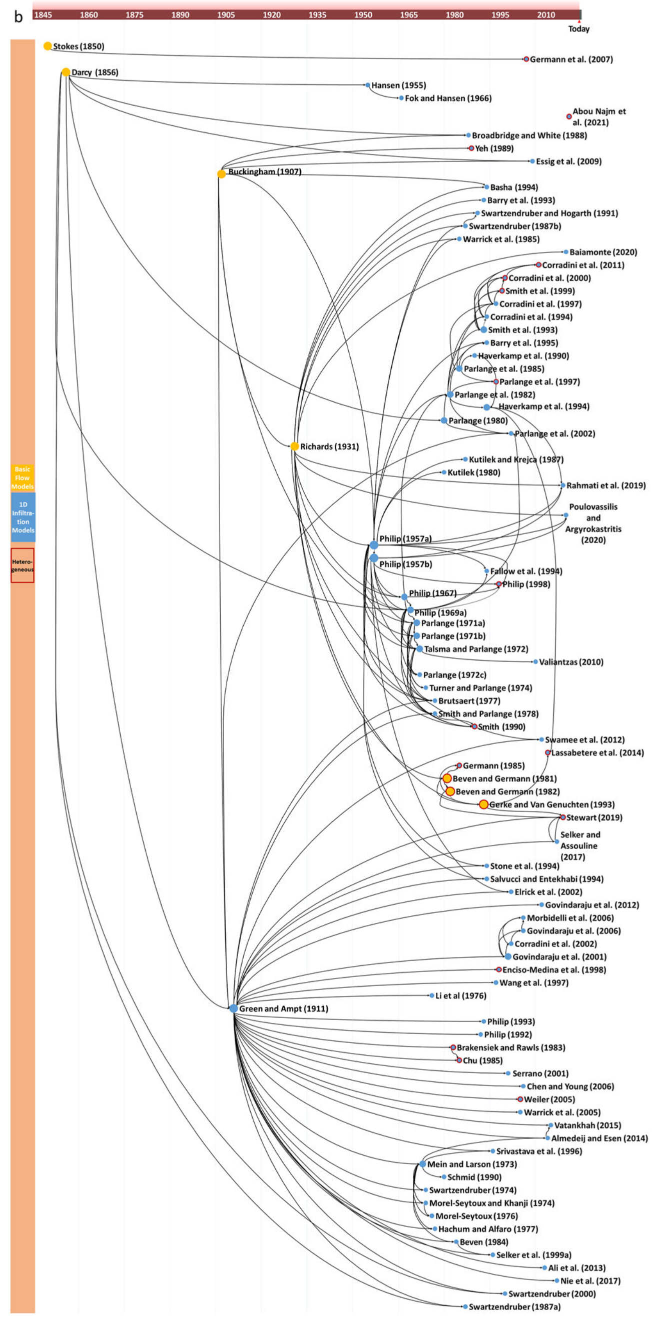

FIGURE 3 Continued

while the keyword “equation” generally represents Richards. On the other hand, soil water characteristic models, which are required to model infiltration processes under the Darcy-BRR paradigm, gained wide adoption. For example, Table 2 shows the citation impact of the van Genuchten model, which had 32,113 citations or nearly four times as many citations than either Darcy (1856) or Richards (1931). This result suggests that the van Genuchten (1980) model was published recently enough in the literature to continue being cited as a primary source.

Therefore, our analysis raises the question of the reliability and validity of citations as quality indicators in infiltration research. We conclude that the assessment of infiltration theory in literature based on citations does not often reflect its impact and relevance for the new concepts being addressed.

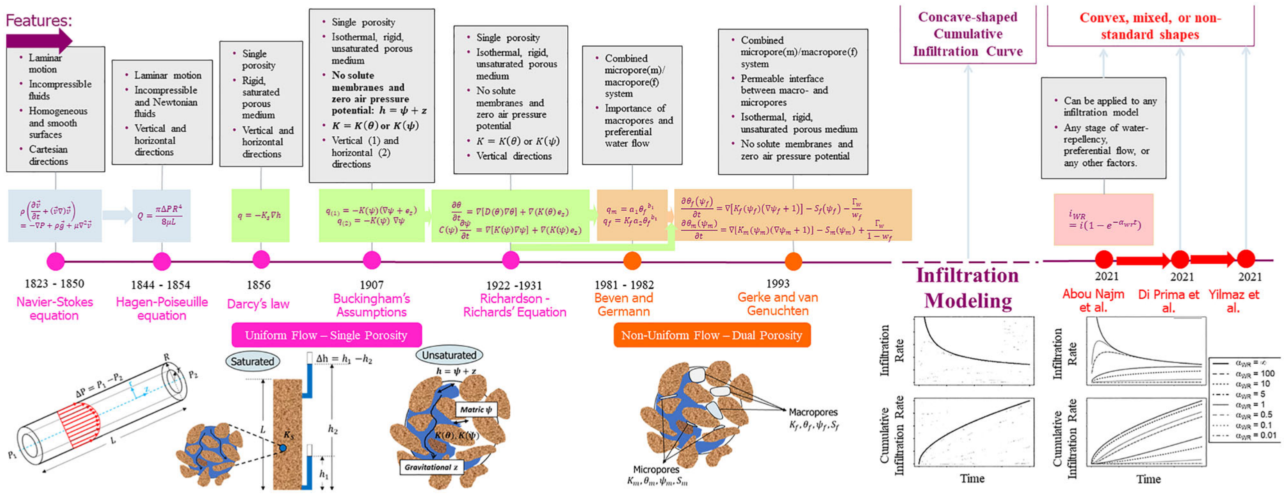

FIGURE 4 All infiltration models are based on the basic physical flow models and mimic the concave shape infiltration curve, except for Abou Najm et al. (2021), Di Prima et al. (2021), and Yilmaz et al. (2021) who captured the multi-shaped cumulative infiltration curve behavior in water-repellent soils.

9 | MODEL PARAMETERIZATION

9.1 | Evolution of the infiltration parameterization

9.1.1 | Hydraulic conductivity K

Saturated hydraulic conductivity, K_s ($\mathrm{L\,T^{-1}}$), was originally defined by Darcy (1856) as a measure of the water infiltration through the soil porous medium and later described as a function of permeability k ($\mathrm{L^2}$), density, ρ ($\mathrm{M\,L^{-3}}$), and kinematic viscosity, η ($\mathrm{M\,L^{-1}T^{-1}}$) of a fluid through an empirical equation developed by Kozeny (1927):

$$K_s = k\left(\frac{\rho g}{\eta}\right) \tag{23}$$

Unsaturated hydraulic conductivity, K ($\mathrm{L\,T^{-1}}$), was introduced by Buckingham (1907), symbolized by $K(\theta)$ ($\mathrm{L\,T^{-1}}$) as a function of the water content θ ($\mathrm{L^3\,L^{-3}}$), and by $K(\psi)$ as a function of the matric potential ψ (L). Accordingly, Wind (1955) proposed an empirical equation relating $K(\psi)$ ($\mathrm{L\,T^{-1}}$) to the matric potential ψ (L) as follows:

$$K\ (\psi) = b^*\ \psi^{-a^*} \tag{24}$$

where b^* ($\mathrm{L^2\ T^{-1}}$) and a^* (dimensionless) are empirical parameters.

Although there exist numerous models in the literature, we only summarize selected empirical and/or theoretical equations developed to illustrate the relationship between the unsaturated ($K(\theta)$ or $K(\psi)$) and the saturated (K_s) hydraulic conductivities, as summarized in Table 3.

Using the exponential equation developed by Gardner (1958) in Table 3, Raats (1971) defined the saturated hydraulic conductivity K_s ($\mathrm{L\,T^{-1}}$), in terms of the matric flux potential, φ ($\mathrm{L^2\ T^{-1}}$), as follows:

$$K_s = \frac{\alpha\varphi}{1 - e^{\alpha\psi_i}} \tag{25}$$

where Ψ_i(L) is the initial matric head.

Considering that the matrix flux potential is defined by:

$$\varphi = \int_{h_i}^{0} K\,(h)\ dh \tag{26}$$

A well-known equation for estimating K_s ($\mathrm{L\ T^{-1}}$) was derived from Equation (25) for soils initially at "field capacity" or drier conditions (Ankeny et al., 1991; Raats, 1971; Reynolds et al., 1985; Scotter et al., 1982; Stewart & Abou Najm, 2018a, 2018b; Wu & Pan, 1997 Yeh, 1989):

$$K_s = \alpha^*\ \varphi \tag{27}$$

Although the hydraulic conductivity improved our understanding of infiltration problem, researchers realized that solving this problem was still so far from complete. Thus, they recognized the need for an additional soil property that could improve the estimation of water infiltration. This major characteristic is the soil sorptivity, symbolized by S, that describes the water absorption by capillarity.

9.1.2 | Sorptivity S

Philip (1957b) introduced the sorptivity S_0 ($\mathrm{L\ T^{-0.5}}$), as the first term in his two-term equation of cumulative, one-

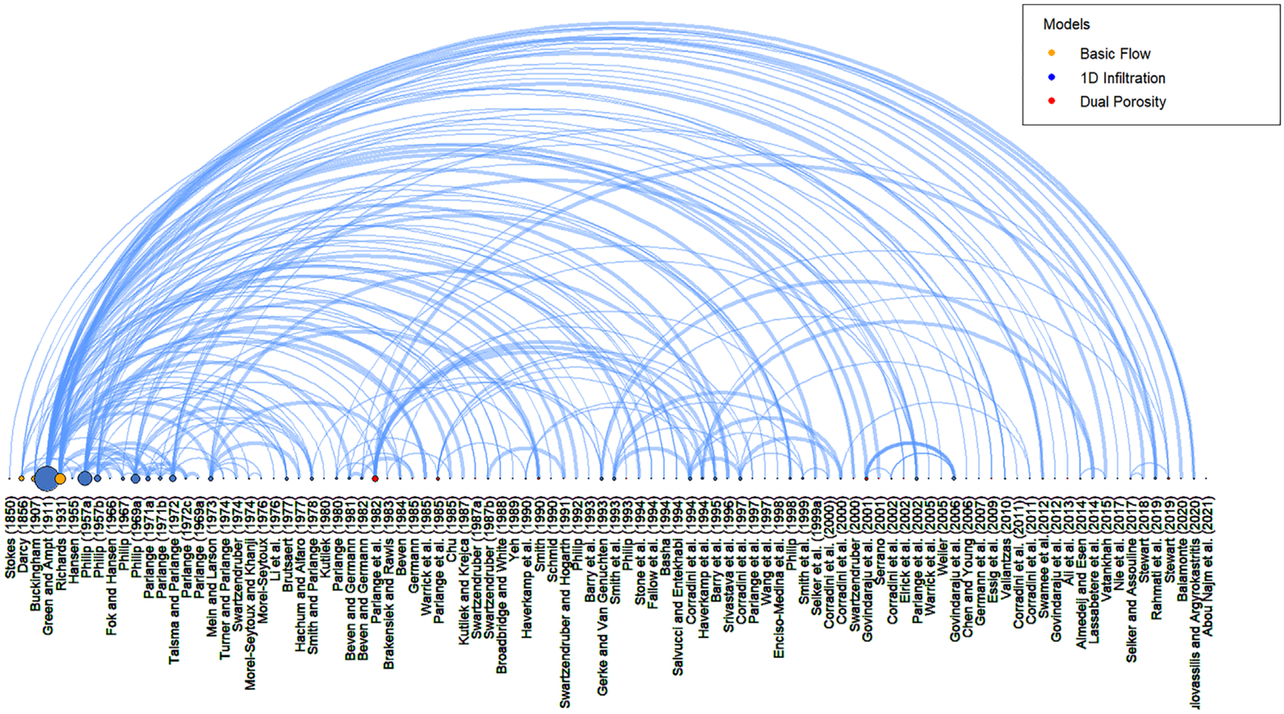

FIGURE 5 Arc diagrams constituted by nodes that represent the conceptual infiltration models displayed as nodes along a single axis in an ascending historical order, and links that show connections between those models. Each node was assigned a size weight based on how many links (N) connect the represented model with the other nodes along each diagram. The diagram connects the represented models to their target sources cited in the corresponding papers.

TABLE 2 Citation analysis involving Darcy, Richards, and van Genuchten theories.

Cited theory (references list)	No. of records by Google scholar [Scopus] (on September 26, 2023)	Theory mentioned in text	No. of records by Scopus (on September 26, 2023)
Darcy (1856)	8067 [ND]	"Darcy* law" or "Darcy* model*" or "Darcy* equation"	**21,381**
		a) "Darcy* law"	16,131
		b) "Darcy* model*"	3603
		c) "Darcy* equation"	3209
Richards (1931)	7936 [4484]	"Richard* model*" or "Richard* equation*"	**8763**
		a) "Richard* model*"	2295
		b) "Richard* equation*"	6584
van Genuchten (1980)	32,113 [19,904]	"van Genuchten model*" or "van Genuchten equation" or "van Genuchten parameter*"	**2068**

Note: The asterisk is used as a trick for a better Scopus search— it can search for the alternative spellings of the same word.

dimensional, unsaturated infiltration I (L) with negative or zero head surface boundary condition ($h_0 \leq 0$ and $\theta_o \leq \theta_s$):

$$I = S_0 t^{0.5} + At \tag{28}$$

After he introduced the term sorptivity in 1957, Philip undertook a series of successive events to entrench soil sorptivity into literature. In 1969, Philip (1969a) first developed an analytical equation describing sorptivity, S_0 (L T$^{-0.5}$), for an unsaturated soil with a constant diffusivity D (L^2 T^{-1}) independent of θ (L^3 L^{-3}):

$$S_0 = 2\left(\theta_o - \theta_i\right)\left(\frac{D}{\pi}\right)^{0.5} \tag{29}$$

Then, in 1973, he introduced the concept of flux concentration function $F(\theta)$ to describe the water absorption by capillary. He defined this concept as the ratio of water flux at any location in the profile $i(t) - K_i$, to the infiltration rate at surface, $i_0(t) - K_i$, where K_i (L T^{-1}) is the initial hydraulic conductivity. In 1974, Philip and Knight further used this concept to estimate the sorptivity S_0 (L T$^{-0.5}$):

TABLE 3 Empirical equations for the unsaturated hydraulic conductivities $K(\theta)$ and $K(\psi)$.

Equation	Reference
Unsaturated hydraulic conductivity $K(\theta)$ or $K(S_e)$	
$K(\theta) = K_s\, S_e^{\,n}\ \theta < \theta_s$ $K(\theta) = K_s\ \theta = \theta_s$	Brooks and Corey (1964)
$K(\theta) = K_s\, e^{\alpha(\theta - \theta_s)}$	Davidson et al. (1969)
$K(\theta) = K_s\, (\frac{\theta}{\theta_s})^n$	Campbell (1974)
$K(\theta) = K_s\, S_e^{\,0.5}[\int_0^{S_e} \frac{dS_e}{\psi} / \int_0^1 \frac{dS_e}{\psi}]^2$	Mualem (1976)
$K(\theta) = K_s\, S_e^{\,2} \int_0^{S_e} \frac{dS_e}{\psi^2} / \int_0^1 \frac{dS_e}{\psi^2}$	Burdine (1953)
$K(\theta) = K_s\, S_e^{\,0.5}[1 - (1 - S_e^{\,1/m_{ssc}})^{m_{ssc}}]^2$	Mualem's model applied on van Genuchten (1980) model for water retention curve (WRC)
$K(\theta) = K_s\, e^{\beta_l(\theta - \theta_s)}$	Libardi et al. (1980)
$K(\theta) = K_s\, (\frac{S_e}{S_{eo}})^{0.5} [\frac{1-(1-S_e^{\,1/m_{\mathrm{ssc}}})^{m_{\mathrm{ssc}}}}{1-(1-S_{eo}^{\,1/m_{\mathrm{ssc}}})^{m_{\mathrm{ssc}}}}]^2$	van Genuchten et al. (1991)
$K(\theta) = K_s\, e^{-a_{sn}(\theta_s - \theta)^{b_{sn}}}$	Setiawan and Nakano (1993)
$K(\theta) = K_s\, S_e^{\,\alpha_k} \{\frac{1}{2} erfc[\mathrm{erfc}^{-1}(2S_e) + \frac{\beta_k \sigma_k}{\sqrt{2}}]\}^{\gamma_k}$	Kosugi (1999)
$K(\theta) = K_s\, [\frac{\frac{\beta_g}{\phi} - 1 + \theta - \theta_c}{\frac{\beta_g}{\phi} - \theta_c}]^{\frac{\lambda_g}{3 - D_g}}\quad \theta_x \le \theta < \phi$ $K(\theta) = K_s\, [\frac{\frac{\beta_g}{\phi} - 1 + \theta_x - \theta_c}{\frac{\beta_g}{\phi} - \theta_c}]^{\frac{\lambda_g}{3 - D_g}} (\frac{\theta - \theta_c}{\theta_x - \theta_c})^2$ $\theta_c \le \theta < \theta_x$	Ghanbarian et al. (2016)
Unsaturated hydraulic conductivity $K(\psi)$	
$K(\psi) = K_s\, S_e^{\,3.5}$	Averjanov (1950)
$K(\psi) = K_s\, e^{\alpha\psi}$	Gardner (1958)
$K(\psi) = K_s\, e^{\alpha(\psi - \psi_{str})}\ \psi < 0 and K(\psi) = K_s\ \psi \ge 0$	Rijtema (1965) (See Fredlund et al., 1994)
$K(\psi) = K_s\, \{\frac{1}{2} erfc[\frac{\ln(\psi/\beta_k) - \gamma_k^{\,2}}{\gamma_k \sqrt{2}}]\}^{\delta_k} \{\frac{1}{2} erfc[\frac{\ln(\psi/\beta_k)}{\gamma_k \sqrt{2}}]\}2$	Das and Kluitenberg (1995) (See Leij et al., 1997)

$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$; $S_{eo} = \frac{\theta_o - \theta_r}{\theta_s - \theta_r}$
θ_o, θ_r, θ_s, θ_c, and θ_x are arbitrary, residual, saturated, critical, and the crossover water content at which fractal scaling from critical path analysis switches to universal percolation scaling from percolation theory, respectively.
ψ_{str} (L) is the suction at the air-entry point.
All other model parameters are fitted parameters.

$$S_0^2 = 2 \int_{\theta_i}^{\theta_0} \frac{[\theta - \theta_i]\, D(\theta)}{F(\theta)} d\theta \tag{30}$$

where θ_i and θ_o ($L^3\ L^{-3}$) are the initial ($t = 0$) and specific ($t > 0$) water contents, respectively, and $F(\theta)$ stands for the flux concentration function.

Many estimates for sorptivity S_0 ($L\ T^{-0.5}$) have relied on the use of the flux concentration function $F(\theta)$ as shown in Table 4.

Another approximation for sorptivity, S_0 ($L\ T^{-0.5}$), was derived by equating the GA equation (considering a surface pressure of zero, i.e., $h_0 = 0$) to Philip's two-term infiltration equation (Collis-George, 1977):

$$S_0^2 = 2\psi_{wf} (\theta_o - \theta_i) K_s \tag{31}$$

where ψ_{wf} (L) is the wetting front potential.

Later, Philip (1985a) introduced the macroscopic capillary length, λ_c (L), typically equivalent to the wetting front potential, ψ_{wf} (L) as follows:

$$\lambda_c = \frac{1}{K_0 - K_i} \int_{\theta_i}^{\theta_o} D(\theta)\, d\theta \tag{32}$$

Based on Philip's definition of λ_c, White and Sully (1987) reformulated the sorptivity, S_0 ($L\ T^{-0.5}$) by substituting Equation (29) into (32), with $F = 0$ for $\theta = \theta_i$ and $F = 1$ for $\theta = \theta_o$:

$$S_0^2 = \frac{\lambda_c (K_0 - K_i)(\theta_o - \theta_i)}{b} \tag{33}$$

TABLE 4 Main approximations of the flux concentration function $F(\theta)$ and sorptivity S_0^2 ($L^2\ T^{-1}$) (See Table 1.1 in Angulo-Jaramillo et al., 2016).

$F(\theta)$	S_0^2 ($L^2\ T^{-1}$)	Reference
$\frac{\theta-\theta_i}{\theta_o-\theta_i}$	$2(\theta_o-\theta_i)\int_{\theta_i}^{\theta_0} D(\theta)d\theta$	Philip and Knight (1974)
$\frac{2(\theta-\theta_i)}{\theta_o+\theta-2\theta_i}$	$\int_{\theta_i}^{\theta_o}(\theta_o+\theta-2\theta_i)D(\theta)d\theta$	Parlange (1975)
$(\frac{\theta-\theta_i}{\theta_o-\theta_i})^{0.5}$	$2(\theta_o-\theta_i)^{0.5}\int_{\theta_i}^{\theta_0}(\theta-\theta_i)^{0.5}D(\theta)d\theta$	Brutsaert (1976)
$\exp\{-[\mathrm{inverfc}(\frac{\theta-\theta_i}{\theta_o-\theta_i})]^2\}$	$2D\int_{\theta_i}^{\theta_o}\frac{\theta-\theta_i}{\exp\{-[\mathrm{inverfc}(\frac{\theta-\theta_i}{\theta_o-\theta_i})]^2\}}d\theta$	Crank (1979)

where b is a dimensionless constant. White and Sully (1987) showed that $b = 0.5$ exactly for soils exhibiting a step-function infiltration front (Philip and Knight, 1974) and $b = \pi/4$ if soil diffusivity $D(\theta)$ is constant (Philip, 1969a). For general field soils, White and Sully (1987) suggested $b = 0.55$, since the actual infiltration fronts do not tend to be as sharp as a step function.

Using the matric flux potential term, φ ($L^2\ T^{-1}$), Reynolds and Elrick (1990) redefined the sorptivity S_0 ($L\ T^{-0.5}$) as follows:

$$S_0^2 = \frac{\varphi\left(\theta_o - \theta_i\right)}{b} \tag{34}$$

Now for the case of ponded head infiltration ($h_0 \geq 0$ and $\theta_o = \theta_s$), the sorptivity, here defined as S_H ($L\ T^{-0.5}$), was related to S_0 ($L\ T^{-0.5}$) (Haverkamp et al., 1990; White and Sully, 1987) by:

$$S_H^2 = S_0^2 + 2h_0K_s\left(\theta_s - \theta_i\right) \tag{35}$$

The Reynolds and Elrick (1990) expression in Equation (34) can also be applied to Equation (35) to define the sorptivity S_H ($L\ T^{-0.5}$) as follows:

$$S_H^2 = \left(\theta_s - \theta_i\right)\left(\frac{\varphi}{b} + 2h_0K_s\right) \tag{36}$$

Recently, Lassabatère et al. (2021) proposed a specific scaling procedure to simplify the computation of sorptivity. In addition, the same authors proposed a specific mixed formulation to ease the numerical computation of sorptivity, when the final state corresponds to water ponding and for very low initial water contents (Lassabatère et al., 2023). These authors demonstrated that the saturated part of sorptivity, $2h_0K_s(\theta_s - \theta_i)$ must not be forgotten and its omission may lead to erroneous modeling of water infiltration.

9.2 | Methods and challenges behind K and S estimation in literature

There are many techniques to measure infiltration in laboratory and in field. Starting with characterization of the $K(\theta)$ or $K(\psi)$ functions, the commonly used methods are the instantaneous profile (Watson., 1966) and the plane of zero flux (Arya et al., 1975) methods. These methods are further divided between direct approaches, in which both θ and ψ are measured within the soil profile, and indirect approaches, in which one variable is measured, while estimating the other variable is determined from a separate water retention curve $\theta(\psi)$. In principle, these methods are simple; however, complications often arise in practice leading to complex and time-consuming measurements. To simplify the field determination of $K(\theta)$ and $K(\psi)$ and therefore the extensive labor requirements, these two functions can be determined using the empirical equations illustrated in Table 3 once the water retention curve $\theta(\psi)$ is determined (Dane & Hruska, 1983; Libardi et al., 1980; Zachmann et al., 1981). However, this simplification can show a disagreement between the $\theta(\psi)$ curves as determined in situ and on undisturbed core samples—a common disadvantage among all described methods. In addition, applying the widely used equations of $K(\theta)$ and $K(\psi)$ (Table 3) involves the determination of the saturated hydraulic conductivity K_s, which gives rise to more efforts and challenges in knowing the soil heterogeneity (Fodor et al., 2011), the spatial and seasonal variability of K_s (Farkas et al., 2006; Gülser et al., 2016), as well as its scale dependency (Lai & Ren, 2007).

To determine the saturated hydraulic conductivity K_s as well as soil sorptivity S_0 or S_H (noted hereafter as $S_{0/H}$), different experimental tools were designed to measure one- or multi-dimensional flow that can include early, transient, and steady-state flow stages depending on the approach being applied. Starting with single and double ring-infiltrometers, these tools were built to determine the saturated hydraulic conductivity either under constant- (Di Prima et al., 2016; Olson & Swartzendruber, 1960; Parr & Bertrand, 1960; Ronnqvist, 2018; Schiff, 1953; Touma et al., 2007; Xu et al., 2012) or falling-head conditions (Angulo-Jaramillo et al., 2003; Bagarello et al., 2004; Elrick et al., 1995). In a series of papers, Bouwer (1960, 1963, 1986) introduced a simple field measurement cylinder, which can be used to determine the two parameters, saturated hydraulic conductivity, K_s, and wetting front potential, ψ_{wf}, required to apply the GA approach. Bouwer's cylinder infiltrometer is discussed in detail in Selker's vadose zone book (Selker et al., 1999b). Moving to the disc infiltrometer developed by Perroux and White (1988), referred to also as a tension infiltrometer (Watson & Luxmoore, 1986), this tool involved supplying water to the soil surface under controlled suction (Latorre et al., 2015; Smettem & Clothier, 1989; Zhang, 1997). These techniques are all non-destructive and allow rapid flow measurements if steady flow can be quickly achieved depending on the ring size, soil texture, and soil structure. However, while driving discs, rings and cylinders into the soil, some disturbance, such as compaction, fracturing, or smearing, almost inevitably occurs (Bouma & Dekker, 1981). In addition to soil disturbance, the large spatial variability of K_s and $S_{0/H}$ can affect the precision and accuracy of the measured parameters (Logsdon & Jaynes, 1996; Sharma et al., 1980). To overcome the challenges of soil heterogeneity and spatial variability of K_s, the rainfall simulator can be used as an alternative experimental tool (Bradford et al., 1987; Lassu et al., 2015; Di Prima et al., 2018). However, infiltrometers are still the most widely used devices for measuring field infiltration rates due to their simple application in the field (Rahmati et al., 2018). However, more complex parameter estimation models need to be developed to consider soil behavior such as swelling (Gérard-Marchant et al., 1997). Indeed, failure to take deviant behavior into account can lead to an underestimation, or even unrealistic values, of both sorptivity and hydraulic conductivity.

Different infiltration models are employed to determine K_s and $S_{0/H}$ from the multi-dimensional infiltration measured using ring, cylinder, and disc infiltrometers. Among these models, numerous conceptual equations relate the one-dimensional cumulative infiltration I_{1D} to both parameters, K_s and either S_0 (Barry et al., 1993; Philip, 1957b, 1969a; Talsma & Parlange, 1972) or S_H (Barry et al., 1993; Parlange, 1980; Smith & Parlange, 1978; Swartzendruber, 2000; Valiantzas, 2010). These relationships reveal the relevance and applicability of estimating these two properties by fitting analytical equations numerically to infiltration data. In this context, the two-term equation derived by Philip (1957b) has been extensively used to determine K_s and S_0 due to its explicit numerical application (Equation 28). In one application, S_0 (L T$^{-0.5}$) is estimated by plotting I_{1D} (L), measured into a uniform unsaturated soil profile, against $t^{0.5}$ (T$^{0.5}$) during the very early stage of infiltration. The slope of the linear relationship (I_{1D} vs. $t^{0.5}$) allows the determination of S_0. A (L T^{-1}), on the other hand, can be estimated by plotting I_{1D} (L) against t (T) for larger times of infiltration. The slope of the linear relationship (I_{1D} vs. t) allows the determination of A, which is further used to estimate the saturated hydraulic conductivity K_s (L T^{-1}). Specifically, $A = mK_s$ with $1/3 \leq m \leq 2/3$ (Philip,1969a; Talsma, 1969; Talsma & Parlange, 1972; Youngs, 1968). A value of $m = 0.363$ may be appropriate for soils with a relatively low initial water content (Philip, 1987), while a value of $m = 2/3$ is often used (Fodor et al., 2011; Whisler and Bouwer, 1970). In some studies, $m = 1$ and hence $A = K_s$ (Davidoff & Selim, 1986; Ghorbani et al., 2009; Swartzendruber and Young, 1974). It is crucial to note that Philip's equation is no longer valid for very large times reaching the steady state and should be restricted to the modeling of the transient state (Philip, 1957b).

A second set of models, based on three-parameter equations, are also fitted to infiltration data measured at any given time. In addition to K_s and $S_{0/H}$, each of these equations depend on a dimensionless constant symbolized by β_0 (Brutsaert, 1977), A_0 (Swartzendruber, 1987b; Swartzendruber & Hogarth, 1991), δ (Parlange et al., 1982, 1985, 2002; Haverkamp et al., 1990), or β (Haverkamp et al., 1994) that also needs to be estimated failing to be able to be calculated simply analytically. Brutsaert (1977) recommended the value $\beta_0 = 2/3$ for practical applications. Swartzendruber (1987b) showed that A_0 can be equal to $\frac{4K_s}{3S_H}$. Parlange et al. (1982, 1985) defined $\delta = \frac{1}{(\theta_s-\theta_i)(K_s-K_i)} \int_{\theta_i}^{\theta_s}(K_s - K)d\theta$, where K_i (L T^{-1}) is the initial soil hydraulic conductivity and δ can take values from 0 to 1, with an approximate used value of 0.8 or 0.85. Haverkamp et al. (1994) further replaced δ by a new dimensionless constant, β, which varies from 0.3 (sand) to 1.7 (silt), with an average value of 0.6 according to Lassabatère et al. (2009) who fitted the analytical model to analytically generated data to quantify the best values of constants β and γ.

A third set of 1D infiltration equations, based on the approach of Green and Ampt (1911), also leads to the estimation of K_s and $S_{0/H}$ under constant- (Almedeij & Esen, 2014; Li et al., 1976; Salvucci & Entekhabi, 1994; Selker & Assouline, 2017; Stone et al., 1994; Swamee et al., 2012; Vatankhah, 2015) or falling-head ponding conditions (Elrick et al., 2002; Philip, 1992). By fitting the equations to one-dimensional infiltration data, we can estimate the parameters, K_s (L T^{-1}), and wetting front potential, ψ_{wf} (L). Then, the parameter S_H can be deduced from Equation (36), with h_0 (L), θ_s, and θ_i (L^3 L^{-3}) measured directly from the field.

In addition, the quasilinear analysis of steady-state infiltration, Q_{inf} (L^3 T^{-1}), using the 3D infiltration equations (Ankeny et al., 1991; Reynolds et al., 1983, 1985; Reynolds & Elrick, 1990; Scotter et al., 1982; Wooding, 1968) allows the determination of parameters K_s and φ, and thus leads to the estimation of S_0 from Equation (34) or S_H from Equation (35) or (36). Haverkamp et al. (1994) proposed an implicit formulation for the 1D infiltration model and Smettem et al. (1994) extended it to define the 3D cumulative infiltration, I_{3D} (L T^{-1}), from a surface disk infiltrometer, adding the term γ (average value $\gamma = 0.75$) to represent 3D geometrical effects as:

$$\frac{2\,\Delta K^2}{S^2}\,t = \frac{1}{1-\beta}\,\frac{2\,\Delta K}{S^2}\left(I\,(t) - K_i t\right) - \frac{1}{1-\beta}\ln\left(\frac{\exp\left(\frac{2\beta\,K}{S^2}\left(I\,(t) - K_i t\right)\right) + \beta - 1}{\beta}\right) \tag{37a}$$

$$I_{3D} = I_{1D} + \frac{\gamma {S_0}^2 t}{r\left(\theta_s - \theta_i\right)} \tag{37b}$$

where $\Delta K = (K_s - K_i)$ and r(L) stands for the disc radius. Equation (37a) defines an implicit formulation that does not ease the computation of the cumulative infiltration. Consequently, Haverkamp et al. (1994) have proposed two simplified expansions that are valid at transient and steady state, respectively (Haverkamp et al., 1994; Lassabatère et al., 2006):

$$I_{3D} = S_0\, t^{0.5} + \left(\frac{2-\beta}{3}\left(K_s - K_i\right) + K_i + \frac{\gamma {S_0}^2}{r\left(\theta_s - \theta_i\right)}\right) t \tag{38}$$

$$I_{3D,\infty} = \left(K_s + \frac{\gamma {S_0}^2}{r\left(\theta_s - \theta_i\right)}\right) t + \frac{1}{2\left(1-\beta\right)} ln\left(\frac{1}{\beta}\right)\frac{{S_0}^2}{K_s - K_i} \tag{39}$$

Note that more precise approximate expansions may be defined for the transient state according to the number of terms considered (Lassabatère et al., 2009; see Appendix).

Based on Haverkamp et al. (1994) approach, BEST methods were designed to estimate the whole set of unsaturated hydraulic parameters, offering a complete hydraulic characterization of soils. The BEST-Slope method was pioneered by Lassabatère et al. (2006) and was followed by BEST-Intercept (Yilmaz et al., 2010) and BEST-Steady methods (Bagarello et al., 2014). The three methods consider the same hydraulic function for defining water retention and hydraulic conductivity functions, with the use of van Genuchten (1980) model along with the Burdine condition and Brook and Corey (1964) model for describing the hydraulic conductivity. The three methods consider the same pedo-transfer functions to relate soil texture (i.e., particle size distribution) to hydraulic shape parameters but differ in the use of experimental infiltration data to estimate the scale of hydraulic parameters. The infiltration data are obtained by a Beerkan infiltration run, which consists of water infiltrating through a single ring under ponded conditions (Aiello et al., 2014; Braud et al., 2005; Di Prima et al., 2016, 2018, 2020; Lassabatère et al., 2019a)

BEST-Slope and BEST-Intercept use the last/steady part of the cumulative infiltration to fit the asymptotic model (Equation 39) to estimate either the slope $a = K_s + \frac{\gamma {S_0}^2}{r(\theta_s - \theta_i)}$ or the intercept $b = \frac{1}{2(1-\beta)}\ \ln(\frac{1}{\beta})\frac{{S_0}^2}{K_s - K_i}$ relating the hydraulic conductivity and the sorptivity. The two methods reduce the number of unknowns from two (K_s, S_0) to one (S_0), which strengthens the robustness of the inversion. Then, the two methods fit the two-term equation (Equation 38) to the first part of the cumulative infiltration (transient part) to derive the value of sorptivity S_0. The part of the curves assigned to the transient state is defined by a specific iterative procedure that defines a validity time as a function of the estimated values of (K_s, S_0). BEST-Steady only considers the steady-state part and fits the asymptotic model (Equation 39) to the steady part of the cumulative infiltration data. The estimation of the intercept and of the slope leads to a system with two equations and two unknowns, leading to simultaneous estimations of the couple (K_s, S_0). Lastly, the scale parameter is estimated from the knowledge of sorptivity and saturated hydraulic conductivity (Lassabatère et al., 2006, Equation 8).

Bagarello et al. (2013, 2014, 2017) used these explicit expansions to propose two simplified versions of the BEST methods, transient (TSBI) and steady (SSBI), to determine K_s of an initially rather nonconductive soil ($K_i = 0$). The TSBI method (Bagarello et al., 2013, 2014) is based on the explicit transient relationship (Equation 38) which, divided by $t^{0.5}$, results in a linear relationship between $I_{3D}/t^{0.5}$ and $t^{0.5}$ with slope b_1 (L T^{-1}), defined by:

$$b_1 = \frac{\gamma {S_0}^2}{r\left(\theta_s - \theta_i\right)} + \frac{2-\beta}{3} K_s \tag{40}$$

The slope b_1 (L T^{-1}) can be estimated by a linear regression analysis of the ($I_{3D}/t^{0.5}$, $t^{0.5}$) data collected during the transient phase of the infiltration run. The intercept of the regression line ($I_{3D}/t^{0.5}$, $t^{0.5}$) indicates the sorptivity S_0 (L T$^{-0.5}$). Then, solving for K_s gives:

$$K_s = \frac{b_1}{\frac{b\gamma}{r\alpha} + \frac{2-\beta}{3}} \tag{41a}$$

$$K_s = \frac{b_1}{\frac{\gamma\gamma_w}{r\alpha^*} + \frac{2-\beta}{3}} \tag{41b}$$

where γ_w is a dimensionless constant (White & Sully, 1987) related to the shape of the wetting (or drainage) front and α^* corresponds to the ratio between the matrix flux potential and the saturated hydraulic conductivity, $\alpha^* = \frac{K_s}{\varphi}$ (Bagarello et al., 2017). Note that the two methods SSBI and TSBI make use of the relation between sorptivity and hydraulic conductivity $S_0^2 = \gamma_w\ (\theta_s - \theta_i)\,\varphi$ (Bagarello et al., 2017).

The SSBI method, on the other hand, is based on the explicit steady-state expansion (Equation 39). The slope, b_2

(L T^{-1}), of the linear relationship between I_{3D} and t collected during the steady-state phase of the infiltration run is defined as:

$$b_2 = K_s + \frac{\gamma S_0{}^2}{r\left(\theta_s - \theta_i\right)} \tag{42}$$

Solving for K_s gives:

$$K_s = \frac{b_2}{\frac{b\gamma}{r\alpha} + 1} \tag{42a}$$

$$K_s = \frac{b_2}{\frac{\gamma\gamma_w}{r\alpha^*} + 1} \tag{42b}$$

BEST methods have been used for many applications and in many contexts and encountered several difficulties with real soils for some types of situations (water repellency, preferential flows, and self-sealing soils). Di Prima et al. (2021) recently pioneered a new BEST method dedicated to water-repellent soil. Lassabatère et al. (2023) proposed an improvement of the same method by developing a three-term expansion to describe the transient state. Regarding preferential flows, Lassabatère et al. (2019b) developed BEST-2K for the hydraulic characterization of dual-permeability soils. This version allows the characterization of the soil as the combination of matrix and fast-flow regions, with the complete characterization of the unsaturated hydraulic parameters of the two regions.

Alternatively, further approaches were developed based on model variations that solve for K_s and include different parameters including Stewart and Abou Najm (2018b), Iovino et al. (2021), and Kargas et al. (2022).

10 | DECISION-MAKING FRAMEWORK FOR MODEL SELECTION

We present a decision-support framework aimed at guiding researchers in selecting suitable infiltration models that align with their specific study conditions, including underlying assumptions, soil types, and available information. This framework includes four interconnected stages, each influencing model selection, and ultimately helping users identify the best-suited infiltration model for their conditions (Figure 6).

In stage 1, researchers should specify soil and field conditions and connect those conditions with the problem statement at hand. This stage sets the expectations from the models given the field conditions and the problem that needs to be addressed. Soils may range from homogeneous to highly heterogeneous, including fine to coarse, water-repellent, sealed, or macroporous types.

In stage 2, researchers should develop, assess, or reevaluate an experimental plan, which can be constrained by budget, personnel, or equipment availability among other factors. The researchers should identify the type, frequency, and scale of data available (or needed) including infiltration tests, boundary conditions (prescribed pressure vs. prescribed flow), and any necessary auxiliary tests to characterize different soil types encountered in the field. For example, characterizing macroporous soils may require a combination of tension infiltrometer and Beerkan experiments to account for both matrix and macropore systems.

In stage 3, researchers should examine data generated from the experimental plan, which can reveal or confirm various soil and field conditions, as well as infiltration behavior. Pachepsky and Karahan (2022) detected 12 major cumulative infiltration curve shapes that indicate different infiltration behaviors. Furthermore, other critical data can be measured, such as soil particle distribution, water content, soil water repellency, and more. For instance, the water drop penetration time test can be combined with organic matter measurements to assess soil water repellency.

In stage 4, researchers should identify which group of infiltration models is the most suitable to describe water infiltration for their soil conditions, along with any auxiliary data or information that is available. For example, when fitting a model to infiltration data without other measurements, it is crucial to limit the number of parameters to two or three at most. Therefore, models with fewer parameters are recommended like Green and Ampt (1911) or Philip (1957b) if only cumulative infiltration data are available. Also, researchers should stick with one-dimensional (1D) in that case, since multi-dimensional models (2D/3D) require accounting for spatial variability in soil properties, moisture content, and infiltration dynamics, which involves the estimation of additional parameters to accurately represent 3D infiltration. More complex models like BEST and multi-dimensional models (2D/3D) are only recommended if additional auxiliary data are available.

The infiltration curve shape should also be considered in stage 4. For example, a concave infiltration curve that is typical in homogeneous and initially dry soils can be modeled using many different infiltration models for homogeneous soils (i.e., option A), such as Green and Ampt (1911), Philip (1957b), Parlange et al. (1982), Haverkamp et al. (1994), Lassabatère et al. (2006), Valiantzas (2010), and Rahmati et al. (2019), among others. However, soils with very low permeability or those requiring long experiments might show exclusively concave shapes, requiring transient-state model expressions to avoid overestimation of infiltration rates. Highly permeable coarse-textured soils or initially wet homogeneous soils may produce linear-shaped curves, best

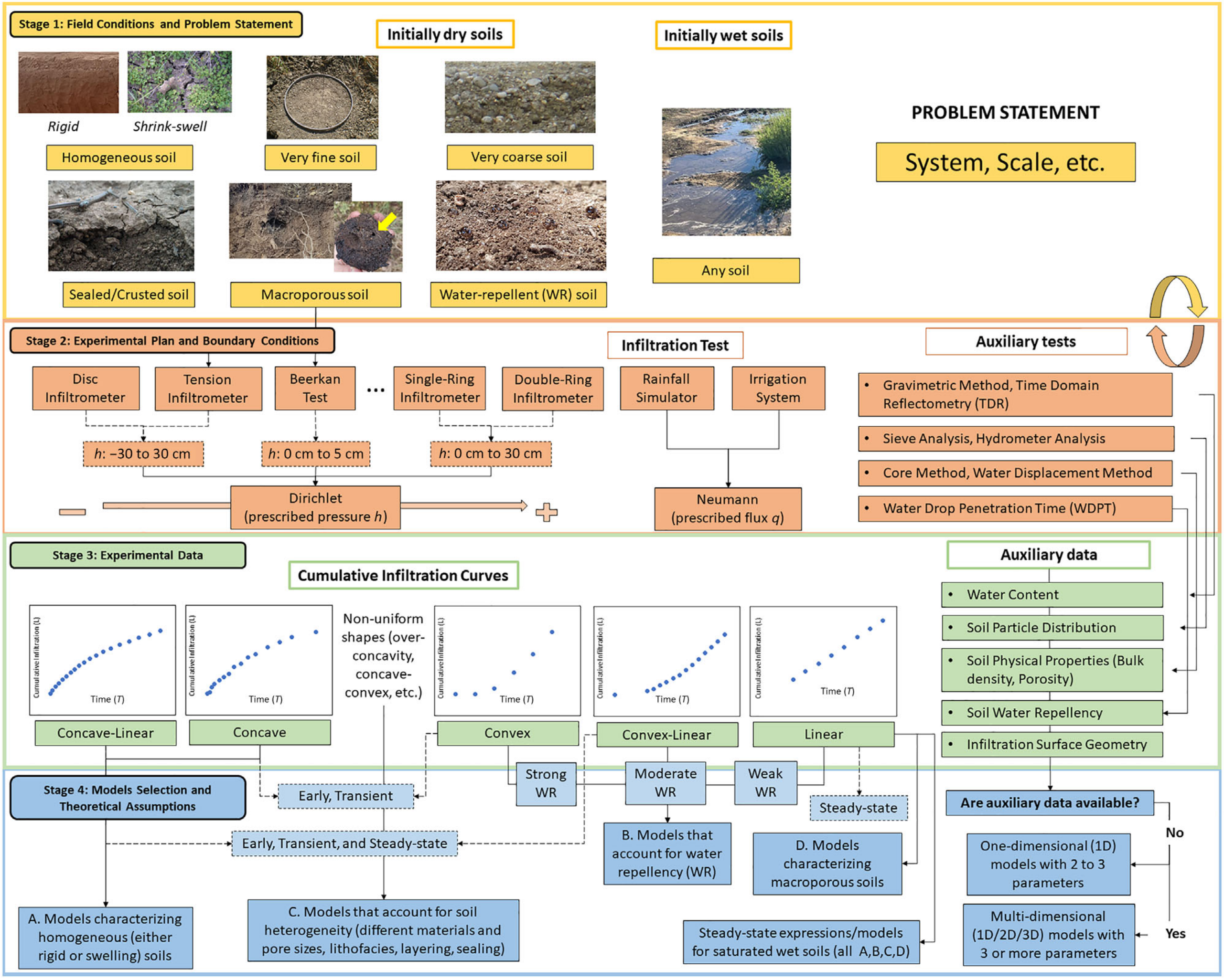

FIGURE 6 A decision-making framework to guide researchers in selecting the most suitable model for their specific study conditions.

modeled using steady-state expressions. Water-repellent soils may display convex curves exhibiting strong WR (convex), moderate WR (convex followed by a linear portion, i.e., convex-linear), or light WR (almost linear), requiring models that account for water repellency such as Abou Najm et al. (2021) and the BEST-WR model of Di Prima et al. (2021) (i.e., option B). Other unique conditions (or their combinations) such as soil heterogeneity, including the presence of macropores, lithofacies with contrasting properties, layering, and sealing, may result in nonuniform infiltration curves (e.g., concave-convex, convex-concave, or extra concavity). Under such conditions, models that characterize infiltration into heterogeneous soils (i.e., option C) are more appropriate. We emphasize that the presence of macropores (due to bioturbation or plant root systems) with high infiltration rates often exhibit a linear shape pointing to the use of models incorporating preferential paths such as Weiler (2005), Germann et al. (2007), Lassabatère et al. (2014), and Stewart (2019), among others (i.e., option D).

11 | CONCLUSION

It is a challenge to choose which infiltration model to use, and which procedures to follow for estimating hydraulic properties. The proliferation of infiltration models has led to better understanding and quantification, but also to confusion regarding the origins, assumptions, and limitations of different approaches. While many reviews have covered different aspects of infiltration processes and modeling, until now there has not been a comprehensive and objective examination of infiltration models in the literature. This gap motivated us to develop a literature review that summarizes and organizes the many distinct conceptual and empirical infiltration models that have been developed over the past two centuries. We identified 138 unique infiltration models and categorized them based on characteristics such as conceptual versus empirical equations, application to rigid versus deformable swelling soils, one-dimensional versus multi-dimensional infiltration, unsaturated to completely saturated porous media, and so on.

Most of the developed infiltration models clustered theoretically around major milestones that were achieved by six or seven major contributions. Our citation analysis determined that Darcy's law, the Richardson–Richards–Buckingham paradigm, and the GA models were very common sources for subsequent advancements, while models building on Stokes' work and other earlier theories represent potential areas for further investigation and advancement. We proposed a framework for the selection of suitable infiltration models given field conditions, data availability, and experimental constraints.

We end our critical review by embracing how the evolution of infiltration models has led to incremental advancements along with limited improvements in characterizing the inherent variability of soil systems, preferential flows, water repellency, and surface processes. With that being said, we should always aspire to develop practical and adaptive models to characterize the infiltration behavior by treating all theories with the critical lens of a curious scientist and step outside the comfort zone secured with some of our basic assumptions, theories, and boundary conditions.

AUTHOR CONTRIBUTIONS

Christelle Basset: Conceptualization; data curation; formal analysis; investigation; methodology; visualization; writing—original draft; writing—review and editing. **Majdi Abou Najm**: Conceptualization; data curation; formal analysis; funding acquisition; investigation; methodology; project administration; resources; supervision; visualization; writing—original draft; writing—review and editing. **Rafael Angulo-Jaramillo**: Formal analysis; investigation; validation; writing—review and editing. **Vincenzo Bagarello**: Formal analysis; investigation; validation; writing—review and editing. **Behzad Ghanbarian**: Formal analysis; investigation; validation; writing—review and editing. **Simone Di Prima**: Formal analysis; investigation; validation; writing—review and editing. **Massimo Iovino**: Formal analysis; investigation; validation; writing—review and editing. **Laurent Lassabatère**: Formal analysis; investigation; validation; writing—review and editing. **Ryan Stewart**: Data curation; formal analysis; investigation; validation; writing—review and editing.

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APPENDIX

TABLE A1 Summary of model parameters.

Symbol	Unit	Description
Characteristics		
A	L^2	Area
$\tilde{A}$	$L^2\ T^{-2}$	Specific Helmholtz free energy
b_R	$L\ T^{-1}$	Conductance of a rivulet (Germann et al., 2007)
C	$M\ L^{-3}$	Concentration
d	L	Depth
D	$L^2\ T^{-1}$	Diffusivity
D_b	Dimensionless	Backbone fractal dimension in three dimensions (Hunt et al., 2014, 2017)
e	$L^3\ L^{-3}$	Void ratio
E	$L^2\ T^{-2}\ T^{o-1}$	Entropy
E	$M\ L^{-1}\ T^{-2}$	Energy
g	$L\ T^{-2}$	Acceleration of gravity
h	L	Total head
i	$L\ T^{-1}$	Infiltration rate
I	L	Cumulative infiltration
$\bar{K}$	$L\ T^{-1}$	Averaged hydraulic conductivity
k_{rc}	Dimensionless	Relative water conductivity accounting for air-confining condition (Wang et al., 1997)
k	L^2	Permeability
K	$L\ T^{-1}$	Hydraulic conductivity
l	L	Thickness
L	L	Length
P	$M\ L^{-1}\ T^{-2}$	Pressure
q	$L\ T^{-1}$	Flux
q_0	$L\ T^{-1}$	Precipitation/rainfall rate
Q_0	$L^3\ T^{-1}$	Volumetric Precipitation/rainfall rate
Q	$L^3\ T^{-1}$	Volumetric flow rate
Q_{inf}	$L^3\ T^{-1}$	Volumetric infiltration rate
r	L	Radius
s	Dimensionless	Slope
s	T^{-1}	Sink-source term
S_0	$L\ T^{-0.5}$	Sorptivity of the soil alone (with $h_0 = 0$)
S_H	$L\ T^{-0.5}$	Sorptivity including the effect of the constant ponded head h_0
S_e	$L^3\ L^{-3}$	Saturation degree
t	T	Time
t_c	T	Critical time between transient and steady-state infiltration
t_s	T	Travel time under steady-state conditions
T_c	T	Sorptive time
T_S	T	Duration of rainfall
T	T°	Temperature

(Continues)

TABLE A1 (Continued)

Symbol	Unit	Description
v	L T^{-1}	Velocity
V	L^3	Volume
$\bar{V}$	Dimensionless	Slope of the shrinkage curve
w	L	Width
w_f	Dimensionless	Ratio of the volume occupied by each fast-flow region to total pore volume (called β in Stewart, 2019)
W	L	Perimeter
W^*	L	Adjusted wetted perimeter (Bautista et al., 2014, 2016; Warrick et al., 2007)
z	L	Vertical distance in the z-direction (+ downward)
ϑ	L^2 T^{-1}	Volumetric infiltration rate per unit length
σ	M T^{-1} T^{-2}	Stress
μ	M T^{-1} T^{-1}	Dynamic viscosity of the fluid
γ_w	Dimensionless	Wet specific gravity
ρ_b	M L^{-3}	Soil bulk density
$\tilde{\mu}$	L^4 M^{-1} T^{-2}	Chemical potential
ν	Dimensionless	Geometrical aspect ratio
ϕ	L^3 L^{-3}	Volume fraction
θ	L^3 L^{-3}	Volumetric water content
ω	M M^{-1}	Gravimetric water content
φ	L^2 T^{-1}	Matric flux potential
u^2	Dimensionless	Anisotropy
λ	Dimensionless	Pore size distribution index
$\emptyset$	L^3 L^{-3}	Porosity
Γ	T^{-1}	Transfer of water between two pore systems
ψ	L	Matric head
ψ_{str}	L	Air entry value
ψ_{wf}	L	Wetting front potential
γ_0	Degrees	Slope angle of the infiltration surface
ε	Dimensionless	Surface roughness coefficient
λ_c	L	Macroscopic capillary length (called λ_s in Wu & Pan, 1997; Wu et al., 1999; Abou Najm et al., 2018a, 2018b)
κ	L^3 T M^{-1}	Second order positive semi-definite tensor (Bennethum & Cushman, 1996)
α_{wr}	L^{-1}	Rate of water repellency attenuation (Abou Najm et al., 2021)
Υ	Dimensionless	Ratio of volume occupied by border cracks to total crack volume (Stewart, 2018)
Model constants		
a	Dimensionless	Wu and Pan (1997); Wu et al. (1999); Stewart and Abou Najm (2018a, 2018b)
a_1	L^{-1}	originally called a (Beven & German, 1981; Germann, 1985)
a_2	Dimensionless	originally called a' (Beven & German, 1981; Germann, 1985)
a_3	Dimensionless	Lambe and Whitman (1979); Su et al. (2020)
a_4	Dimensionless	Originally called a (Govindaraju et al., 2012)
a^*	Dimensionless	Originally called a (Basha, 1994)
a'	Dimensionless	Originally called a (Wind, 1995)
a_{sn}	Dimensionless	Originally called a (Setiawan & Nakano, 1993)
A	L T^{-1}	Mezencev (1948); Philip (1957a, 1957b)

(Continues)

TABLE A1 (Continued)

Symbol	Unit	Description
A_0	Dimensionless	Swartzendruber (1987b); Swartzendruber and Hogarth (1991)
A_1	T^{-1}	Originally called A (Holtan, 1961; Huggins & Monke; 1966; Overton, 1964)
A^*	Dimensionless	Originally called A (Smith, 1972)
A'	Dimensionless	Originally called A (Warrick et al., 1985)
A''	Dimensionless	Originally called A (Lambe & Whitman, 1979; Su et al., 2020)
b	Dimensionless	White et al. (1992); Fallow et al. (1994); Wu and Pan (1997); Wu et al. (1999); Stewart and Abou Najm (2018a, 2018b)
b_0	Dimensionless	Originally called b (Broadbridge & White, 1988)
b_1	Dimensionless	Originally called b (Beven & German, 1981; Germann, 1985)
b_2	Dimensionless	Originally called b (Swartzendruber, 1974)
b^*	Dimensionless	Originally called b (Basha, 1994)
b'	T^{-1}	Originally b (Wind, 1995)
b_{sn}	Dimensionless	Originally called b (Setiawan & Nakano, 1993)
B	Dimensionless	Chu (1985)
B'	Dimensionless	Originally B (Warrick et al., 1985)
B''	Dimensionless	Originally called B (Lambe & Whitman, 1979; Su et al., 2020)
c	L	Wu et al. (1999)
c_0	Dimensionless	Originally called c (Poulovassilis & Argyrokastritis, 2020)
c_1	Dimensionless	Corradini et al. (1997, 2011)
c_2	Dimensionless	Corradini et al. (1997, 2011)
c_3	Dimensionless	Corradini et al. (1997, 2011)
c_4	Dimensionless	Originally called c (Swartzendruber, 1974)
C	Dimensionless	Reynolds et al. (1983, 1985)
C'	Dimensionless	Originally C (Warrick et al., 1985)
C_1	$L\ T^{-0.5}$	Kutilek and Krejca (1987)
C_2	$L\ T^{-1}$	Kutilek and Krejca (1987)
C_3	$L\ T^{-1.5}$	Kutilek and Krejca (1987)
d_1	Dimensionless	Corradini et al. (2011)
D_g	Dimensionless	Originally called D (Ghanbarian et al., 2016)
f	L^{-1}	Beven (1984)
f_i	Dimensionless	Morel-Seytoux and Khanji (1976)
F_1	Dimensionless	Ali et al. (2013)
F_2	Dimensionless	Ali et al. (2013)
F_3	Dimensionless	Ali et al. (2013)
G	Dimensionless	Reynolds and Elrick (1990)
k_1	Dimensionless	Originally called k (Overton, 1964)
k'	$L\ T^{-1}$	Originally called k (Kostiakov, 1932)
k''	Dimensionless	Originally called k' (Mezencev, 1948)
K_F	Dimensionless	Horton (1940, 1941)
m_{ssc}	Dimensionless	Originally called m (Su et al., 2020; van Genuchten, 1980)
m'	Dimensionless	Lambe and Whitman (1979); Su et al. (2020)
n	Dimensionless	Brooks and Corey (1964); Lassabatère et al. (2006); Essig et al. (2009)
n_1	Dimensionless	Originally called n (Swartzendruber, 1974)

(Continues)

TABLE A1 (Continued)

Symbol	Unit	Description
n_{ssc}	Dimensionless	Originally called n (Su et al., 2020; van Genuchten, 1980)
n'	Dimensionless	Originally n (Kostiakov, 1932)
n''	Dimensionless	Originally called n' (Mezencev, 1948)
n^*	Dimensionless	Originally called n (Selker et al., 1999a)
p	Dimensionless	Philip (1993); Corradini et al. (1994, 1997)
P	Dimensionless	Holtan (1961); Huggins and Monke (1966)
t_0	T	Smith (1972)
t_S	T	Srivastava et al. (1996); Chen and Young (2006)
α	L^{-1}	Gardner (1958); Philip (1968); Warrick et al. (1985); Ankeny et al. (1991)
α_1	Dimensionless	Originally called α (Corradini et al., 2000, 2011; Smith et al., 1999)
α_3	Dimensionless	Lambe and Whitman (1979); Su et al. (2020)
α_{swell}	Dimensionless	Originally called α (Smiles & Raats, 2005; Su, 2009, 2010)
α_k	Dimensionless	Originally called α (Das & Kluitenberg, 1995; Kosugi,1999)
α^*	Dimensionless	Originally called α (Smith, 1972)
α'	Dimensionless	Originally called α (Srivastava et al., 1996)
β_0	Dimensionless	Brutsaert (1977); Selker and Assouline (2017); called A and α in Stewart (2018, 2019), respectively
β	Dimensionless	Haverkamp et al. (1994); Lassabatère et al. (2006, 2014); Rahmati et al. (2019)
β_1	T^{-1}	Originally called β (Novak & Soltesz, 1984)
β_2	Dimensionless	Originally called β (Philip, 1972)
β_3	Dimensionless	Originally called β (Morel-Seytoux & Khanji, 1974, 1976)
β_4	Dimensionless	Originally called β (Su, 2010)
β^*	Dimensionless	Originally called β (Selker et al., 1999a)
β'	Dimensionless	Originally β (Srivastava et al., 1996)
β_{swell}	Dimensionless	Originally called β (Smiles & Raats, 2005; Su, 2009, 2010)
β_l	Dimensionless	Originally called β (Libardi et al., 1980)
β_g	Dimensionless	Originally called β (Ghanbarian et al., 2016)
β_k	Dimensionless	Originally called β (Das & Kluitenberg, 1995; Kosugi, 1999)
δ	Dimensionless	Parlange et al. (1985, 2002); Haverkamp et al. (1990); Smith et al. (1993); Corradini et al. (1994), originally called α in Parlange et al. (1982)
δ'	Dimensionless	Originally called δ (Srivastava et al., 1996)
γ	Dimensionless	Haverkamp et al. (1994); Lassabatère et al. (2006, 2014); Warrick and Lazarovitch (2007); Warrick et al. (2007)
γ_k	Dimensionless	Originally called δ (Das & Kluitenberg, 1995
γ_k	Dimensionless	Originally called γ (Das & Kluitenberg, 1995; Kosugi, 1999)
λ_g	Dimensionless	Originally called λ (Ghanbarian et al., 2016)
$\mathcal{B}$	Dimensionless	Originally called β (Corradini et al., 1994, 1997; Corradini et al., 2000, 2011; Smith et al., 1993; Smith et al., 1999)
σ_k	Dimensionless	Originally called σ (Das & Kluitenberg, 1995; Kosugi, 1999)

TABLE A2 Detailed summary of infiltration models.

Model	Year	Equation(s)		Applied concepts
Green and Ampt	1911	$t = \frac{I}{K_s} - \frac{(\psi_{wf} + h_0)(\theta_s - \theta_i)}{K_s} \ln\left(1 + \frac{I}{(\theta_s - \theta_i)(\psi_{wf} + h_0)}\right)$		Green and Ampt equation is derived from the following expression of the infiltration rate, I (L T^{-1}), with respect to time: $i = K_s \left[\frac{\psi_{wf} + z_w + h_0}{z_w}\right] = \frac{d[z_w(\theta_s - \theta_i)]}{dt}$
Kostiakov	1932	$I = k' t^{n'}$		Kostiakov proposed the empirical equation from field-measured data, which predicts physically a zero-infiltration capacity for prolonged rainfall events. (See Eq. 3 in Ghorbani Dashkati et al., 2009). $k' > 0$ and $0 < n' < 1$
Horton	1940	$i = i_\infty + (i_i - i_\infty) e^{-K_F t}$		Horton derived the empirical equation from field-measured data, assuming that the reduction in the infiltration capacity is directly proportional to the rate of infiltration and is applicable only when the effective rainfall intensity is greater than i_∞. (See Eq. I in Horton, 1941). $i_\infty = K_s$ and $K_F > 0$
Mezencev	1948	$I = k'' t^{n''} + At$		Mezencev modified the Kostiakov's model (1932) by including a linear term with a coefficient A (L T^{-1}) that is equal to the saturated hydraulic conductivity, K_s (L T^{-1}), at infinite times. (See Eq. 5 in Ghorbani Dashkati et al., 2009). $k'' > 0$ and $0 < n'' < 1$
Hansen	1955	$x_w = \left(\frac{2Kh}{\emptyset \Delta S}\right)^{0.5} t^{0.5}$ $t = \frac{\emptyset \Delta S}{K} [h\{1 - \ln(h - z_w)\} - z_w]$ $t = \frac{\emptyset \Delta S}{K} [h\{1 - \ln(h + z_w)\} + z_w]$	Horizontal Upward Downward	Hansen related the soil-water movement and one-dimensional infiltration in the horizontal, upward, and downward directions and noted that the nature of flow from a free water surface through unsaturated soil exhibited three distinct zones, namely, the transmission zone, the wetting zone, and the wetting front.
Philip	1957a	$I = \sum_{i=1}^{\infty} A_n(\theta)\, t^{n/2} + K_i t$		A time series solution for $x(\theta, t)$ is developed in powers of $t^{1/2}$ as: $x(\theta, t) = \varphi(\theta) t^{\frac{1}{2}} + \chi(\theta) t + \psi(\theta) t^{\frac{2}{3}} + \omega(\theta) t^2 + \cdots$ The first term, $X_1(\theta)$, reflects the influence of the capillary forces on the flow process, and the following terms, $X_2(\theta)$ and $X_3(\theta)\ldots$, reflect the gravity effect on infiltration. The cumulative infiltration equation, I (L), was obtained by integrating the time series solution for $z(\emptyset, t)$, resulting in: $I = A_1(\theta) t^{1/2} + (A_2(\theta) + K_i) t + A_3(\theta) t^{\frac{2}{3}} + A_4(\theta) t^2 + \cdots$ (Implicit) One broad equation describes early and transient infiltration behaviors.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Philip	1957b	$I = S_0 t^{0.5} + At \quad 0 \le t \le t_c$ $I = K_s t + \frac{S_0^{\,2}}{4(K_s - A)} \quad t > t_c$ Where $t_c = \frac{S_0^{\,2}}{4(K_s - A)^2}$	The Philip's time series solution for $x(\theta, t)$ is approximated by the two first terms for small times. The parameter A is an estimate of ($A_2 + K_i$ + the truncation error ε) such that $A = mK_s$ where $1/3 \le m \le 2/3$ (Philip, 1969a; Talsma, 1969, Talsma & Parlange, 1972; Youngs, 1968). $m = 0.363$ may be appropriate for soils with a relatively low initial moisture content (Philip, 1987), while a value of $m = 2/3$ is often used (Fodor et al., 2011; Whisler & Bouwer, 1970). In some studies (Davidoff & Selim, 1986; Ghorbani Dashkati et al., 2009; Swartzentruber & Young, 1974), $m = 1$ and hence $A = K_s$. Then, for large times ($t > t_c$), $I = K_s$ t which is nothing but the linear term in the equation valid for short times ($0 \le t \le t_c$). (See Eqs. 39, 40, and 45 in the handbook of Groundwater Engineering by Delleur, 1998). (Explicit) Two specific equations were developed for each infiltration behavior, the first describing the early-state ($0 \le t \le t_c$), while the second describing steady-state behavior ($t > t_c$).
Holtan	1961	$i = i_\infty + A_1(s_0 - I)^P$ Where $s_0 = \emptyset - \theta_i$	Holtan expressed the infiltration capacity as a function of the residual potential storage in the upper soil layers. The advantage of Holtan's approach is that it can compute the infiltration rate for both periods of rain intensity lower than infiltration capacity and periods of no rain. (See Eq.10 in Huggins and Monke, 1966).
Overton	1964	$I = k_1 s_0 - \sqrt{\frac{i_\infty}{A_1}} \tan[\sqrt{A_1 i_\infty}(t_c - t)]$ Where $t_c = \frac{1}{\sqrt{A_1 i_\infty}} \tan^{-1}\left(\sqrt{\frac{A_1}{i_\infty}} k_1 s_0\right)$ and $s_0 = \emptyset - \theta_i$	Using the Holtan model with P = 2, Overton derived an infiltration equation which computes the infiltration rate at any time during a storm, even when rainfall does not exceed the infiltration capacity or when there is a temporary interruption in rainfall. k_1 is based on vegetation, varying between 0.3 for weeds and 1 for bluegrass.
Huggins and Monke	1966	$i = i_\infty + A_1\left(\frac{s_0 - I}{\emptyset}\right)^P$	Huggins and Monke introduced porosity Ø ($L^3\ L^{-3}$) in the Holtan model. As s_0 (L) takes the dimension of length, it cannot be equal to a non-dimensional quantity, i.e., $(\emptyset - \theta_i)$ as originally hypothesized, rather it is equal to $(\emptyset - \theta_i)$ times the depth of soil stratum above the impeding layer.
Fok and Hansen	1966	$\frac{I}{h\emptyset \Delta S} - ln\left(1 + \frac{I}{h\emptyset \Delta S}\right) = \frac{Kt}{h\emptyset \Delta S}$	Fok and Hansen derived an equation for one-dimensional downward infiltration from furrow irrigation based on Hansen (1955) who described soil-water movement through homogeneous unsaturated soils exhibiting three distinct zones, the transmission zone, the wetting zone, and the wetting front. Within the transmission zone (T), the hydraulic conductivity, K ($L\ T^{-1}$), is essentially constant. The moisture content, and thus the hydraulic conductivity within the wetting zone, decreases toward the wetting front; however, the energy consumed within this zone is approximately constant. The wetting front is, in effect, a capillary fringe; the moisture content of the foremost part is approximately the same.
Philip	1967	$I = S_0 t^{0.5} + A_2 t + A_3 t^{3/2} + A_4 t^2 + \cdots$	Philip developed the following one-dimensional infiltration equation for an unsaturated heterogeneous medium: $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K(\theta, x)\frac{\partial}{\partial x}\{\psi(\theta, x)\}\right) - \frac{\partial}{\partial x}\{K(\theta, x)\}$. To solve this equation, Philip (1967) applied a quasi-analytical method previously established in Philip (1957a) for small and moderate times: $x(\psi, t) = \phi_1(\psi)t^{1/2} + \chi_1(\psi)t + \cdots$ The expression for cumulative infiltration I(L) into this subclass of scale-heterogeneous media is formally similar to that for infiltration into homogeneous media (Philip, 1957a, 1957b).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Philip	1968	For buried point source, $i = \frac{\alpha^2 Q_0 i^*}{8\pi}$ Where $i^* = \frac{1}{2T} e^{Z-R}\left[1 + \frac{Z}{T} + \frac{Z}{T^2}\right]$ direction of z $i^* = \frac{R}{2T^2} e^{Z-R}\left[1 + \frac{1}{T}\right]$ direction of r $R = 0.5\alpha r$; $Z = 0.5\alpha z$ and $T = \sqrt{R^2 + Z^2}$ For spherical cavity, $Q_{inf} = 4\pi r\varphi V^*$ $V^* = (1 - R)^{-1}$ (1) $V^* = 1$ (2) $V^* = R$ (3)	Philip treated 3D steady infiltration from buried point sources and spherical cavities of continuous supply Q_0 (L^3 T^{-1}). Hydraulic conductivity K (L T^{-1}) depends exponentially on moisture (capillary) potential ψ (L) ➔ $K(\psi) = K_s\, e^{\alpha\psi}$; For soils initially at "field capacity" or drier, $e^{\alpha\psi_i} \ll 1$, and thus $\varphi = \frac{K_s}{\alpha^*}$. The fundamental point source solution is first explored and then is used as the basis of an analysis of steady infiltration from spherical cavities. Philip analyzed steady infiltration from spherical cavities using the dimensionless infiltration V^* for small R (1), capillarity dominant ($R \to 0$) (2), and for gravity dominant ($R \to \infty$) (3).
Wooding	1968	$Q_{inf} = \pi r^2 \alpha\varphi + 4r\varphi$	Wooding approximated a steady-state infiltration of water from a circular ring of radius r (L) assuming no surface ponding and the ratio $K(\psi)/\varphi(\psi)$ is equal to the constant parameter α (Philip,1968). (See Eq.4 in Scooter et al., 1982).
Philip	1969a	1D Absorption and Infiltration (1): For absorption (1.1): $i = \frac{1}{2} S_0 t^{-1/2}$ (1.1.A) $i = \sqrt{\frac{D^*}{\pi}}(\theta_s - \theta_i)t^{-1/2}$ (1.1.B) $i = \frac{1}{2}\sqrt{2K_s C(\theta_s - \theta_i)}t^{-1/2}$ (1.1.C) Where $D^* = \frac{\pi S_0{}^2}{4(\theta_s - \theta_i)^2}$ and $C = \frac{S_0^2}{2K_s(\theta_s - \theta_i)}$ For Infiltration (1.2): (1.2.A) $i = \frac{1}{2} S_0 t^{-1/2} + (A_2 + K_i) + \frac{3}{2} A_3 t^{1/2} + \cdots$ $t < \infty$ $i = K_s$ $t \to \infty$ $i = (K_s - K_i)\, i^* + K_s$ (1.2.B and C) $i^* = \frac{1}{2}\left(\frac{1}{\sqrt{\pi T}} - 1 + \cdots\right)$ small T (B) $i^* = \frac{1}{4\sqrt{\pi}T^{\frac{3}{2}}} e^{-T}$ large T (B) $T = \frac{1}{2\pi}(\frac{1}{i^*} - \ln(1 + \frac{1}{i^*}))$ small T (C) $i^* = \frac{1}{2\pi T}$ large T (C) Where $T = \frac{(K_s - K_i)^2 t}{\pi S_0{}^2}$ 2D and 3D Absorption and Infiltration (2): For 2D and 3D absorption (2.1): $i = \frac{(\theta_s - \theta_i) D^* i^*}{r}$	A. Philip expressed the dynamics of cumulative 1D surface absorption (1.1) and surface infiltration rate i (L T^{-1}) (1.2), as well as two- and three- dimensional subsurface infiltration (2.2) with non-linear exact series solutions. In two-dimensions, there is no steady-state (i.e., $t \to \infty$), unless diffusivity D (L^2 T^{-1}) takes particular values, e.g., for constant D or approaching delta function. B. Philip also expressed the dynamics of 1D surface absorption (1.1) and surface infiltration rate i (L T^{-1}) (1.2), as well as two- and three- dimensional subsurface absorption (2.1) and infiltration (2.2) in terms of dimensionless functions for constant diffusivity D (L^2 T^{-1}) with hydraulic conductivity K (L T^{-1}) depending exponentially on moisture (capillary) potential ➔ $K(\psi) = K_s\, e^{\alpha\psi}$; For soils initially at "field capacity" or drier, $e^{\alpha\psi_i} \ll 1$, and thus $\varphi = \frac{K_s}{\alpha}$. C. Philip also expressed the dynamics of 1D cumulative surface absorption (1.1) and surface infiltration i (L T^{-1}) (1.2), as well as two- and three- dimensional subsurface absorption i (L T^{-1}) (2.1) in terms of dimensionless function i^* for diffusivity D (L^2 T^{-1}) approaching delta function. K_0 *is the modified Bessel function of the second kind of order 0.*

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)		Applied concepts
		Where $D^* = \frac{\pi S_0{}^2}{4(\theta_s - \theta_i)^2}$	(2.1.B)	
		For cylindrical cavity:		
		$i^* = \frac{1}{\sqrt{\pi T}} + \frac{1}{2} - \frac{1}{4}\sqrt{\frac{T}{\pi}} + \frac{T}{8} - \ldots$	small T	
		$i^* = \frac{2}{lnT}$	large T	
		Where $T = \frac{\pi t}{t_{\text{geom}}}$; $t_{\text{geom}} = \left(\frac{2r(\theta_s - \theta_i)}{S_0}\right)^2$ and $\gamma = 0.57722$		
		For spherical cavity:		
		$i^* = \frac{1}{\sqrt{\pi T}} + 1$	large T	
		Where $T = \frac{\pi t}{4t_{\text{geom}}}$ and $t_{\text{geom}} = \left(\frac{r(\theta_s - \theta_i)}{S_0}\right)^2$	(2.1.C)	
		For cylindrical cavity:		
		$T = \frac{\pi}{8}\left[\left(\frac{4}{\pi i^*} - 1\right)e^{4/\pi i^*} + 1\right]$	small T	
		$i^* = \frac{4}{\pi \ln(T)}$	large T	
		Where $T = \frac{\pi t}{t_{\text{geom}}}$ and $t_{\text{geom}} = \left(\frac{2r(\theta_s - \theta_i)}{S_0}\right)^2$		
		For spherical cavity:		
		$T = \frac{\pi}{12}\left[2\left(1 - \frac{2}{\pi i^*}\right)^{-3} - 3(1 - \frac{2}{\pi i^*})^{-2} + 1\right]$	small T	
		$i^* = \frac{2}{\pi} + \left(\frac{4\pi^2}{3T}\right)^{1/3}$	large T	
		Where $T = \frac{\pi t}{4t_{\text{geom}}}$ and $t_{\text{geom}} = \left(\frac{r(\theta_s - \theta_i)}{S_0}\right)^2$		
		For 2D and 3D infiltration (2.2):	(2.2.A)	
		$i = \frac{1}{2} S_0 t^{-1/2} + \frac{A_2^*}{r} + \frac{3A_3^*}{2r^2} t^{1/2} + \cdots$	$t < \infty$	
		$i_{3D} = \int_{\theta_i}^{\theta_s} \frac{D(\theta)}{r} d\theta$	$t \rightarrow \infty$	
		$i = k\left(\theta_s - \theta_i\right) i^*$	(2.2.B)	
		For cylindrical cavity:		
		$i^* = \frac{1}{4R\text{K}_0(2R)}$	$t \rightarrow \infty$	
		For spherical cavity:		
		$i^* = 1/2[\frac{1}{\sqrt{\pi T^*}} e^{-T^*} + \frac{1}{2R} + erf(\sqrt{T})]$	$t < \infty$	
		$i^* = \frac{1}{2} + \frac{1}{4R}$	$t \rightarrow \infty$	
		Where $k = \frac{(K_s - K_i)}{(\theta_s - \theta_i)}$; $R = \frac{kr}{4D^*}$; $T^* = \frac{k^2 t}{4D^*}$ and $D^* = \frac{\pi S_0{}^2}{4(\theta_s - \theta_i)^2}$		
Parlange	1971a	$I = \int_{\theta_i}^{\theta_s} z_w d\theta$ $z_w = \frac{\sqrt{2t}\int_0^{\theta_s} D(\alpha)d\alpha}{\sqrt{\int_{\theta_i}^{\theta_s} \alpha D(\alpha)d\alpha}}$		An accurate, analytic representation of one-dimensional horizontal infiltration was developed by Parlange et al. where absorption is dominant and gravity effect is omitted.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Parlange	1971b	$I = \int_{\theta_i}^{\theta_s} z_w d\theta$ $z_w = K_s\,(t - t_i) + \int_{\theta}^{\theta_s - \Delta\theta} \frac{D(\alpha)d\alpha}{K_s\alpha - K(\alpha)}$ $\quad t < t_\infty$	The theory by (Parlange, 1971a) was extended to develop an accurate, analytic representation of one-dimensional infiltration, valid for all times, where gravity plays a role. Diffusivity D ($L^2\,T^{-1}$) and hydraulic conductivity K ($L\,T^{-1}$) vary rapidly near saturation.
Parlange	1971c	2D Absorption: $I = \frac{r\sqrt{4D_i\tau}\int_{\theta}^{\theta_s} D(\alpha)d\alpha}{\int_{\theta_i}^{\theta_s} \alpha D(\alpha)d\alpha}$ 3D Absorption: $1 - \frac{r}{I} = g(\tau)\int_{\theta}^{\theta_s} D(\alpha)d\alpha$ Where $\tau = \frac{t}{r^2}$ and $g(\tau)$ is given by Equation (20)	An analytic solution for absorption (i.e., movement without gravity) in two and three dimensions is obtained, valid for all times, for arbitrary diffusivity D ($L^2\,T^{-1}$). For simplicity, the two- and three-dimensional problems are treated for the case of a cylinder and a sphere, respectively.
Philip	1971	2D Infiltration: $\vartheta = \frac{\alpha Q_0 \vartheta^*}{2\pi}$ For surface line source: $\vartheta^* = \frac{Z}{T}\, e^Z K_1(T)$ $\quad$ direction of x $\vartheta^* = X e^Z \left[\frac{K_1(T)}{T} - e^Z \int_Z^{\infty} e^{-Z}\frac{K_1(T)}{T} dZ\right]$ $\quad$ direction of z For buried line source: $\vartheta^* = \frac{1}{2}\, e^Z \left[K_0(T) + \frac{Z}{T} K_1(T)\right]$ $\quad$ direction of x $\vartheta^* = \frac{X}{2T}\, e^Z K_1(T)$ $\quad$ direction of z Where $X = 0.5\alpha x$; $Z = 0.5\alpha z$ and $T = \sqrt{X^2 + Z^2}$ 3D Infiltration: $i = \frac{\alpha^2 Q_0 i^*}{8\pi}$ For surface point source: $i^* = \frac{Z(1+T)}{T^3}\, e^{Z-T}$ $\quad$ direction of z $i^* = \frac{R^2 + ZT(T-Z)}{RT^3}\, e^{Z-T}$ $\quad$ direction of r For buried point source: $i^* = \frac{1}{2T}\, e^{Z-R}[1 + \frac{Z}{T} + \frac{Z}{T^2}]$ $\quad$ direction of z $i^* = \frac{R}{2T^2}\, e^{Z-R}[1 + \frac{1}{T}]$ $\quad$ direction of r Where $R = 0.5\alpha r$, $Z = 0.5\alpha z$, and $T = \sqrt{R^2 + Z^2}$	Philip established the solution of the quasilinear steady infiltration equation for any distribution of surface and buried sources of continuous supply Q_0 ($L^3\,T^{-1}$). Hydraulic conductivity K ($L\,T^{-1}$) depends exponentially on moisture (capillary) potential ➔ $K(\psi) = K_s\, e^{\alpha\psi}$; For soils initially at "field capacity" or drier, $e^{\alpha\psi_i} \ll 1$, and thus $\varphi = \frac{K_s}{\alpha^*}$. *$K_0$ and K_1 are the modified Bessel function of the second kind of order 0 and 1.*
Parlange	1972a	Spherical cavity: $Q_{inf} = 4\pi r\varphi V^*$ Where $V^* = 1 + \alpha r/2$	The solution for the steady state infiltration from a cavity is obtained analytically in two and three dimensions by a singular perturbation technique i.e., a solution valid near the cavity is matched to a solution valid far from it. Details of the method are given for a spherical cavity. The form of the solution depends upon the product $\alpha r \leq 1$ where α (L^{-1}) is a soil property that characterizes the relative importance of gravity and capillarity, and r (L) characterizes the size of the cavity. When $\alpha r \to 0$, gravity effects become negligible in three dimensions. (See Eq. 33 in Parlange et al., 1972b).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Parlange	1972b	Spherical cavity: $Q_{inf} = 4\pi r\varphi V^*$ Where $V^* = \frac{1}{g(t)\varphi} + \frac{r^3}{r_1^2\varphi}\int_{r/r_1}^{1} \gamma^{-4} K d\gamma$ $g(t)$ and $r_1(t)$ are given by Equations (10) and (22) in Parlange's paper, respectively.	An analytical technique is presented to study unsteady infiltration from multidimensional cavities for moderate times. The solution reduces to known expressions for very short (Parlange, 1971c) and very long times (Parlnage, 1972b). Only the simplest geometry, that is, a spherical cavity, is considered.
Smith	1972	$i = i_\infty + A^*(t - t_0)^{-\alpha^*}$	Richards' equation for unsaturated soil moisture flow is solved by extensive numerical simulation of infiltration for various patterns of rainfall to develop a single dimensionless formula that describes the infiltration decay curves for all soils, initial conditions, and rainfall rates.
Philip	1972	2D Infiltration: $\vartheta = \frac{\alpha Q_0 \vartheta^*}{2\pi}$ For surface line source: $\vartheta^* = (1+\beta_2)\frac{Z}{T} e^{(1+\beta_2)Z} K_1[(1+\beta_2)T]$ direction of x $\vartheta^* = (1+\beta_2) X e^{(1+\beta_2)Z}\{\frac{1}{T} K_1[(1+\beta_2)T]$ $-(1+\beta_2)e^{(1+\beta_2)Z}\int_Z^{\infty}\frac{1}{T}e^{-(1+\beta_2)Z}K_1[(1+\beta_2)T]dZ\}$ direction of z For buried line source: $\vartheta^* = \frac{1}{2}(1+\beta_2)e^{(1+\beta_2)Z}\{K_0[(1+\beta_2)T] + \frac{Z}{T}K_1$ $[(1+\beta_2)T]\}$ direction of x $\vartheta^* = \frac{1}{2}(1+\beta_2)\frac{X}{T}e^{(1+\beta_2)Z}K_1[(1+\beta_2)T]$ direction of z Where $X = 0.5\alpha x$; $Z = 0.5\alpha z$ and $T = \sqrt{X^2 + Z^2}$ 3D Infiltration: $i = \frac{\alpha^2 Q_0 i^*}{8\pi}$ For buried point source: $i^* = \frac{1}{2T} e^{(1+\beta_2)(Z-T)}\left[\frac{(1+\beta_2)(Z+T)}{T} + \frac{Z}{T^2}\right]$ direction of z $i^* = \frac{R}{2T^2} e^{(1+\beta_2)(Z-T)}[1 + \beta_2 + \frac{1}{T}]$ direction of r For surface point source: $i^* = \frac{Z}{T^3}[1 + (1+\beta_2)T]e^{(1+\beta_2)(Z-T)}$ direction of z $i^* = \frac{1}{RT^3}[R^2 + (1+\beta_2)ZT(T-Z)]e^{(1+\beta_2)(Z-T)}$ direction of r Where $R = 0.5\alpha r$; $Z = 0.5\alpha z$, and $T = \sqrt{R^2 + Z^2}$	Philip developed a quasi-linearized steady infiltration equation for heterogeneous soils under buried and surface sources of continuous supply Q_0 ($L^3\ T^{-1}$). Hydraulic conductivity K ($L\ T^{-1}$) depends exponentially on both moisture (capillary) potential ψ (L) and depth z (L) ➔ $K(\psi) = K_s\ e^{[\alpha(\psi + \beta_2 z)]}$. β_2 is the dimensionless coefficient of dependence of conductivity K ($L\ T^{-1}$) on depth z (L), $\beta_2 \geq 0$ K_0 *and* K_1 *are the modified Bessel function of the second kind of order 0 and 1.*
Talsma and Parlange	1972	$i^* = \frac{i - K_s}{K_s}$ $2\pi T = \ln(\frac{i^*}{1+i^*}) + \frac{1}{i^*}$ (1) $2\pi T = \ln(\frac{1+i^*}{i^*}) - \frac{1}{1+i^*}$ (2) $2\pi T = 3\{(1+2i^*)ln(\frac{1+i^*}{i^*}) - 2\}$ (3) Where $T = \frac{K_s{}^2 t}{\pi S_0{}^2}$	Taslma and Parlange used the linearized infiltration equations (Philip, 1969a) to develop 1D infiltration equation assuming the diffusivity D ($L^2\ T^{-1}$) is (1) a delta function, (2) proportional to the variation rate of the hydraulic conductivity in respect to the volumetric moisture content $\partial K/\partial\theta$ or (3) $(\theta_s - \theta_i)\frac{D}{K^2}$ is proportional to $\frac{\partial K}{\partial \theta}$

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)		Applied concepts
Parlange	1972c	$D_0(t)\theta_0(t)\,\frac{d\theta_0}{dt} = i[i - K_0(t)]$ $z_w = \int_{\theta}^{\theta_s} \frac{D(\alpha)d\alpha}{i-K(\alpha)}$	$t < t_\infty$ (1) $t \to t_\infty$ (2)	The one-dimensional infiltration was solved analytically. If $i > K_s$ ($t \to t_\infty$), the surface becomes saturated with water and ponding occurs, then equation (1) ceases to apply. Equation (2) (Parlange, 1971b) shows that the limiting profile is identical to the "profile at infinity" and therefore can be applied at infinite times.
Mein and Larson	1973	$t = \frac{I}{K_s} - \frac{(\theta_s-\theta_i)\psi_{wf}}{K_s}\ln\left(1 + \frac{I}{(\theta_s-\theta_i)\psi_{wf}}\right)$ $I_p = i_p\, t_p$ $t - t_p = \frac{I}{K_s} - \frac{i_p t_p}{K_s} - (\psi_{wf} + h_0)(\theta_s - \theta_i)$ $\ln\left(\frac{(\psi_{wf} + h_0)(\theta_s - \theta_i) + I}{(\psi_{wf} + h_0)(\theta_s - \theta_i) + i_p t_p}\right)$ Where $t_p = \frac{K_s \psi_{wf}(\theta_s-\theta_i)}{i_p(i_p-K_s)}$	$0 \le t < t_p$ $t = t_p$ $t > t_p$	Mein and Larson published an extension of the Green and Ampt (1911) model of ponded infiltration, which applied the same piston flow concept to the case of constant flux surface conditions. Prior to the ponding time ($0 \le t < t_p$), the rainfall intensity is less than the potential infiltration rate and the soil surface is unsaturated. Ponding ($t = t_p$) begins when the rainfall intensity exceeds the potential infiltration rate; the soil surface becomes saturated. As rainfall continues ($t > t_p$), the saturated zone extends deeper into the soil. (See Eqs. 8, 9, and 10 in Almedeij and Esen, 2014).
Smiles	1974	$I = S_0 t^{0.5} + 0.5[(1 - \gamma_{w,i})K_i - (1 - \gamma_{w,n})K_n]t$		Smiles approximated infiltration into swelling soils into a two-term infiltration equation in which S_0 (L $T^{-0.5}$) is the analogue of sorptivity in the swelling system and hydraulic conductivity K (L T^{-1}) reflects gravity. As for $(1 - \gamma_w)$, this term may be interpreted as if it reduced the effect of gravity to $(1 - \gamma_w)$ times its values in a rigid soil, and thus increases the time before which the effect of gravity is unimportant when compared with that of the moisture potential gradient.
Turner and Parlange	1974	$I = AS_0\, t^{1/2} + 1/3[AK_s + \frac{0.72 W S_0^2}{\theta_s-\theta_i}]t$		An analytical solution for the lateral movement at the periphery of a one-dimensional flow of water is derived, valid for short times when water infiltrates through a region having an area A (L^2) and a perimeter W (L).
Swartzen-druber	1974	$t = \frac{I}{K_s} - \frac{(\theta_s-\theta_i)\psi_{wf}}{K_s}\ln\left(1 + \frac{I}{(\theta_s-\theta_i)\psi_{wf}}\right)$ $I_p = \frac{(\theta_s-\theta_i)\psi_{wf} K_s}{q_0-K_s}$ $t > t_p$ $I - I_p = \left[\frac{(q_0-K_s)t_p}{1-n_1}\right]\left[(\frac{t}{t_p})^{1-n_1} - 1\right] + K_s(t - t_p)$ $I - I_p = 2(q_0 - K_s)(t_p - c_4)\{[(t - c_4)/(t_p - c_4)]^{0.5} - 1\}$ $+ K_s(t - t_p)$	$0 \le t < t_p$ $t = t_p$ (1) (2)	Swartzendruber presented a theoretical study of one-dimensional infiltration under constant-flux rainfall q_0 (L T^{-1}) into a uniform soil by treating the soil-water profile as step function, as assumed in Green and Ampt (1911). The analysis is similar to that of Mein and Larson (1973) for $t \le t_p$. For describing the infiltration characteristics after the ponding time t_p (L), two explicit-form equations, (1) and (2), are proposed for i versus t, both equations being integrable into explicit forms of I versus t under constant-flux rainfall. Both equations are somewhat like that of Smith (1972) but containing one less characterizing constant. Equation (1) may be viewed as an altered Kostiakov (1932) form, modified to contain the feature of an asymptotically-approached the saturated hydraulic conductivity, K_s (L T^{-1}). In Equation (2), the exponent of $(t - c_4)$ is taken as $-1/2$ rather than the more general $-\alpha^*$ (Smith, 1972). (1) $i = at^{-n_1} + K_s$ where $a = (q_0 - K_s)t_p^{n_1}$ (2) $i = b_2\,(t - c_4)^{-1/2} + K_s$
Morel-Seytoux and Khanji	1974	$t = \frac{\beta_3}{K_s}\left\{I - (\psi_{wf} + h_0)(\theta_s - \theta_i)ln\left[1 + \frac{I}{(\psi_{wf}+h_0)(\theta_s-\theta_i)}\right]\right\}$		Morel–Seytoux and Khanji modified Green and Ampt equation to account for the effects of air viscous resistance to water flow where β_3 (originally called β, dimensionless) is a correction factor for viscosity.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Morel-Seytoux	1976	$I_p = i_p t_p \; t = t_p$ $\frac{K_s}{\beta_3}(t - t_p) = I - I_p - \left[\frac{(\theta_s-\theta_i)(\psi_{wf}+h_0)}{1-f_i} + I_p(1-\frac{1}{\beta_3})\right]$ $ln\left[\frac{1+(1-f_i)I/(\theta_s-\theta_i)(\psi_{wf}+h_0)}{1+(1-f_i)I_p/(\theta_s-\theta_i)(\psi_{wf}+h_0)}\right]$ $t > t_p$ Where $t_p = \frac{\psi_{wf}(\theta_s-\theta_i)}{(1-f_i)i_p}\left(e^{\frac{1}{\beta_3 i_p/K_s-1}} - 1\right)$ constant rainfall rate Check Eq. (55) in Morel-Seytoux's paper for t_p variable rainfall rate	Formulas were derived for prediction of ponding time and cumulative infiltration following ponding under the influence of rainfall. The derivations use a piston profile but with corrections for air flow and resistance effects, that is, do not assume immediate saturation at the surface nor a piston displacement of air by water.
Li et al.	1976	$I = \frac{1}{2}(\psi_{wf} + h_0)(\theta_s - \theta_i)(t^* + \sqrt{(t^*)^2 + 8t^*})$ Where $t^* = \frac{K_s}{(\psi_{wf}+h_0)(\theta_s-\theta_i)} t$	One of the simplest explicit solutions of the implicit Green and Ampt model was derived by Li et al. based on the approximation of the logarithmic term in GA equation using the first term of the power series expansion.
Brutsaert	1977	$I = K_s t + \frac{{S_0}^2}{\beta_0 K_s}[1 - \frac{1}{1+\frac{\beta_0 K_s t^{0.5}}{S_0}}]$	Brutsaert presented a closed form solution for each of the functions $X_1(\theta)$, $X_2(\theta)$, $X_3(\theta)$, and $X_4(\theta)$ in Philip's time series solution for $z(\emptyset, t)$. The method of obtaining these solutions assumes that these functions are near-step functions. $0 \le \beta_0 \le 1$; $\beta_0 = 2/3$ is sufficiently accurate for most soils. However, for soils with a very wide distribution of pore sizes, a larger value such as $\beta_0 = 1$, may yield a better result. For most practical purposes, $\beta_0 = 1$ is recommended (Kutilek et al., 1991).
Collis-George	1977	$I = i_0(tanhT)^{0.5} + i_\infty t$ Where $i_0 = S_0(t_c)^{0.5}$ and $T = t/t_c$	An empirical equation is proposed by Collis–George which satisfies the conditions that cumulative infiltration is proportional to (time)$^{1/2}$ at short times and reaches a steady state infiltration rate at long times.
Hachum and Alfaro	1977	$I = \int_0^t q_0(t)dt - K_i t \qquad 0 < t \le t_p$ $I = I_p + \frac{(\psi_{wf}+h_0)K_s(\theta_s-\theta_i)}{(K_s-K_i)} ln\left\{\frac{K_s(\psi_{wf}+h_0)(\theta_s-\theta_i)+(K_s-K_i)[I-K_i t]}{K_s(\psi_{wf}+h_0)(\theta_s-\theta_i)+(K_s-K_i)[I_p-K_i t_p]}\right\}$ $+ K_s(t - t_p) \qquad t > t_p$	By extending the analysis of Mein and Larson (1973), Hachum and Alfaro developed a physical model for studying one-dimensional infiltration under variable application rainfall rate patterns q_0 (L T^{-1}), subjected to the assumptions of an abrupt wetting front, constant saturated hydraulic conductivity in the wetted zone, K_s (L T^{-1}), and constant wetting front potential ψ_{wf} (L).
Smith and Parlange	1978	$i = K_s(\frac{C}{K_s I} + 1)$ (1) $i = K_s \frac{e^{K_s I/C}}{e^{K_s I/C}-1}$ (2) Where $C = (\psi_{wf} + h_0)(\theta_s - \theta_i)$	By adopting two extreme assumptions concerning the behavior of unsaturated soil hydraulic conductivity K (L T^{-1}) near saturation, Smith and Parlange derived a two-branched model for infiltration rate i (L T^{-1}) for arbitrary rainfall rates. Assumption (1) was that diffusivity D (L^2 T^{-1}) is a step function (i.e., $D(\theta)$ varies rapidly with water content θ (L^3 L^{-3})). For initially ponded conditions, $t_p \rightarrow 0$, and with rainfall rate $q_0 \rightarrow \infty$, the familiar Green and Ampt (1911) expression results. Assumption (2) was that D and $dK/d\theta$ are closely proportional. This model also holds for both rainfall and ponded surface conditions.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Batu	1978	$i = \frac{q_0 \alpha L i^*}{2\pi}$ For single strip source: Check Equations (25) and (26) in Batu's paper, for horizontal and vertical infiltration, respectively. For periodic strip sources equally spaced by 2D (L): Check Eq. (48) and (49) in Batu's paper, for horizontal and vertical infiltration, respectively. Where F_{2N}, F_{2n}, F_{2n-1} and I_n from Equations (20), (21), (22), and (47), in Batu's paper, respectively. $X_0 = \frac{\alpha L}{2}; X_1 = \frac{\alpha D}{2}; X = \frac{\alpha x}{2}; Z = \frac{\alpha z}{2}; L = \frac{w}{2}$	Batu presented analytical solutions for steady 2D infiltration from single and periodic strip sources of width w (L) and constant flux q_0 (L T^{-1}) using Fourier analysis techniques with no flow outside the strip. The theory assumes that the hydraulic conductivity K (L T^{-1}) is an exponential function of the soil water potential ψ (L).
Kutilek	1980	$i = \frac{1}{2} S_0 t^{-1/2}$ $\quad 0 \le t < t_p$ $i = i_p$ $\quad t = t_p$ $i = \frac{1}{2} S_0 \left[t - \frac{S_0^2}{4A^2 b(b-1)}\right]^{-1/2} + A$ $\quad t > t_p$ Where $b = \frac{q_0}{i_\infty}$	An approximate solution of the infiltration equation for rain of constant intensity q_0 (L T^{-1}) is developed. The solution of infiltration for $t > t_p$ is obtained for "delta function" soil.
Parlange	1980	$t = \frac{I}{(K_s - K_i)} - \frac{S_H{}^2}{2(K_s - K_i)^2} \ln\left(1 + \frac{2(K_s - K_i)I}{S_H{}^2}\right)$ (1) $t = \frac{I}{(K_s - K_i)} + \frac{S_H{}^2}{2(K_s - K_i)^2}\left(e^{-\frac{2(K_s - K_i)I}{S_H{}^2}} - 1\right)$ (2)	Using Darcy's law and the conservation of mass, Parlange solved the following partial differential equation governing water movement: $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}[D \frac{\partial \theta}{\partial z}] - \frac{\partial K}{\partial z}$ under ponding conditions ($h_0 \ge 0$), assuming that: (1) D (L^2 T^{-1}) increases rapidly with water content, θ (L^3 L^{-3}), while the conductivity, K (L T^{-1}), varies much less rapidly near saturation and (2) D and $\frac{\partial K}{\partial z}$ increase rapidly, and in similar fashion. (See Eqs. 1 and 2 in Parlange et al., 1982)
Parlange et al.	1982	$t = \frac{S_0{}^2}{2(K_s - K_i)^2(1-\delta)}\left[2\frac{(K_s - K_i)}{S_0{}^2} I - ln \frac{e^{2\delta \frac{(K_s - K_i)}{S_0{}^2} I} + \delta - 1}{\delta}\right]$ $I = S_0\, t^{0.5} + \frac{1}{3}(2 - \delta)(K_s - K_i)t$ $\quad$ small t $I = (K_s - K_i)t + \frac{S_0{}^2}{2(K_s - K_i)(1-\delta)} \ln(\frac{1}{\delta})$ $\quad$ large t	A new infiltration equation is obtained by introducing a new parameter δ to embrace different variations of the diffusivity term D (L^2 T^{-1}) and the hydraulic conductivity K (L T^{-1}) under zero ponding ($h_0 = 0$). D is considered as a delta function. δ (originally called α in Parlange et al., 1982) takes values from 0 to 1, with an approximate used value of 0.85. Equations (1) and (2) by Parlange (1980) can be obtained by setting δ equal to 0 and equal to 1, respectively.
Scotter et al.	1982	$i = K_s + \frac{4\varphi}{\pi r}$	Wooding's equation is rewritten assuming the exponential relationship between $K(\psi)$ and ψ as defined by Gardner (1958): $K(\psi) = K_s\, e^{\alpha\psi}$; For soils initially at "field capacity" or drier, $e^{\alpha\psi_i} \ll 1$, and thus $\varphi = \frac{K_s}{\alpha^*}$.
Fok et al.	1982	$V = \frac{\pi}{2}\, x_w z_w l \emptyset \Delta S_e$ Where $x_w = (\frac{2Kh_x}{\emptyset \Delta S})^{0.5} t^{0.5}; \frac{z_w}{h} - \ln(1 + \frac{z_w}{h}) = \frac{Kt}{\emptyset h \Delta S_e}$	A physical two-dimensional infiltration equation is developed assuming that the loci of the wetting pattern for two-dimensional furrow infiltration are half-ellipses, and their vertical and horizontal flow components can be described as one-dimensional infiltration derived in an earlier study (Fok & Hansen, 1966; Hansen, 1955).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)		Applied concepts
Brakensiek and Rawls	1983	$i = \frac{K_{prec}}{2}\left(\frac{z_w+\psi_{wf,prec}}{z_w}\right)$ $z_{w,p} = \frac{\psi_{wf,subc}-(\frac{2q_0}{K_{s,subc}})(\frac{1-q_0}{q_0})l_c}{\frac{2q_0}{K_{s,subc}}-1}$ $i = \frac{K_e}{2}\left(\frac{z_w+\psi_{wf,subc}+h_0}{z_w}\right)$ Where $K_e = \frac{z_w}{\frac{z_w-l_c}{K_{s,subc}}+\frac{l_c}{K_{s,c}}}$; $r = \frac{K_{s,c}}{K_{s,subc}}$ and $t_p = z_{w,p}\,\emptyset/q_0$	$0 \le t < t_p$ $t = t_p$ $t > t_p$	Brakensiek and Rawls developed a two-layer Green–Ampt model with transient crust conductivity to describe infiltration on crusted soils during uniform rainfall intensity. Infiltration for soil crust modeling may proceed during two periods, pre-ponding, and post-ponding. Crusting of thickness l_c (L) is assumed to start at the time of surface ponding.
Reynolds et al.	1983	$Q_{inf} = \frac{2\pi h_0^2 K_s}{C} + \pi r^2 K_s$		The flow out of a well into the surrounding soil is a three-dimensional infiltration process that achieves steady state rapidly and is described in terms of pressure- and gravity-induced fluxes. The steady flow is affected by weak capillarity assuming semi-infinite, field-saturated flow (i.e., $\varphi_m = 0$). The water head inside the well remains constant. Analytical solutions are described for the estimation of the parameter C.
Novak and Soltesz	1984	$i_f = q_0$ $i_f = K_{s,f} + (q_0 - K_{s,f})e^{-\beta_1(t-t_p)}$	$0 \le t \le t_p$ $t > t_p$	An empirical method is presented to determine the rate of infiltration into heavy, swelling soils, assuming water infiltrates into heavy soils primarily through the cracks, infiltration through the soil matrix is relatively small, and the geometry of cracks is a function of the water content.
Fok and Chiang	1984	$V = \left[2x_w d + wz_w + (\frac{\pi}{2})x_w z_w\right] l\emptyset\Delta S_e$ Where $x_w = \left(\frac{2Kh_x}{\emptyset\Delta S_e}\right)^{0.5} t^{0.5}$; $\frac{z_w}{h} - ln\,(1+\frac{z_w}{h}) = \frac{Kt}{\emptyset h\Delta S_e}$		Fok and Chiang estimated the total volume of water infiltrated into the soil from irrigation furrow of depth d (L), width w (L), and thickness l (L), using developed 1-D and 2-D infiltration equations obtained from previous studies (Fok et al., 1982; Hansen, 1955).
Philip	1984a	$i = 0.5\alpha\varphi i^*$ $i^* = -[R(\gamma + \ln(0.5R))]^{-1}$ $i^* = \frac{2}{\pi}(1 + R^{-2/3})$ Where $\gamma = 0.57722$ and $R = 0.5\alpha r$	 (1) (2)	Philip analyzed steady 2D infiltration from cylindrical cavities averaged over the whole cavity surface using the dimensionless infiltration rate i^* for very small R (1), and for gravity dominant ($R \to \infty$) (2).
Philip	1984b	$i = \frac{\varphi V^*}{r}$ $V^* = 1$ $V^* = (1 - R)^{-1}$ $V^* = 1 + R$ $V^* = 0.5R + R^{1/3}$ Where $R = 0.5\alpha r$	 (1) (2) (3) (4)	Philip analyzed steady 3D infiltration from spherical cavities averaged over the whole cavity surface using the dimensionless infiltration V^* for capillarity dominant ($R \to 0$) (1), very small R (2), small R (3), and for gravity dominant ($R \to \infty$) (4).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Beven	1984	$i = \frac{f K_i(\psi_{wf}+\frac{I}{\theta_s-\theta_i})}{1-e^{-fI/(\theta_s-\theta_i)}}$ $\quad 0 \le t \le t_p$ $t - t_p = \frac{1}{f^* K_i}\left[\ln(I+C) - \frac{1}{e^{f^*C}}\ln(I+C) + \sum_{m=1}^{\infty}\frac{\{f^*(I+C)\}^m}{m!m} - \lambda\right]$ $t > t_p$ Where $t_p = \frac{I_p(1-e^{-fI_p/(\theta_s-\theta_i)})}{fK_i(\psi_{wf}+\frac{I_p}{\theta_s-\theta_i})}$; $C = (\psi_{wf} + h_0)(\theta_s - \theta_i)$; $\lambda = \ln(I_p + C) - \frac{1}{e^{f^*C}}\left[\ln(I_p+C) + \sum_{m=1}^{\infty}\frac{\{f^*(I_p+C)\}^m}{m!m}\right]$ and $f^* = \frac{-f}{\theta_s-\theta_i}$	Beven developed an infiltration model based on the Green-Ampt assumptions on a class of non-uniform soils in which saturated hydraulic conductivity decreases as an exponential function of depth z ($K_s = K_i\, e^{fz}$.
Germann	1985	$i = 0$ $\quad t < 0$ $c = b_1^{1/a_1}\, i^{(1-1/a_1)}$ $\quad 0 \le t \le T_s$ $i = 0$ $\quad t > T_s$	Kinematic wave theory [1] was applied to the infiltration of water in the non-capillary macropore system of a porous medium. Water sorbance from the macropores into the matrix was ignored, limiting the approach to nearly saturated porous media. [1]Kinematic wave theory: $\frac{\partial q}{\partial t} + c\,\frac{\partial q}{\partial z} = 0$ where c (L T^{-1}) is the kinematic wave velocity
Warrick et al.	1985	$I = \frac{(\theta_s-\theta_i)I^*}{\alpha}$ $I^* = A'T^{0.5} + B'T + C'T^{1.5}$ $\quad 0 \le T \le T_g$ $I^* = I_g^* + T - T_g^{\,2}$ $\quad T > T_g$ $I_g^* = A'T_g^{\,0.5} + B'T_g + C'T_g^{\,1.5}$; Where $T = \frac{\alpha K_s t}{\theta_s-\theta_i}$; $T_g = \left(\frac{A'}{1-K_i/K_s}\right)^2$	A generalized solution to the moisture flow Richards' (1931) equation is developed for vertical infiltration using the solution of Philip (1957a) by applying reduced forms of time, depth, water moisture and infiltration.
Reynolds et al.	1985	$Q_{inf} = \frac{2\pi h_0}{C}\,(K_s h_0 + \varphi) + \pi r^2 K_s$	The original theory, as presented in Reynolds et al. (1983), is extended to account for the matric effects of the unsaturated soil (the initial suction head, ψ_i (L), and the capillarity of the outer unsaturated envelope, φ (L^2 T^{-1})). By assuming an exponential relationship between $K(\psi)$ and ψ as defined by Gardner (1958): $K(\psi) = K_s\, e^{\alpha\psi}$, then $\varphi = \int_{\psi_i}^{0} K(\psi)d\psi = \frac{K_s}{\alpha}\,[1 - e^{\alpha\psi_i}]$. For soils initially at "field capacity" or drier, $e^{\alpha\psi_i} \ll 1$, and thus $\varphi = \frac{K_s}{\alpha^*}$ (Scotter et al., 1982). (See Eq. 1 in Reynolds and Elrick, 1985).
Parlange et al.	1985	$t = \frac{S_0^{\,2}}{2\delta(1-\delta)(K_s-K_i)^2}\, ln(1 + \delta\frac{(K_s-K_i)}{i-K_s}) + \frac{K_s h_0(\theta_s-\theta_i)}{(i-K_s)(K_s-K_i)} - \frac{S_0^{\,2}+2\delta(1-\delta)K_s h_0(\theta_s-\theta_i)}{2(1-\delta)(K_s-K_i)^2}\, ln(\frac{i-K_i}{i-K_s})$	Parlange et al. extended the work developed under zero surface head condition (Parlange et al., 1982) to positive head boundary condition for arbitrary diffusivity D (L^2 T^{-1}). (See Eq. 16 in Haverkamp et al., 1988). δ takes values from 0 to 1, and changes slightly with the type of soil. For heavy clay soils, δ tends to be equal to 1, and for coarse-structured soils, the value of δ decreases below the value of $\delta = 0.8$ or 0.85. For the case of $\delta = 0$, the equation takes the Green and Ampt form.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Chu	1985	Constant rainfall: $I = \theta_c\, l_c + \theta_{till}(z_w - l_c)$ $\frac{dz_w}{dt} = \frac{K_e}{2\theta_{till}}\left(\frac{z_w + \psi_{wf,till}}{z_w}\right)$ Where $K_e = \frac{z_w}{\frac{z_w - l_c}{K_{till}} + \frac{l_c}{K_c}}$; $K_c = K_{s,c} + (K_{i,c} - K_{s,c})e^{-CE}$; $C = ln\left(\frac{K_{s,c}}{K_{i,c} - K_{s,c}}\right)B\left(1 - \frac{\varepsilon}{4}\right)E_o$; $E_o = [0.02062 + 0.00379\ln(q_0)]\ q_0 t_{1/2}$; Variable rainfall: $\frac{dz_w}{dt} = \frac{\frac{z_w + \psi_{wf,till}}{2\theta_{till}}}{\frac{l_c}{K_c} + \frac{z_w - l_c}{K_{till}}}$ $i = \min\left(q_0, \frac{\theta_{till} D_L}{T_s}\right)$ Where D_L from Equation (14) in Chu's paper; $K_c = K_{s,c} + (K_{i,c} - K_{s,c})e^{ln\left(\frac{K_{s,c}}{K_{i,c} - K_{s,c}}\right)B\left(1 - \frac{\varepsilon}{4}\right)\frac{E(t)}{E_o}}$; $E\ (t) = E_i + [0.02062 + 0.00379\ln(q_0)]q_0 T_s$; Ponding on the interface: $i = \min\left(q_0, \frac{(l_c + l_{till} - l_{interpond})/2}{\frac{l_c}{K_c} + \frac{l_{till} - l_c}{K_{till}}}\right)$	The layered Green-Ampt model proposed by Brakensiek and Rawls (1983) was extended to include the effect of non-uniform rainfall of intensity q_0 (L T^{-1}) and crusting energy E (M L^{-1} T^{-2}), as well as the water ponding on the interface between the tilled layer and the subsoil in the infiltration process. A tilled soil profile is assumed to consist of three layers - the surface crust, the tilled layer, and the subsoil, each characterized by specific thickness l (L), hydraulic conductivity K (L T^{-1}), wetting front potential ψ_{fp} (L), and water storage capacity θ (dimensionless). A ponded depth or zone of positive water pressure of thickness $l_{\text{inter pond}}$ (L) may exist in the tilled layer above the interface with the subsoil. The Runge-Kutta method was used to conduct the calculations.
Waechter and Philip	1985	$i = 0.5\alpha\varphi i^*$ $i^* = \frac{1}{s}\ \sum_{n=-\infty}^{\infty}(-1)^n \frac{\mathrm{I_n}}{\mathrm{K_n}}$ $i^* = \frac{2}{\pi}\ [1 + 0.996R^{-2/3} + 0.02235s^{-4/3} + O(s^{-2})]$; for large s Where $s = 0.5\alpha r$	Waechter and Philip obtained the asymptotic expansion of the mean infiltration rate i (L T^{-1}) for large s (dominated by gravity, with capillary effects weak but nonzero) from buried circular cylindrical and spherical cavities using a scattering analog. *$\mathrm{I_n}$ and $\mathrm{K_n}$ are the modified Bessel functions of the first and second kinds of order* n.
Philip	1985	2D Infiltration: $\vartheta = s\varphi\vartheta^*$ Check Equation (34) in Philip's paper for ϑ^* $\vartheta = 4\varphi\xi_0$ for large s 3D Infiltration: $Q_{inf} = \varphi sLV^*$ Check Equation (53) in Philip's paper for V^* $Q_{inf} = \frac{8\pi\varphi}{\alpha}\ \xi_0$ for large s Where $\xi_0 = \sum_{n=0}^{\infty} a_n$ and $s = 0.5\alpha L$	Philip analyzed 2D and 3D steady infiltration from cavities of arbitrary size and shape (characteristic length L (L)) using Van de Hulst theorem which connects 2D and 3D infiltration, ϑ (L^2 T^{-1}) and Q_{inf} (L^3 T^{-1}) respectively, to dimensionless "downward wetting function" ξ_0 at large R (i.e., $R \to \infty$).
Philip	1986a	$Q_{inf} = 8\pi\varphi\alpha^{-1}\xi_0(w, s)$ Where $\xi_0\ (w, s) = s\sum_{n=0}^{5} a_n(w)s^n + O(s^7)$ for small s Check Equation (31) to (33e) in Philip's paper for a_n $\xi_0\ (w, s) = \frac{1}{2}\ s^2[1 + 1.99230638w^{2/3}s^{-2/3}] + O(s^{-4/3})$ for large s $s = 0.5\alpha ur$ and $w = u * v$	Solutions are given for quasilinear 3D steady infiltration from spheroidal cavities of arbitrary aspect ratio v and radius r (L) into isotropic and anisotropic homogeneous soils where anisotropy is defined by u^2. The isotropic case is simply that of $u^2 = 1$. For prolate spheroids $v > 1$, for spheres $v = 1$, and for oblate spheroids $v\ <\ 1$.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Philip	1986b	Buried disc, $Q_{inf} = 8\pi\varphi\alpha^{-1}\xi_0(s)$ Where $\xi_0(s) = \frac{2s}{\pi}\left[\sum_{n=0}^{7} a_n(\frac{\pi}{2})s^n + As^8\right]$ for small s Check Equation (30) in Philip's paper for a_n with $\phi = \frac{\pi}{2}$ $\xi_0(s) = \frac{1}{2}s^2\left[1 + \frac{1}{s} + \frac{1}{4s^2} - \frac{1}{8s^3} + \frac{1}{16s^4} + \frac{B}{s^5}\right]$ for large s A and B are estimated by matching the two above expansions of $\xi_0(s)$ $s = 0.5\alpha r$ Circular cylinder, $Q_{inf} = \varphi V^*(s)$ Check Equation (60) in Philip's paper for $V^*(s)$ Sphere, $Q_{inf} = 4\pi r\varphi V^*(s)$ Check Equation (65) in Philip's paper for $V^*(s)$	Philip studied steady quasilinear infiltration from buried discs and other sources of radius r (L) by expressing the volumetric infiltration rate Q_{inf} ($L^3\ T^{-1}$) as function of the far-field wetting function ξ_0 and the dimensionless discharge function V^*.
Kutilek and Krejca	1987	$I = C_1 t^{0.5} + C_2 t + C_3 t^{1.5}$	C_1 is an estimate of sorptivity S, C_2 is an estimate of $(A_2 + K_i)$, and C_3 is an estimate of (A_3 + the truncation error ε). The saturated hydraulic conductivity is: $K_s = (3C_1C_3)^{1/2} + C_2$
Swartzendruber	1987a	$I = K_s t + \frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)K_s}{(K_s-K_i)}\ln\left(1 + \frac{(I-K_i t)(K_s-K_i)}{(\psi_{wf}+h_0)(\theta_s-\theta_i)}\right)$	Using the following expression of the infiltration rate i: $i = K_s\left[\frac{\psi_{wf}+z+h_0}{z}\right] = \frac{d[z(\theta_s-\theta_i)]}{dt} + K_i$ By setting K_i equal to 0, the equation becomes nothing but Green and Ampt equation.
Swartzendruber	1987b	$I = K_s t + \frac{S_H}{A_0}(1 - e^{-A_0 t^{0.5}})$	Based on Swartzendruber, Philip's time series solution for $z(\emptyset, t)$ is unaffected by the pressure head condition at the soil surface h_0; each function $[X_1(\theta),\ X_2(\theta),\ X_3(\theta)\cdots]$ is still the solution of its own ordinary differential equation that can be solved by relatively numerical methods. A_0 is a fitting parameter depending on the initial water content θ_i. $A_0 = \frac{4K_s}{3S_H}$ (Stroosnijder, 1976). As $A_0 \to 0$, it reduces to a form of the Philip's model (1957b) with K_s as the coefficient of the linear term.
Broadbridge and White	1988	$I = q_0\, t$ $\quad t < \infty$ $I = \lambda_c\Theta_e^{-1}\left[\ln\{\frac{C(\Theta_e-\Theta)}{\Theta(C-\Theta_e)}\} - C^{-1}\ln\{\frac{\Theta_e(C-\Theta)}{\Theta(C-\Theta_e)}\} + R^*t^*\right]$ $\quad t \to \infty$ Where $\Theta_e = 2C\rho\left[(1+\rho^{-1})^{\frac{1}{2}} - 1\right]$; $\rho = R^*/m$; $m = 4C(C-1)$; $C = (b_0 - \theta_i)/(\theta_s - \theta_i)$; $\Theta = (\theta - \theta_i)/(\theta_s - \theta_i)$; $t^* = t/T_c$; $R^* = (q_0 - K_i)/(K_s - K_i)$	Broadbridge and White described constant rate rainfall infiltration q_0 ($L\ T^{-1}$) in uniform soils and other porous materials. The model is based on Darcy–Buckingham approach to unsaturated water flow and assumes simple functional forms for the soil water diffusivity $D(\theta)$ and hydraulic conductivity $K(\theta)$.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Yeh	1989	$\psi = \frac{1}{\alpha}\, ln\{e^{-\alpha(z_w-\psi_0)} + \frac{i}{K_s}e^{-\alpha z} - \frac{i}{K_s}\}$	Analytical solution to one-dimensional, steady state infiltration in heterogeneous soils is developed, only valid for $\psi \leq 0$. Yeh integrated Buckingham equation $q = -K(\psi)(\frac{\partial\psi}{\partial z} + 1)$ using the exponential model by Gardner (1958) which describes the unsaturated hydraulic conductivity by $K(\psi) = K_s e^{\alpha\psi}$.
Reynolds and Elrick	1990	$Q_{inf} = \frac{r}{G}(K_s h_0 + \varphi) + \pi r^2 K_s$	Reynolds and Elrick derived the steady-state infiltration by summing the steady flow out of the ring due to hydrostatic pressure of the ponded water in the ring and capillarity of the unsaturated flow, and the steady flow out of the ring due to gravity. The depth of the water ponding inside the ring remains constant. For ponding depths ranging from 5 to 25 cm, G factor in can be approximated by the average shape factor, Ge, of an infiltrometer, for different textured soils equal to: $Ge = 0.316(d/r) + 0.184$; Wooding's solution (1968) may be considered when $H = 0$ and $G = 0.25$.
Haverkamp et al.	1990	$I^* = (I - K_i t)\,\frac{2(K_s-K_i)}{S_0{}^2+2K_s h_0(\theta_s-\theta_i)}$ $I^* = \frac{\gamma}{q^*-1} + (1-\gamma)ln(1 + \frac{1}{q^*-1})$ $t^* = \frac{2(K_s-K_i)2}{S_0{}^2+2K_s h_0(\theta_s-\theta_i)}$ $t\gamma = \frac{2K_s(h_0+\psi_{str})(\theta_s-\theta_i)}{S_0{}^2+2K_s h_0(\theta_s-\theta_i)}$ $q^* = \frac{q_0-K_i}{K_s-K_i}$	The infiltration equation of Parlange et al. (1985) is improved in such a way that it applies equally well for infiltration and capillary rise. This new equation considers the possibility of an infinite diffusivity near saturation. One can set $\delta = 1$ for all practical purposes (Barry et al., 1995).
Smith	1990	$i = \frac{K_s G}{I}$ $t < t_p$ $I_p = \int_{\theta_i}^{\theta_s}(\theta - \theta_i)\,\frac{D(\theta)}{q_0-K(\theta)}d\theta$ $t = t_p$ $i = K_s\,(\frac{G}{I} + 1)$ $t > t_p$ (1) $i = K_s\,\frac{e^{I/G}}{e^{I/G}-1}$ $t > t_p$ (2) Where $G = (\psi_{wf} + h_0)(\theta_s - \theta_i)$ (1) $G = \frac{S_H^2}{2K_s}$ (2)	The resulting analytic expressions relate the infiltration rate i (L T^{-1}) to infiltrated cumulative water I (L). This type of expression is uniquely general in describing the decay of i with I for either saturation or flux boundary conditions, and in describing the onset of ponding for flux boundary conditions (i.e., $t < t_p$, $t = t_p$ and $t > t_p$). Specific cases are for D (L^2 T^{-1}) following a step function (1) or D (L^2 T^{-1}) and $dK/d\theta$ (L T^{-1}) are closely proportional (2) (Smith & Parlange, 1978). This study explored the applicability of this theory when the soil profile is composed of two layers. The behavior of the layered system exhibits the same unifying $i(I)$ response to flux boundary conditions as does a homogeneous profile, which lends unexpected generality to the I-based analytical infiltration mode.
Schmid	1990	$I = \int_0^{t_p} q_0(t)dt + q_{0,i}(t - t_p) +$ $K_s(q_{o,p} - q_{0,i})\frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)}{(q_{o,p}-K_s)^2}\{[1 + 2\frac{(q_{o,p}-K_s)^2}{K_s(\psi_{wf}+h_0)(\theta_s-\theta_i)}(t - t_p)]^{0.5} - 1\}$ Where $\int_0^{t_p}[q_0(t) - q_{0,i}]dt = K_s\,\frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)}{(q_{0,p}-K_s)}$	Based on Mein and Larson (1973), an explicit equation for time-dependent cumulative infiltration following ponding was proposed in this paper considering time-varying rainfall intensity $q_0(t)$ (L T^{-1}).
Ankeny et al.	1991	$Q_{inf} = K_s\, e^{\alpha\psi}[\pi r^2 + \frac{4r}{\alpha}]$	Wooding's equation is rewritten assuming: • No water ponding occurs inside the ring • Exponential relationship between $K(\psi)$ and ψ as defined by Gardner (1958): $K(\psi) = K_s\, e^{\alpha\psi}$ • The ratio $K(\psi)/\varphi(\psi)$ is equal to the constant parameter α

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Swartzendruber and Hogarth	1991	$I = \frac{I^* S_0^2}{K_s}$ $I^* = n^{-1}\,[1 - e^{-\alpha t^{*0.5}}] + t^*$ Where $n = \frac{\alpha}{(1+p)^{0.5}}$; $\alpha = \frac{A_0 S_0}{K_s}$; $p = \frac{S_H^2}{S_0^2}$ and $t^* = \frac{K_s^2 t}{S_0^2}$	Swartzendruber and Hogarth developed a new three-parameter infiltration equation based on Swartzendruber's equation (1987b) to describe the effect of soil-surface-ponded water head, h_0 (L), on the cumulative quantity of water infiltrated into the soil with time.
Philip	1992	$\frac{[1-(\theta_s-\theta_i)]\bar{K}}{(\theta_s-\theta_i)} t = \frac{I}{(\theta_s-\theta_i)} - (\frac{\psi_{wf}+h_0}{1-(\theta_s-\theta_i)}) ln(1 + \frac{[1-(\theta_s-\theta_i)]I}{(\psi_{wf}+h_0)(\theta_s-\theta_i)})$ $h_0 = cst$ $\frac{\bar{K}}{(\theta_s-\theta_i)} t = \frac{I}{(\theta_s-\theta_i)} - (\psi_{wf} + h_0) ln(1 + \frac{I}{(\psi_{wf}+h_0)(\theta_s-\theta_i)})$ $h_0 = cst$	Philip develops the solution for falling-head infiltration, which takes precisely the same functional form as that for constant ponded depth; only the values of the constants change.
White et al.	1992	$i = K_s + \frac{4bS_0^2}{(\theta_s-\theta_i)\pi r}$ $h_0 \leq 0$ $i = K_s + \frac{4bS_H^2}{(\theta_s-\theta_i)\pi r}$ $h_0 > 0$	White et al. combined the early-time transient and steady flow phases to estimate three-dimensional infiltration. The parameter b is constrained to $0.5 \leq b \leq \pi/4$. A typical "average" value for b is 0.55 (White & Sully, 1987).
Barry et al.	1993	$t = \frac{I}{K_s} - \alpha^* \ln(1 + \frac{I}{\alpha^* K_s})$ Where $\alpha^* = \frac{S_0{}^2}{2K_s^2}$ no ponding ($h_0 \leq 0$) Or $\alpha^* = \frac{S_H{}^2}{2K_s^2}$ under ponding ($h_0 > 0$)	Barry et al. expressed the cumulative infiltration in terms of the Lambert W-function as $I = -\alpha^* K_s[1 + W(-e^{-1-\frac{t}{\alpha^*}})]$. As such, the new infiltration solution was derived, which has the general form of the Green and Ampt (1911) law without requiring a sharp wetting front. The saturated soil surface can be either ponded or not.
Fonteh and Podmore	1993	$V = [2x_w d + wz_w + (\frac{\pi}{2})x_w z_w]\; l(\theta_s - \theta_i)$ (1) $V = [2(d + z_c)x_{w,max} + wz_w + (\frac{\pi}{2})x_{w,max}(z_w - z_c)]\; l(\theta_s - \theta_i)$ (2) Where $i_c = \frac{K_s(\psi_{wf}+h_0+z_c)}{z_c}$	Fonteh and Podmore developed a simple two-dimensional infiltration model for rectangular furrows for depth d (L), width w (L), and thickness l (L), based on the Green and Ampt equation and Fok and Chiang (1984) assuming the cumulative infiltrated volume is divided into two parts: before the wetting fronts meets (1) and after they meet (2). After the fronts meet, it is assumed that the region of two-dimensional flow shifts downwards due to vertical downward infiltration I_c (L) estimated using Green and Ampt (1911).
Smith et al.	1993	$0 < t \leq t_h$ (1) $i = q_0$ $t \leq t_p$ $K_s(t - t_p)\,(1 - \delta) = (I - I_p) - \frac{(\theta_s-\theta_i)G(\theta_i,\theta_s)}{(K_s-K_i)}$ $ln\left(\frac{e^{\delta I'/(\theta_s-\theta_i)G(\theta_i,\theta_s)}-1+\gamma}{e^{\delta I'_p/(\theta_s-\theta_i)G(\theta_i,\theta_s)}-1+\gamma}\right)$ $t > t_p$ Where $I' = I - K_i t$, $G(\theta_i, \theta_s) = \frac{S(\theta_i,\theta_s)^2}{2K_s} = \frac{S_H^2}{2K_s}$ and $\gamma = \frac{\delta K_s}{(K_s-K_i)}$ $I = I_h + (q_0 - K_i)(t - t_h)$ $t_h < t \leq t_n$ (2) $K_s(t - t_p)\,(1 - \delta) = (I - I_p) - \frac{(\theta_s-\theta_n)G(\theta_i,\theta_n)K_s}{(K_s-K_n)}$ $ln(\frac{F_G(I)-1+\gamma}{F_G(I_p)-1+\gamma})$ $t > t_n$ (3) Where $F_G\,(I) = e^{\frac{\delta(I-I_n)'}{G(\theta_s-\theta_n)}}$, $(I - I_n)' = I - I_n - K_n(t - t_n)$ and $\gamma = \frac{\delta K_s}{K_n-K_i}$	The main purpose of this paper is to propose a simple conceptual model of rainfall infiltration which continuously describes the infiltration/redistribution/infiltration cycle. Smith et al. considered three sequential periods of time, with the first (1), assuming an initial rainfall pattern having $q_0 > K_s$, lasting to a time t_h, which at some time $t_p < t_h$ causes ponding. The infiltration model (1) is based on the three-parameter analytic model of Parlange et al. (1982), extended to treat soils with very high initial water content θ_i. Then, during any significant interval within a storm in which q_0 (L T^{-1}) falls below i (L T^{-1}) at some time $t > t_h$ (t_h = time of hiatus) (2), $\theta_0 < \theta_s$. In this case, the redistribution model is based on profile extension with shape similarity defined by the scale factor $\mathcal{B}$, originally called β, and a shape factor, p. The similarity condition would be satisfied by a rectangle ($\mathcal{B} = 1$) which distorts in time, or the quadrant of an ellipse ($\mathcal{B} = \frac{\pi}{4}$). After the rainfall hiatus (posthiatus period), q_0 (L T^{-1}) is assumed to increase at a time $t > t_n$ causing a second ponding (3). A secondary wetting profile of accumulated water $(I - I_n)' = I - I_n - K_n(t - t_n)$ advances alongside the earlier profile computed at $\theta_0 = \theta_n$ with $\mathcal{B} = 1$. A correction to $G(\theta_n, \theta_s)$ for $\beta < 1$ was also incorporated.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Philip	1993	Constant rainfall rate $q_0(t)$: $I = (\theta_s - \theta_i)[z(t) - z_0]$ $t < t_\infty$ To calculate $z(t)$, check Eq. (17), (20), and (26) in Philip's paper $i = 0.5\bar{K}[1 - (\theta_s - \theta_i)][1 + \{1 + \frac{4q_0(\theta_s-\theta_i)}{\bar{K}[1-(\theta_s-\theta_i)]^2}\}^{0.5}]$ $t \to t_\infty$ Variable rainfall rate q_0(t): $I = (\theta_s - \theta_i)[z(t) - z_0]$ To calculate $z(t)$, differentiate with respect to time t: $q_0\,(t) = \frac{(\theta_s-\theta_i)}{\bar{K}}\,\frac{d}{dt}(z\frac{dz}{dt}) - [1 - (\theta_s - \theta_i)]\frac{dz}{dt}$	Philip established the study of the effect of excess rainfall, ponded without runoff, on the dynamics of infiltration into a deep homogeneous soil using Green and Ampt, or delta function, approximation under constant and variable rainfall rates.
Stone et al.	1994	$I = (\psi_{wf} + h_0)\Delta\theta(t^* + \sqrt{2t^*} - 0.2987t^{0.7913})$ Where $t^* = \frac{K_s}{(\psi_{wf}+h_0)\Delta\theta}\ t$	Stone et al. developed an explicit approximation of the Green–Ampt equation (one-stage infiltration) by rewriting it in the form of the Philip's equation.
Mandal and Waechter	1994	$i = 0.5\alpha\varphi i^*$ $i^* = \frac{1}{s}\ \sum_{n=-\infty}^{\infty} (-1)^n \frac{\mathrm{I_n}}{\mathrm{K_n}}$ Top half: $i^* = \frac{2}{\pi}\ (0.69553s^{-2/3})$ Bottom half: $i^* = \frac{2}{\pi}\ (1 + 0.30066s^{-2/3})$ Semicircular trench: $i^* = \frac{4}{\pi}\ (1 + 0.69553s^{-2/3})$ Where $s = 0.5\alpha r$	Mandal and Waechter obtained the separate contributions to the mean infiltration rate i (L T^{-1}) from the top and the bottom halves of the buried circular cylinders of radius r (L). *I_{n} and K_{n} are the modified Bessel functions of the first and second kinds of order n.*
Fallow et al.	1994	$I = [2(\theta_s - \theta_i)K_s h_0 + \frac{(\theta_s-\theta_i)\varphi}{b}]^{0.5}\ t^{0.5}$	Fallow et al. presented an analytical equation for determining early-time, transient, and one-dimensional infiltration under both, constant head, and falling-head conditions, in low-permeability soils, not significantly perturbed by gravitational effects. The parameter b is constrained to $0.5 \le b \le \pi/4$. A typical "average" value for b is 0.55 (White & Sully, 1987, White et al., 1992).
Basha	1994	$Q_{inf} = \frac{4K_S V^*}{\alpha^2}$ 2D shallow ditch: $V^* = \frac{2a^*}{1-b^*}$ 3D shallow pond: $V^* = \frac{2\pi a^* s}{(1-b^*)(2-b^*)}$ Where $s = 0.5\alpha r$	The Green's function method was used to derive a general analytical model which can handle multidimensional steady infiltration problems in a semi-infinite medium with arbitrary boundary conditions and root uptake forcing functions and for various simple source geometries. Two special kinds of boundary conditions were considered in this work, 2D shallow ditch, and 3D shallow pond of radius r (L).
Salvucci and Entekhabi	1994	$I = K_s\ \{(1 - \frac{\sqrt{2}}{3})t + (\frac{\sqrt{2}}{3})(\chi t + t^2)^{0.5} +$ $(\frac{\sqrt{2}-1}{3})\chi[\ln(t + \chi) - \ln(\chi)] + (\frac{\sqrt{2}}{3})\chi[\ln(t + \frac{\chi}{2} + (\chi t + t^2)^{0.5})$ $- \ln(\frac{\chi}{2})]\}$ Where $\chi = \frac{(\psi_{wf}+h_0)\Delta\theta}{K_s}$	An explicit expression for Green-Ampt cumulative infiltration is presented for any soil type definition and all times using Philip's time series solution (Philip, 1957a).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Corradini et al.	1994	$I' = I - K_i t$ $I' - \frac{I'^2}{2(q_0-K_i)(\theta_0-\theta_i)} \frac{d\theta_0}{dt} = \frac{(\theta_0-\theta_i)G(\theta_i,\theta_0)K_s}{\delta K_0} \ln(1 + \frac{\delta K_0}{q_0-K_0})$ (1) $(q_0 - K_i - K_{0*})\, I' = B q_0 K_s(\theta_{0*} - \theta_i)G(\theta_i, \theta_{0*})$ (2) Where $G(\theta_i, \theta_0) = \frac{1}{K_0} \int_{\theta_i}^{\theta_0} D(\theta)d\theta$	Corradini et al. extended the conceptual model earlier developed by Smith et al. (1993) towards further generality treating an arbitrary sequence of rainfall rates. They included the representation of a sequence of infiltration/redistribution cycles with situations of infiltration not leading to soil surface saturation (i.e., $t < t_p$) (1), and wetting profile reshaping under reduced rainfall rates (i.e., $q_0 < i$) (2). Corradini et al. used a slightly modified version of Parlange et al. (1985) model for description of increases in the surface water content (1) and the Smith et al. (1993) redistribution equation for decreases (2). Equation (1) gives the limited case of soil surface saturation (Smith et al., 1993) for $\theta_0 = \theta_s$ and $\frac{d\theta_0}{dt} = 0$. In Equation (2), the evolution of θ_{0*} will be estimated by Runge-Kutta integration of (1). p is a shape factor (Smith et al., 1993) approaching 1 for q_0 (L T^{-1}) near K_s (L T^{-1}). $\pi/4 \leq B \leq 1$ (Smith et al., 1993).
Smettem et al.	1994	$I = I_{1D} + \frac{\sqrt{0.3\pi r S_0{}^2 t}}{(\theta_s-\theta_i)}$	Smetten et al. developed an analytical expression for three-dimensional unsteady, unconfined, gravity-free flow out of a disc infiltrometer from one-dimensional confined infiltration.
Haverkamp et al.	1994	1D Infiltration: $\frac{2(K_s-K_i)^2}{S_0{}^2} t = \frac{2}{1-\beta} \frac{(K_s-K_i)(I-K_i t)}{S_0{}^2} - \frac{1}{1-\beta} \ln\left[\frac{1}{\beta} e^{2\beta\frac{(K_s-K_i)(I-K_i t)}{S_0{}^2}} + \frac{\beta-1}{\beta}\right]$ (1) $I = S_0 t^{0.5} + \frac{2-\beta}{3} K_s t$ (2) 3D Infiltration: $I = I_{1D} + \frac{\gamma S_0{}^2 t}{r\Delta\theta}$ $I = S_0\, t^{0.5}$ very short t $I = S_0\, t^{0.5} + \left[\frac{2-\beta}{3}(K_s - K_i) + K_i + \frac{\gamma S_0{}^2}{r(\theta_s-\theta_i)}\right] t$ short t $I = \left(K_s + \frac{\gamma S_0{}^2}{r(\theta_s-\theta_i)}\right) t + \frac{1}{2(1-\beta)} \ln(\frac{1}{\beta}) \frac{S_0{}^2}{(K_s-K_i)}$ steady state	(1) Parlange's equation is redefined such that δ is replaced by new dimensionless constant β. (2) Haverkamp's equation (1) is simplified into two-term approximate expansion, for short infiltration times and K_i close to zero. β ranges between 0.3 and 1.7 for sand to silty soils (Lassabatère et al., 2009, Rahmati et al., 2019). $\beta = 0.6$ is an average value. A 3D cumulative infiltration, I_{3D}, from a disk source, is related to 1D cumulative infiltration, I_{1D}, particularly for water infiltration experiments that make use of disk or ring infiltrometers. An average value of 0.75 can be used for γ (Haverkamp et al., 1994; Lassabatère et al., 2006).
Barry et al.	1995	$I^* = (I - K_i t) \frac{2(K_s-K_i)}{S_0{}^2+2K_s h_0(\theta_s-\theta_i)}$ Where $I^* = t^* + 1 - \gamma^* - exp(-\frac{\sqrt{2t^*}}{1+\frac{\sqrt{2t^*}}{6}} - \frac{2t^*}{3}) +$ $\frac{\gamma}{1+t^*}\{exp(-\frac{2t^*}{3})[1 - (1-\gamma)^8 t^{*\frac{5}{2}}] + (2\gamma + t^*)ln(1 + \frac{t^*}{\gamma})\};$ $t^* = \frac{2(K_s - K_i)^2}{S_0{}^2 + 2K_s h_0(\theta_s - \theta_i)} t;$ $\gamma = \frac{2K_s(h_0 + \psi_{str})(\theta_s - \theta_i)}{S_0{}^2 + 2K_s h_0(\theta_s - \theta_i)}$	Barry et al. transformed the implicit infiltration formula presented by Haverkamp et al. (1990) into an explicit infiltration equation, which improved the applicability of the approach in both the short- and long-term limits.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Elrick et al.	1995	$I = \frac{A_{\text{pond}}}{A}\,[h_0 - h(t)]$	One-dimensional infiltration experiments conducted in the laboratory using undisturbed soil cores have been analyzed using numerical inversion procedures of a finite solution of Richards' equation. For falling-head conditions, the cumulative infiltration is a function of the ponded head $h(t)$, the cross-sectional area of the infiltrating surface, A (L^2), and that of the falling head tube, A_{pond} (L^2).
Sommer and Mortensen	1996	$I = [1 + s(0)]\, S_0 t^{0.5}$ $\theta'(\chi) = \frac{\theta \mu S_0{}^2}{2D(\theta)}\,[l(\chi) - s(\chi)]$ Where $\chi = \frac{x - x_0}{S_0 t^{0.5}}$ $l'(\chi) = -\frac{\theta'(\chi)}{\theta}[l(\chi) - s(0) - \chi]$; $s'(\chi) = \frac{\theta'(\chi)}{(1-\theta)}\,[s(\chi) - s(0) - \chi]$ under pressure liquid Or $s'(\chi) = \frac{\theta'(\chi)}{3(1-\theta)}\,[s(\chi) - s(0) - \chi]$ under hydrostatic pressure	Using mixture theory, Sommer and Mortensen treated unidirectional infiltration of an initially dry deformable porous medium under constant liquid pressure, assuming the porous medium goes from completely dry to fully saturated (slug-flow assumption) and neglecting gravity and inertial forces. To solve for S_0 ($L\,T^{-0.5}$) and the dimensionless water content, liquid, and solid velocities, $\theta(\chi)$, $l(\chi)$, and $s(\chi)$ respectively, Sommer and Mortensen set the following boundary conditions: $\theta = \theta_i$ at $\chi = 0$ $\theta = \theta_s$ at $\chi = 1$ $l(1) = 1 + s(0)$ $s(1) = [1 + s(0)]\,\frac{\theta_s - \theta_c}{1-\theta_s}$ Where θ_c ($L^3\,L^{-3}$) the water content of the porous material immediately ahead of the infiltration front at time t. A simpler limiting case is obtained if there is no capillary pressure drop across the infiltration front, then, $\theta_c = \theta_s$ and therefore $s(1) = 0$.
Srivastava et al.	1996	$I = \alpha'\,(\theta_s - \theta_i)(\psi_{wf} + h_0)\emptyset^{[\beta' + \delta' \ln(\emptyset)]}$ Where $\emptyset = \frac{K_s(t - t_p + t_S)}{(\theta_s - \theta_i)(\psi_{wf} + h_0)}$	Srivastava et al. developed an explicit equation to the modified Green-Ampt equation presented by Mein and Larson (1973) to represent cumulative infiltration at any time t greater than ponding time.
Preziosi et al.	1996	1D Infiltration: $\rho_{solid}\phi_{solid}\,(\frac{\partial v_{solid}}{\partial t} + v_{solid}\frac{\partial v}{S}\partial x) =$ $-\frac{\mu}{k}[v_{solid} - C(t)] + \frac{\partial \sigma}{\partial x} - \phi_{solid}(\rho_{solid} - \rho_{liquid})g$ 3D Infiltration: Equations (1), (7) and (9) in Preziosi et al.'s paper	Preziosi et al. presented a mathematical model for unidirectional infiltration of an incompressible liquid into an initially deformable porous material assuming slug-flow in the absence of inertial forces. The quantity $C(t)$ depends on how the liquid constituent is pushed into the porous medium. The simplest case is when we are completely able to govern the inflow, for instance, we are able to steadily push liquid into the porous medium, which means $C(t)$ = const. A more interesting situation for determining $C(t)$ arises in Equation (23) in Preziosi et al.'s paper.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)		Applied concepts
Corradini et al.	1997	$I = \int_0^t (q_0 - K_i)\, dt$ $I_p = \int_0^{t_p} (q_0 - K_i)\, dt = \frac{(\theta_s-\theta_i)G(\theta_i,\theta_s)\mathcal{B}pK_s}{q_0-K_s}$ $i = K_s + \frac{(\theta_s-\theta_i)G(\theta_i,\theta_s)\mathcal{B}pK_s}{I-K_i t}$ Where $G(\theta_i, \theta_s) = \frac{1}{K_s}\int_{\theta_i}^{\theta_s} D(\theta)d\theta$ $i_1 = (-\frac{1}{\mathcal{B}}\frac{d\mathcal{B}}{d\theta_0} - \frac{1}{\theta_0-\theta_i})(I_1 - K_i t_1)\frac{d\theta_0}{dt} - K_i$ $\frac{d\theta_0}{dt} = \frac{(\theta_0-\theta_i)\mathcal{B}(\theta_0)}{[I - K_i(t-t_1)][(\theta_0-\theta_i)\frac{d\mathcal{B}(\theta_0)}{d\theta_0} + \mathcal{B}(\theta_0)]}$ $\left\{ q_0 - K_0 - \frac{\mathcal{B}(\theta_0)pK_0(\theta_0-\theta_i)G(\theta_i,\theta_0)}{I - K_i(t-t_1)} \right\}$ $\frac{d\theta_0}{dt} = \frac{(\theta_0-\theta_1)\mathcal{B}(\theta_0)}{[I - I_1 - K_1(t-t_1)][(\theta_0-\theta_1)\frac{d\mathcal{B}(\theta_0)}{d\theta_0} + \mathcal{B}(\theta_0)]}$ $\left\{ q_0 - K_0 - \frac{\mathcal{B}(\theta_0)pK_0(\theta_0-\theta_1)G(\theta_1,\theta_0)}{I - I_1 - K_1(t-t_1)} \right\}$	$0 \le t < t_1$ $t < t_p$ $t = t_p$ $t > t_p$ $t > t_1$ $t = t_1$ if $q_0 \le i_1$ if $q_0 >$ i_1	A relatively simple conceptual model for infiltration during complex rainfall sequences is presented as a reformulation of an analytically derived model developed earlier by Smith et al. (1993) and Corradini et al. (1994) in a more homogeneous version suitable for hydrologic applications. During the first rainfall period ($0 \le t \le t_1$), infiltration and redistribution rates are described prior to and during the rainfall hiatus, respectively. As for parameters, p approaches 1 near K_s (L T^{-1}) and $\pi/4 \le \mathcal{B} \le 1$ (Corradini et al., 1994; Smith et al., 1993). Explicit relations for $\mathcal{B}$ and p were given as follows: $\mathcal{B} = c_1 \frac{\theta_0-\theta_i}{\theta_s-\theta_r} + c_2$, $\mathcal{B}p = f(q_0/K_s)$, and $\mathcal{B}p = c_3$ (during redistribution), where c_1, c_2, and c_3 are constants to be determined, (q_0/K_s) is an implicit function through which the variation of the profile shape with q_0/K_s expressed in terms of values of p and $\mathcal{B}$ is invariant with time during redistribution. After a limited period of redistribution, a new rainfall period (post-hiatus, $t > t_1$) produces re-infiltration into the soil. If $q_0 \le i_1$, the reinfiltrating water is approximately distributed through the whole dynamic profile. If $q_0 > i_1$, re-infiltration creates a new $\theta(z, t)$ profile in a soil with fictitious initial water content $\theta(0, t_1) = \theta_1$ and initial hydraulic conductivity $K_i(t = t_1) = K_1$.
Parlange et al.	1997	For a linear soil, $I = 2\sqrt{t/\pi} + \frac{t}{2} + O(t^{\frac{3}{2}})$ $I = 1 + t - (1 + \frac{t}{2})erfc(\frac{\sqrt{t}}{2}) + \sqrt{t/\pi}e^{-t/4}$ For a near- linear soil, $t = I - 2/3\ln(1 + \frac{3I}{2})$ $I = 2\sqrt{t/3} + \frac{2t}{3} + O(t^{\frac{3}{2}})$	 short times long times short times long times	A general approximation for the solution[2] to the one-dimensional Richards equation is presented for arbitrary soil properties and boundary conditions, exact for short times, in general, and for all times as D approaches a delta function. The approximation becomes increasingly accurate for "linear" and "near-linear" soils. [2] $2M = \frac{q_0}{\theta_0 D_0} - \frac{d\theta_0}{dt}\frac{1}{q_0-K_s}$ Where M (L^{-1}) is an unknown function of time.
Wu and Pan	1997	$i = i_c\ [a + b(\frac{t}{T_c})^{-0.5}]$ Where $i_c = fK_s$; $T_c = \frac{\Delta\theta\lambda_c}{K_s}$; $f = \frac{h_0+\lambda_c}{d+r/2} + 1$		Wu and Pan developed a scaling method for axisymmetric, three-dimensional infiltration from a single ring infiltrometer, which resulted in a generalized solution to infiltration that can be applied to different soil conditions and ring geometry using two dimensionless parameters, a and b. The essential part of this method was to model the macroscopic capillary length, λ_c(L), the sorptive time, T_c(T) and the parameter i_c (L T^{-1}) used to scale the infiltration rate. λ_c(L) is defined as being equal to the matric flux potential, φ(L^2 T^{-1}), scaled by K_s (L T^{-1}): $\lambda_c = \varphi/K_s$. a and b were determined through curve-fitting to be approximately equal to 0.91 and 0.17, respectively.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Wang et al.	1997	Without air compression: $t = \frac{\emptyset(1-S_{ei}-S_{eair})}{K_s}\left[\frac{I}{\emptyset(1-S_{ei}-S_{eair})} - (\psi_{wf}+h_0)ln(1+\frac{I}{\emptyset(1-S_{ei}-S_{eair})(\psi_{wf}+h_0)})\right]$ With air compression: $t = \frac{\emptyset(1-S_{ei}-S_{eair})}{k_{rc}K_s}[\frac{I}{\emptyset(1-S_{ei}-S_{eair})} - (\psi_{wf}+h_0+h_{af})$ $ln(1+\frac{I}{\emptyset(1-S_{ei}-S_{eair})(\psi_{wf}+h_0+h_{af})})]$	One-dimensional infiltration was derived on the basis of the Green and Ampt (1911) assumptions (infiltration process is isothermal, the porous medium is homogeneous, and the wetting front is sharp) by including the term h_{af} (L), air pressure immediately below the wetting front, to account for air entrapment effects. When soil air is not compressed during infiltration ➔ $h_{af} = 0$.
Enciso-Medina et al.	1998	Zone (1): $i = \left(\frac{B+l_{seal}+(\psi_{wf}+h_0)}{\frac{l_{seal}}{K_{seal}}+\frac{B}{K_{till}}}\right)$ $z \le l_{seal}+l_{till}$ $i = \left(\frac{C+l_{seal}+l_{till}+(\psi_{wf}+h_0)}{\frac{l_{seal}}{K_{seal}}+\frac{l_{till}}{K_{till}}+\frac{C}{K_{seal}}}\right)$ $z > l_{seal}+l_{till}$ Where $B = \frac{I-l_{seal}\Delta\theta}{\Delta\theta_{till}}$; $C = \frac{I-l_{seal}\Delta\theta_{seal}-l_{till}\Delta\theta_{till}}{\Delta\theta}$; Zone (2): $i = \frac{\psi_{wf}+h_0}{\frac{x_{seal}}{K_{seal}}+\frac{x_w-x_{seal}}{K_{till}}}$ Zone (3): $\vartheta = [2x_w d + wz_w + (\frac{\pi}{2})x_w z_w]\ \Delta\theta/t$	To simulate the effects of surface sealing, soil cracking, and initial soil water content, Enciso-Medina et al. developed a model for a three-layered soil system consisting of a surface seal of thickness l_{seal} (L), a tillage layer of thickness l_{till} (L), and the subsoil. The portion of the soil profile that receives water during infiltration is represented by three zones. The first zone (1) is directly below the wetted furrow and only involves vertical one-dimensional infiltration predicted using Green-Ampt method for vertical flow. Enciso-Medina et al. assumed that the time required to wet the seal is negligible; therefore, infiltration starts when the wetting front reaches the bottom of the seal, l_{seal} (L). Thus, the infiltration rate was predicted when the wetting front is within the tillage layer ($l \le l_{seal}+l_{till}$) or extends into the subsoil ($l > l_{seal}+l_{till}$) The second zone (2) represents one-dimensional horizontal flow of water through the sides of the wetted furrow using Green-Ampt method for horizontal infiltration. The third zone (3) is represented by a semielliptical shape that connects the corners of the first two zones predicting quasi-two-dimensional infiltration ϑ ($L^2\ T^{-1}$) for a rectangular furrow based on the model by Fok and Chiang (1984).
Philip	1998	$I = S_{0,c}\ t^{0.5}$ small t $I = \int_0^{l_c}(\theta_c-\theta_{c,i})dz + \int_{l_c}^{\infty}(\theta-\theta_i)dz$ intermediate t $I = K_s\ t$ large t	Quasi-analytic methods are used to analyze ponded infiltration into crusted soils for both small and large times and a good approximation for intermediate times is anchored at both ends by these results. Philip (1998) assumed a uniform soil of initial water content, θ_i ($L^3\ L^{-3}$) and saturated hydraulic conductivity K_s ($L\ T^{-1}$) lying underneath the crusted soil of initial water content $\theta_{c,i}$ ($L^3\ L^{-3}$).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Smith et al.	1999	$0 \le t < t_{crust}$ $q_0 = \frac{dI_1}{dt} + K_{1,i}$ $\quad t \le t_p$ $\frac{d\theta_{1,s}}{dt} = \frac{(\theta_{1,s}-\theta_{1,i})B_1(\theta_s)[q_0 - K_{1,s} - (\theta_{1,s}-\theta_{1,i})B_1(\theta_s)p_1 G_1(\psi_i,\psi_0)K_{1,s}/I_1]}{I_1[(\theta_{1,s}-\theta_{1,i})\gamma_1 + B_1(\theta_s)]}$ Where $G_1(\psi_i,\psi_0) = \frac{1}{K_s}\int_{\psi_i}^{\psi_0} K(\psi)d\psi$ and $\gamma_1 = \frac{dB_1}{d\theta_0}$ $\quad t > t_p$ $t \ge t_{crust}$ $\frac{d\psi_c}{dt} = \frac{1}{P(\psi_0,\psi_c,t)}\left\{K_0 + \frac{G_1(\psi_c,\psi_0)K_{1,s}}{l_c} - K_{2,c} - \frac{B_2(\theta_{2,c})p_2(\theta_{2,c}-\theta_{2,i})K_{2,s}G_2(\psi_i,\psi_c)}{I - l_c[\theta_{1,c}-\theta_{1,i}+\alpha_1(\theta_0-\theta_{1,c})] - K_{2,i}t}\right\}$ Where $P(\psi_0,\psi_c,t)$ from Eq. (22) in Smith et al.'s paper $G_2(\psi_i,\psi_c) = \frac{1}{K_s}\int_{\psi_i}^{\psi_c} K(\psi)d\psi$, $\theta_{1,c} = \theta_1(l_c)$, $\theta_{2,c} = \theta_2(l_c)$, $K_{2,c} = K_2(l_c)$, $\gamma_2 = \frac{dB_2}{d\theta_{2,c}}$	Smith et al. proposed a relatively simple analytical/conceptual model for infiltration in crusted soils treating the crust (1) as a single layer of thickness l_c (L) overlaying the subsoil (2) where the upper layer is always the most restrictive, i.e., $K_{1,s} < K_{2,s}$. Cumulative dynamic infiltration comprises that in the crust layer I_1 (L) plus that in the subsoil I_2 (L). For $t < t_{crust}$, $\theta(z)$ evolves within the crust layer, and cumulative infiltration in the crust upper layer is described by the model for homogeneous soils developed earlier (Corradini et al., 1994, 1997; Smith et al., 1993). For $t \ge t_{crust}$, the dynamic water content in the crust is expressed as fraction α_1 associated with the surface and the remaining $(1-\alpha_1)$ associated with the interface. Also, for $t \ge t_{crust}$, the previous model is extended in order to represent the effects of water flow in the subsoil, I_2 (L), but with the upper boundary at the crust-subsoil interface. The system of equations may be solved by a standard Runge–Kutta method. Explicit relations for B and p were given by Corradini et al. (1997): $B_1(\theta_0) = 0.6\frac{\theta_0-\theta_{1,i}}{\theta_{1,s}-\theta_{1,r}} + 0.4$, $B_1\ p_1 = 0.98 - 0.87e^{-q_0/K_{1,s}}$, and $B_1\ p_1 = 1.7$ (during redistribution). A constant value of 0.86 is assumed for α_1 (originally called α).
Selker et al.	1999a	$i = (\theta_s - \theta_i)\frac{dz_w}{dt}$ $i = \frac{-3K_i(\beta^*\psi_{wf}-1)}{z_w^2\beta^{*2}}$ $\quad (1)$ $i = K_i\beta^*(\frac{2n^*-1}{n^*})[\frac{\psi_{wf}(1-\frac{\beta^*}{n^*}z_w)^{-n^*} - z_w}{1-(1-\frac{\beta^*}{n^*}z_w)^{1-2n^*}}]$ $\quad (2)$ $i = \frac{2\beta^*K_i(\psi_{wf}e^{\beta^* z_w} - z_w)}{1-e^{2\beta^* z_w}}$ $\quad (3)$	Selker et al. developed three expressions for the time rate of infiltration using Green and Ampt approach for soils with hydraulic properties (K_s (L T^{-1}) and ψ_{wf} (L)) decreasing with depth following linear (1), power law (2), and exponential (3) relationships. (1) $K_s(z) = K_i\ \beta^{*-2}z^{-2}$; $\psi_{wf}(z) = \psi_{wf,0}\ \beta^* z$ (2) $K_s(z) = K_i\ (1-\frac{\beta^* z}{n^*})^{2n^*}$; $\psi_{wf}(z) = K_i\ (1-\frac{\beta^* z}{n^*})^{-n^*}$ (3) $K_s(z) = K_i\ e^{-2\beta^* z}$; $\psi_{wf}(z) = \psi_{wf,0}\ e^{\beta^* z}$
Wu et al.	1999	$I = afK_st + 2bfK_s(T_ct)^{0.5}$ $\quad$ short t $I = afK_st + c$ $\quad$ steady state Where $T_c = \frac{(\theta_s-\theta_i)\lambda_c}{K_s}$; $f = \frac{h_0+\lambda_c}{d+r/2} + 1$	Wu et al. integrates the generalized equation in Wu and Pan (1997) for any time, including early-, transient- and steady-state infiltration behavior.
Novak et al.	2000	$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}[K(\psi)(\frac{\partial\psi}{\partial z}+1)] - s_r + s_f$ where $s_f = (K_{s,h}\frac{\psi_{wf,m}+h_0}{z_w})\ A_f$ and $z_w = \sqrt{2K_{s,h}\frac{\psi_{wf,m}+h_0}{(\theta_s-\theta_i)}T_s}$	Novek et al. developed an infiltration model for swelling, cracked fine-textured soils that permits changes in the dimensions of the cracks during the infiltration process. The extended Richards' equation by Feddes et al. (1988) was solved, subject to three upper boundary conditions were considered: (1) unsaturated soil surface, (2) soil surface is saturated and a sub-critical surface layer ($h \le \boldsymbol{h}_{0,max}$) is being formed, and (3) surface water layer reaches its maximum and water flows into cracks. $i_m = -K_m(\psi)(\partial\psi/\partial z + 1)$ $\quad h < 0$ (1) $i_m = \partial\psi/\partial t - K_{s,m}(\partial\psi/\partial z + 1)$ $\quad 0 \le h \le h_{0,max}$ (2) $i_m = i_f - K_{s.m}(\partial\psi/\partial z + 1)$ $\quad h > h_{0,max}$ (3)

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Corradini et al.	2000	$0 \le t < t_{\text{crust}}$ $q_0 = \frac{dI_1}{dt} + K_{1,i}$ $\quad t \le t_p$ $i = K_{1,s} + \frac{(\theta_{1,s} - \theta_{1,i}) G_1(\psi_i, 0) B_1(\theta_s) p_1 K_{1,s}}{I_1}$ $\quad t > t_p$ Where $G_1(\psi_i, 0) = \frac{1}{K_s} \int_{\psi_i}^{0} K(\psi) d\psi$ $t \ge t_c$ $I = [\alpha_1(\theta_0 - \theta_{1,i}) + (1 - \alpha_1)(\theta_{1,c} - \theta_{1,i})]\, l_c + I_2 + K_{2,i} t$ Where I_2 from Eq. (20) in Corradini et al.'s paper $P_L(\psi_c, t)$ from Eq. (19) in Corradini et al.'s paper $\theta_{1,c} = \theta_1(l_c)$, $\theta_{2,c} = \theta_2(l_c)$, $K_{1,c} = K_1(l_c)$ and $K_{2,c} = K_2(l_c)$	Corradini et al. proposed an analytical/conceptual model, which is formulated by an extension of the Smith et al. (1999) model, for the solution of the infiltration and reinfiltration problem into any horizontal two layered soil where either layer may be less permeable under any real rainfall pattern. The model represents both the infiltration rate i (L T^{-1}), cumulative infiltration I (L), and soil water potentials ψ_0 and ψ_c (L) at the surface and the interface, respectively. For $0 \le t < t_c$, the solution is that of the vertically homogeneous case described by Corradini et al. (1997). As to the parameters incorporated in the system, α_1 (originally called α) is considered to be a constant, while $\mathcal{B}_2$ and p_2 are given, by analogy with the relations by Corradini et al. (1997) and Smith et al. (1999): $\mathcal{B}_2(\theta_{2,c}) = 0.6 \frac{\theta_{2,c} - \theta_{2,i}}{\theta_{2,s} - \theta_{2,r}} + 0.4$, $\mathcal{B}_2\, p_2 = 0.98 - 0.87 e^{-v_{12}/K_{2,s}}$, Where $v_{12} = \frac{G_1(\psi_c, \psi_0) K_{1,s}}{l_c} + K_{1,c}$ and $\mathcal{B}_2\, p_2 = 1.7$ (during redistribution). In case of re-infiltration following redistribution from a rainfall hiatus, Corradini et al. (2000) adopted the same concept proposed by Corradini et al. (1997).
Swartzendruber	2000	$I = K_s t + E \ln(1 + \frac{(I - K_i t)}{E})$ $E = \frac{{S_H}^2}{2(K_s - K_i)}$;	Using the following expression of the infiltration rate i: $i = K_s\ [\frac{\psi_{wf} + z_w + h_0}{z_w}] = (\theta_s - \theta_i) \frac{dz_w}{dt} + K_i$ where the head h_0(t) at the soil surface increases with time t according to: $h_0(t) = h_0 + p(t)$ such that $p(t) = \frac{S_H (K_s - K_i) t^{1/2}}{2 K_s (\theta_s - \theta_i)}$
Govindaraju et al.	2001	Constant rainfall: Local-scale Infiltration: $i = q_0$ $\quad 0 \le t \le t_p$ $i = K_s \frac{(\psi_{wf} + h_0)(\theta_s - \theta_i) + I}{I}$ $\quad t > t_p$ Field-scale Infiltration: $I = q_0\, t$ $\quad 0 \le t \le t_p$ $t > t_p$ $I = I_p + \sqrt{2 K_s (\psi_{wf} + h_0)(\theta_s - \theta_i)}(t^{\frac{1}{2}} - t_p^{\frac{1}{2}}) + \frac{2}{3} K_s (t - t_p) + \frac{1}{18} (\frac{2 K_s^3}{(\psi_{wf} + h_0)(\theta_s - \theta_i)})^{1/2} (t^{3/2} - t_p^{3/2})$ Where $t_p = I_p / q_0$ and $I_p = \frac{K_s \psi_{wf} (\theta_s - \theta_i)}{q_0 - K_s}$ Time-dependent rainfall: Local-scale Infiltration: $t = \frac{I - I_i}{q_0} + t_i$ $\quad 0 \le t < t_p$ $t > t_p$ $t = t_i - (\frac{(\psi_{wf} + h_0)(\theta_s - \theta_i) + I_i}{q_0}) + \frac{1}{K_s} [I - (\psi_{wf} + h_0)(\theta_s - \theta_i) \ln(I + (\psi_{wf} + h_0)(\theta_s - \theta_i))] + \frac{(\psi_{wf} + h_0)(\theta_s - \theta_i)}{K_s} \ln(I_p + (\psi_{wf} + h_0)(\theta_s - \theta_i))$	Govindaraju et al. studied the problem of field-scale infiltration over soils where spatial variability of saturated hydraulic conductivity K_s (L T^{-1}) is represented by a homogeneous correlated lognormal random field $Y = \ln(K_s)$ with mean μ_Y and standard deviation σ_Y. The cumulative infiltration, I (L) (symbolized as F in the paper), was used as an independent variable to develop expressions for the averaged field-scale infiltration under both constant and time-dependent rainfall rates. However, Govindaraju et al. also developed an explicit expression for the field-scale infiltration rate at any given time t (T) under continuously ponded conditions of constant rainfall q_0 (L T^{-1}) (Green & Ampt, 1911).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
		Where $t_p = \frac{K_s \psi_{wf}(\theta_s-\theta_i)}{q_0(q_0-K_s)} - \frac{I_i}{q_0} + t_i$ and $I_p = I_i + q_0(t_p - t_i)$ Field-scale Infiltration: $\bar{i} = q_0\,[1-\mu_\Omega] + \{1 + \frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)}{I}\}G(K_c,1)$ Where $t = t_i - \frac{I}{q_0}(1-\mu_\Omega) - \frac{I_i}{q_0} +$ $[I + (\psi_{wf}+h_0)(\theta_s-\theta_i)\ln(\frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)}{I+(\psi_{wf}+h_0)(\theta_s-\theta_i)})]G(K_c,-1) +$ $(\psi_{wf}+h_0)(\theta_s-\theta_i)\sum_{i=1}^{\infty}\frac{G(K_c,i)}{(1+i)q_0{}^{i+1}}$ $G(K_c,\xi)$ from Equation (7), K_c from Equation (14) and μ_Ω from Eq. (16) in Govindaraju et al.'s paper.	
Serrano	2001	$I = I_0\,(t) + a ln[\frac{I_0(t)+a}{I_p+a}][\frac{I_0(t)+a}{I_0(t)}] - \frac{a}{2}ln^2[\frac{I_0(t)+a}{I_p+a}][\frac{I_0(t)+a}{I_0^3(t)}] +$ $\frac{a}{3}ln^3[\frac{I_0(t)+a}{I_p+a}]\{\frac{[I_0(t)+a]2}{I_0^5(t)}\} - \cdots$ Where $a = (\psi + h_0)(\theta_s - \theta_i)$	Serrano developed an explicit solution of Green and Ampt infiltration equation by constructing a decomposition series, valid for deep homogeneous soils under ponding conditions resulting from intense rainfall events.
Corradini et al.	2002	$\bar{i} = q_0\,[1 - prob(K_s \le K_c)] +$ $\sum_{j=1}^{3}(1 + \frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)}{I^*})[G(K_j) - G(K_{j-1})]$ (1) Where $I^* = \overline{K_j}\,(\frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)}{q_0} + t) +$ $4\sqrt{\overline{K_j}}((\psi_{wf}+h_0)(\theta_s-\theta_i))^{0.00022\psi_{wf}}(t - \frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)\overline{K_j}}{q_0(q_0-\overline{K_j})})^{0.5}$; $G\,(K_j) = \int_0^{K_j} K f_{K_s}(K)dK$ and $K_c = \frac{q_0 I}{I+(\psi_{wf}+h_0)(\theta_s-\theta_i)}$ $\bar{I} = I_p\,(1 - \frac{K_{se}}{q_0}) + K_{se}T_s + \psi_{wf}(\theta_s-\theta_i)\ln(\frac{I-(\psi_{wf}+h_0)(\theta_s-\theta_i)}{I_p-(\psi_{wf}+h_0)(\theta_s-\theta_i)})$ (2) Where $I_p = \frac{K_{se}\psi_{wf}(\theta_s-\theta_i)}{q_0-K_{se}}$; $t_p = \frac{\overline{K_s}\psi_{wf}(\theta_s-\theta_i)}{q_0(q_0-\overline{K_s})}$; $K_{se} = \overline{K_s}\{0.89 - 0.0831\ln \mathrm{CV}(K_s) - 0.8e^{(0.429\mathrm{CV}(K_s)-1.629)\frac{T_s}{t_p}}\}$;	Two models for estimating expected areal-average infiltration rate, $\bar{i}$ (L T^{-1}), at the hillslope scale were presented, under the condition of a negligible infiltration of surface water running downslope (run-on process) into soils. The soil is considered to be vertically homogeneous but the saturated hydraulic conductivity K_s (L T^{-1}) is assumed as a lognormally distributed random variable with PDF, $f_{K_s}(K)$, characterized by its mean value $\overline{K_s}$ and coefficient of variation CV(K_s). The first model (1) assumes that the areal-average infiltration is partly rainfall-controlled, and partly soil-controlled. It was developed based on the formulation of the field-scale infiltration rate for a given realization of K_s and following Govindaraju et al. (2001). The second model (2) incorporates the run-on process and is based on an empirical approach that uses an effective saturated hydraulic conductivity K_{se} (L T^{-1}), derived for each specific rainfall event of duration T_s (T).
Elrick et al.	2002	$I = S_H\,t^{1/2} + 1/3K_s t$ $0 \le t \le t_c$ $t = t_c - \frac{(\theta_s-\theta_i)}{CK_s}\{\frac{I-I_c}{(\theta_s-\theta_i)} - \frac{h_0+\psi_{wf}}{C}\ln(\frac{(h_0+\psi_{wf})+CI/(\theta_s-\theta_i)}{(h_0+\psi_{wf})+CI_C/(\theta_s-\theta_i)})\}$ $t > t_c$ Where $C = 1$ Constant head $C = 1 - \frac{(\theta_s-\theta_i)}{R}$; $R = \frac{\mathrm{A_{pond}}}{\mathrm{A}}$ Falling head	Elrick et al. used the GA approach to deduce a new implicit equation for constant- and falling-head conditions by using the term R (Elrick et al., 1995) which is the ratio of the cross-sectional area of the falling head tube, A_{pond} (L^2), to the infiltrating surface, A (L^2).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Parlange et al.	2002	$I = \frac{S_0^{\ 2}}{2K_s}\{t^* + \frac{1}{1-\delta} ln[1 + \frac{1-\delta}{\delta}\sqrt{1-f}]\}$ Where $t^* = \frac{S_0^{\ 2}}{K_s^2}$; $f = e^{\{-2\delta^2 t[\frac{1+A\sqrt{2t+2Bt}}{1+C\sqrt{2t+2Bt\sqrt{2\delta}}}]^2\}}$; $A = \frac{1}{2} + \frac{\lambda-2\delta}{3}$; $B = \frac{1+\sqrt{2\delta}}{12}(\frac{4\lambda-11\delta}{3} + 1)$; $C = \frac{1}{6} + \frac{\lambda}{3}$; $\lambda = \frac{35}{17}\delta - \frac{3}{2}\delta^{1/4} e^{(\frac{-15}{4}\sqrt{\delta})}$	Parlange et al. developed an explicit approximation to Parlange's equation (1982) assuming an initially dry, homogeneous soil where the surface of the soil is saturated, but not ponded. δ varies between 0 (Green & Ampt, 1911) and 1 (Talsma & Parlange, 1972). Average value $\delta = 0.8$ or 0.85 (Haverkamp et al., 1990; Parlange et al., 1985).
Warrick et al.	2005	$I = (\theta_s - \theta_i)\ z$ $\frac{K_s(t-t_i)}{(\theta_s-\theta_i)} = z - z_i - (\psi_{wf} + h_i) ln(\frac{z+\psi_{wf}+h_i}{z_i+\psi_{wf}+h_i})$ $\quad t_i \le t \le t_{i+1}, i = 0, 1, \ldots$	Cumulative infiltration was computed as a function of time-varying ponded water depths h_i (L) using a Green and Ampt analysis; the ponded depth h_i (L) changes stepwise with time.
Weiler	2005	$I_m = \frac{1}{2}\frac{b^{1/3}}{(\theta_s-\theta_i)} + \frac{1}{2}\frac{a}{b^{1/3}} + \frac{1}{2} r_f$ Horizontal I Where $a = (\theta_s - \theta_i)\ r_f^2$; $b = r_f\ (\theta_s - \theta_i)^2[12c - a + 2\sqrt{6c(6c-a)}]$; $c = K_{s,m}\ (\psi_{wf,m} + h_0)$ $\frac{dI_m}{dt} = K_{s,m}\ (\frac{\psi_{wf,m}+h_0}{I_m} + 1)$ Vertical I	Weiler developed an infiltration model which combines macropore flow variability to model dual- permeability soils using analytical solutions of the Green–Ampt equation which serve as the basis for the model.
Govindaraju et al.	2006	$\bar{i}$ from Equations (9) and (13) in Govindaraju et al.'s paper. Where $F_c = \frac{I}{I+(\psi_{wf}+h_0)(\theta_s-\theta_i)}$ and $G_{K_s}\ (K_1, \xi) = \int_0^{K_1} K^{\xi} f_K(K) dK$	A semi-analytical model estimating the areal-average infiltration rate at hillslope scale was presented, which neglects the infiltration of surface water running downslope into pervious soils (run-on process). It accounts for spatial heterogeneity of the saturated hydraulic conductivity, K_s (L T^{-1}), and rainfall rate, q_0 (L T^{-1}). The K_s field is characterized by a lognormal probability density function with PDF, $f_{K_s}(K)$, characterized by its mean value $\overline{K_s}$ and coefficient of variation CV(K_s), while the rainfall rate q_0 (symbolized as r in the paper) is represented by a uniform distribution between two extreme values, r_{min} and $r_{min} + R$. The model formulation relies upon the use of cumulative infiltration (symbolized as F in the paper) as the independent variable which is expressed as function of time for use in practical applications.
Morbidelli et al.	2006	$\bar{i} = \frac{1}{A}\int_{A_1}\int_t (q_0 - \frac{s^{1/2}}{\varepsilon_{Manning}}\frac{\partial h^{5/3}}{\partial x}) dA_1 + \int_{A_2}\int_t i dA_2$ $i = K_s + \frac{K_s(\theta_s-\theta_i)(\psi_{wf}+h_0)}{I}$	Morbidelli et al. explored the role of spatial heterogeneity, in both the saturated hydraulic conductivity K_s (L T^{-1}) and rainfall intensity q_0 (L T^{-1}), on the integrated hydrological response of a natural slope. On this basis, Morbidelli et al. (2006) developed a model which estimates the areal-average infiltration rate $\bar{i}$ (L T^{-1}) in the plane area A (L^2), partly controlled (over the unsaturated area A_1 (L^2)) by the rainfall and run-on, and partly controlled by the soil (saturated area A_2 (L^2)).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Lassabatère et al.	2006	$I = S_0\, t^{0.5} + [A(1-B)S_0{}^2 + Bi_\infty]t$ small t $I = (AS^2 + K_s)\, t + C\frac{S^2}{K_s}$ large t Where $A = \frac{\gamma}{r(\theta_s - \theta_i)}$; $B = \frac{2-\beta}{3}\,[1 - (\frac{\theta_i}{\theta_s})^n] + (\frac{\theta_i}{\theta_s})^n$; $C = \frac{1}{2[1-(\frac{\theta_i}{\theta_s})^n](1-\beta)}\; ln(\frac{1}{\beta})$; $i_\infty = AS_0{}^2 + K_s$	Lassabatère et al. adopted the BEST approach; this latter uses the following equation developed in Haverkamp et al. (1994): $I = S_0\, t^{0.5} + [AS_0{}^2 + BK_s]t$ for short times and expresses an equivalent equation by replacing the hydraulic conductivity K_s (L T^{-1}) by the formula of the steady-state infiltration rate i_∞. $\beta \approx 0.6$ and $\gamma \approx 0.75$, which applies for most soils when $\theta_i < 0.25\theta_s$ (Haverkamp et al., 1994; Smettem et al., 1994).
Chen and Young	2006	Steady rainfall $p(t)$: $i = q_0\, cos(\gamma_0)$ $\quad t \le t_p$ $K_s[t - (t_p - t_s)]cos\,(\gamma_0) = I - \frac{(\psi_{wf}+h_0)(\theta_s-\theta_i)}{cos(\gamma_0)} ln[1 + \frac{I\,cos(\gamma_0)}{(\psi_{wf}+h_0)(\theta_s-\theta_i)}]$ $t > t_p$ Unsteady rainfall $p(t)$: Detailed derivation is presented in the paper's Appendix.	Chen and Young quantified and explained the effects of slope angle γ_0(degrees) on infiltration into homogeneous and isotropic soils by extending the Green-Ampt equation onto sloping surfaces. For the steady rainfall case, the infiltration rate is determined by the rainfall intensity q_0 (L T^{-1}) before ponding (i.e., $t \le t_p$) and is calculated using the GA model after ponding (i.e., $t > t_p$). For unsteady rainfall, detailed derivation is presented in the appendix.
Warrick and Lazarovitch	2007	$I = I_{1D} + \frac{\gamma S_0^2 t}{w(\theta_s-\theta_i)}$ $i = K_s + \frac{\gamma S_0^2}{w(\theta_s-\theta_i)}$ $\quad t \to \infty$	Warrick and Lazarovitch addressed two-dimensional infiltration from water strip sources of width w (L) on the soil surface. The assumption is that when the cumulative infiltration is expressed per unit area of the wetted strip, the difference of that value and one-dimensional infiltration is linear with time.
Warrick et al.	2007	$\frac{I}{W^*} = I_{1D} + \frac{\gamma S_0^2 t}{W(\theta_s-\theta_i)}$ $i = W^* K_s + \frac{W^* \gamma S_0^2}{W(\theta_s-\theta_i)}$ $\quad t \to \infty$	Warrick addressed infiltration from furrows or narrow channels of wetted perimeter W (L). The basic approach was to develop the two-dimensional infiltration I (L) as a combination of the corresponding one-dimensional vertical I_{1D} (L) and an edge effect. Also, a simplified expression was found for the limiting steady-state case, which is analogous to Wooding's equation for infiltration from a shallow pond. Haverkamp et al. rationalized that a reasonable bound on γ was 0.6–0.8, but the results of Warrick and Lazarovitch (2007) for the strip showed generally a wider range from 0.6 to 1.1.
Assouline et al.	2007	$i = q_0\,(t)$ $\quad t \le t_p$ $i = i_{cap}\,(I_{cap})$ $\quad t > t_p$	A simple accurate method was presented to study infiltration under variable intensity rainfall. Before ponding ($t \le t_p$), infiltration rate is equal to the rainfall rate. After ponding ($t > t_p$), the infiltration rate i_{cap} (L T^{-1}) is a unique function of cumulative infiltration I_{cap} (L). Many mathematical expressions can be readily adapted to represent available i_{cap} (e.g., Green & Ampt, 1911; Kostiakov, 1932; Horton, 1940; Philip, 1957a; Smith & Parlange, 1978).
Germann et al.	2007	$i = 0$ $\quad t < t_w(z)$ $i = b_R\, \theta_R^3$ $\quad t_w(z) \le t \le t_d(z)$ $i = b_R\, \theta_R^3 (\frac{t_d(z)-T_s}{t-T_s})^{3/2}$ $\quad t > t_d(z)$ Where $t_w(z) = \frac{z}{b_R \theta_R^2}$; $t_d(z) = T_s + \frac{t_w(z)}{3}$	German et al. introduced a rivulet of conductance b_R (L T^{-1}) modeled as Stokes flow assuming steady and gravity-driven flow in which the effects of capillarity on infiltration are negligible, while viscous momentum dissipation impedes it.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Essig et al.	2009	$\bar{i}\, z_w = cos\gamma_0 \bar{K} z_w - \frac{\psi_{wf}}{(1-n)}(K_i S_i^{-1/\lambda} - \overline{KS}^{-1/\lambda})$	Essig et al. developed a simplified 1-D sharp-front model for sloping surfaces of slope angle γ_0 (degrees) using the Brooks and Corey (1964) relationship in combination with Darcy's law (1856), applicable for different boundary conditions: concentration (constant water content at the soil surface), flux (constant rainfall rate at the soil surface), and constant pressure head.
Valiantzas	2010	$I = S_0\, t^{1/2} + \frac{K_s}{2} t + \frac{K_s^2}{8S_0} t^{3/2}$ no ponding $I = S_H\, t^{1/2} + \frac{K_s}{2} t + \frac{K_s^2}{8S_H} t^{3/2}$ ponding	A simple mathematical form of an infiltration equation is developed using the two-algebraic equation (Philip, 1957b), which applies for the complete time range.
Su	2010	$I = S_0\, t^{\beta_4/2} + At$	A new equation of 1D cumulative infiltration into swelling soils is derived from the fractional Fokker–Planck equation (fFPE) of flow in swelling porous media, following the form of Philip's two-parameter infiltration equation (Philip, 1957b).
Corradini et al.	2011	$K_{1,s} < K_{1,h}$ $i_0 = q_0 + K_{1,i}$ $\quad t \le t_p < t_c$ $I_1 = I_{1,p} + \sqrt{2K_{1,s} G_1(\psi_i, \psi_0)}[t^{1/2} - t_p^{1/2}] + \frac{2}{3} K_{1,s}[t - t_p] + \frac{1}{18}[\frac{2K_{1,s}^3}{(\theta_{1,s}-\theta_{1,i})G_1(\psi_i,\psi_0)}]^{1/2} [t^{3/2} - t_p^{3/2}]$ $\quad t_p < t < t_c$ $i = K_{1,s}\,[1 + \frac{G_1(\psi_c,0)}{l_c}]$ $\quad t \ge t_c$ Where $I_{1,p} = [1 + \frac{B_1 p_1 K_{1,s}(\theta_{1,s}-\theta_{1,i}) G_1(\psi_i,0)}{q_0 - K_{1,s}}]$; $t_p = I_{1,p}\,/q_0$; $K_{1,h} = \frac{q_0}{\frac{p_1 G_1(\psi_i,0)}{l_c}+1}$; $G_1\,(\psi_i, 0) = \frac{1}{K_{1,s}} \int_{\psi_i}^{\psi_0} K_1(\psi) d\psi$; $K_1\,(\psi) = K_{1,s}\,[1 + (\frac{\psi - d_1}{\psi_{1,str}})^{c_1}]^{-\frac{(3n+2)}{c_1}}$ $K_{1,s} \ge K_{1,h}$ (i.e., $t_p > t_c$) $i = q_0$ $\quad t \le t_p$ $i = K_{1,s}\,[1 + \frac{G_1(\psi_c,0)}{l_c}]$ $\quad t > t_p$ $t_p = (1.5 - S_{2i})^{0.7}\,(2.5K_{2,s})^{0.12}(25 + 0.4 l_c) q_0^{-(1+1.5/l_c^{0.5})}$ Where $S_{2i} = \frac{\theta_{2,i} - \theta_{2,r}}{\theta_{2,s} - \theta_{2,r}}$; $G_1\,(\psi_c, 0) = (\frac{4}{5} n - 1.7)\,(\psi_{1,str} + d_1)(1 - e^{-\frac{\psi_c}{\psi_{1,str}}})$ $\quad$ for $\psi_c \le 0$ $G_1\,(\psi_c, 0) = -\psi_c$ $\quad$ for $\psi_c > 0$	A simple conceptual model was proposed for local infiltration into a two-layered soil profile with the upper layer much more permeable than the subsoil under any rainfall pattern. It is a reformulation of a conceptual model of Corradini et al. (2000). Two conditions exist, based on whether ponding occurs while the wetting front is entirely within the surface layer (i.e., $K_{1,s} < K_{1,h}$) or has moved below the interface between layers (i.e., $K_{1,s} \ge K_{1,h}$). In this notation $K_{1,s}$ (L T-1) represents the saturated hydraulic conductivity of the upper soil layer and $K_{1,h}$ (L T-1) represents the saturated hydraulic conductivity value that is needed to achieve ponding by the time the wetting front reaches the interface. For $K_{1,s} < K_{1,h}$, the dynamic wetting front is entirely within the upper layer, yielding the ponding time tp (T). For the case when the front extends into the lower layer ($t \ge t_c$) and $\theta_{1,c}$ is rather close to $\theta_{1,s}$, it is assumed that $K_{1,c} \approx K_{1,s}$ and $d\psi_c/dt$ is negligible because $d\psi_1/\theta_1 \to 0$ as $\theta_1 \to \theta_{1,s}$. Then, parameterized forms for the time when the wetting front reaches the interface is determined based on the time of ponding t_p (T) associated with ($K_{1,s} \ge K_{1,h}$).
Swamee et al.	2012	$I = (\psi_{wf} + h_0)(\theta_s - \theta_i)[1.94(t^*)^{0.74} + (t^*)^{1.429}]^{0.7}$ (1) $I = (\psi_{wf} + h_0)(\theta_s - \theta_i)[2.66(t^*)^{1.124} + (t^*)^{2.174}]^{0.46}$ (2) Where $t^* = \frac{K_s}{(\psi_{wf}+h_0)(\theta_s-\theta_i)}\, t$	Based on (1) the Green-Ampt (1911) and (2) the Talsma– Parlange (1972) implicit equations, Swamee et al. generated two explicit expressions for cumulative infiltration using a curve-fitting method. The main difference between the Green–Ampt and Talsma–Parlange assumptions is that (1) considers a soil which has a rapidly varying diffusivity D (L^2 T^{-1}) and a near-constant hydraulic conductivity $K = K_s$ (L T^{-1}) while (2) assumes that D and $\partial K/\partial\theta$ are proportional.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Govindaraju et al.	2012	Local-scale Infiltration: $I = q_0\, t \qquad 0 \le t \le t_p$ $A(I^3 - I_p^3) + B(I^2 - I_p^2) + C(I - I_p) + D ln\, \frac{I + \Delta\psi\Delta\theta}{I_p + \Delta\psi\Delta\theta} = P(t - t_p)$ $t > t_p$ Where $A = 1/3$; $B = \Delta\theta/2(3a_4 - \Delta\psi)$; $C = (\Delta\psi\Delta\theta)^2 - 3a_4\Delta\psi\Delta\theta^2 + 3(a_4\Delta\theta)^2$; $D = -(\Delta\psi\Delta\theta)^3 + 3a_4\Delta\psi^2\Delta\theta^3 - 3a_4^2\Delta\psi\Delta\theta^3$; $P = 3(a_4\Delta\theta)^2 K_{s,0}$ $t_p = I_p\,/p$ and $p = \frac{3(a_4\Delta\theta)^2 K_s(I_p + \Delta\psi\Delta\theta)}{I_p^3 + 3a_4\Delta\theta I_p^2 + 3(a_4\Delta\theta)^2 I_p}$ Field-scale Infiltration: $\bar{i} = q_0\,[1 - G(K_c, 0)] + \frac{3(a_4\Delta\theta)^2(I + \Delta\psi\Delta\theta)}{I^3 + 3a_4\Delta\theta I^2 + 3(a_4\Delta\theta)^2 I} G(K_c, 1)$ Where I from Equations (12) and (13), $G(K_c, \xi)$ from Equation (9) and K_c from Equation (10) $\Delta\psi = \psi_{wf} + h_0$ and $\Delta\theta = \theta_s - \theta_i$	Govindaraju et al. studied the problem of local- and field-scale infiltration over a particular class of heterogeneous soils with the dimensionless parameter a_4 (originally called a) governing the non-uniformity in the saturated hydraulic conductivity K_s (L T^{-1}). At the local scale, the theory for infiltration refers to vertically nonuniform soils (K_s (L T^{-1}) decreasing with depth according to a power law) and is developed using a sharp front (Green–Ampt) approach. To determine field-scale infiltration properties, the spatial variability in the surface saturated hydraulic conductivity is represented by a log-normal random field $Y = ln(K_{s,0})$ with mean μ_Y and standard deviation σ_Y.
Ali et al.	2013	$I = (\psi_{wf} + h_0)(\theta_s - \theta_i)[\sqrt{\frac{F_1 K_s}{(\psi_{wf} + h_0)(\theta_s - \theta_i)} t} + F_2 + F_3]$	By replacing the logarithmic term of the GA model by sequential segmental second order polynomials, Ali et al. (2013) developed a generalized explicit model for estimation of cumulative infiltration, varying the length of wetting front, z_w(L). The universal values of the model's coefficients change with different ranges of $z_w/(\psi_{wf} + h_0)$.
Almedeij and Esen	2014	$I = \psi_{wf}\,(\theta_s - \theta_i)(0.65t^* + \sqrt{0.25t^{*2} + 2t^*}) \qquad 0 \le t \le t_p$ Where $t^* = \frac{K_s}{\psi_{wf}(\theta_s - \theta_i)}\, t$ $I = \Psi_{wf}(\Theta_s - \Theta_i)\{\frac{1}{2}(t^* + \sqrt{(t^*)2 + 8t^*}) + 0.15t^*\} \qquad t > t_p$ Where $t^* = \frac{K_s}{\Psi_{wf}(\Theta_s - \Theta_i)}\, t$; $t_p = \frac{K_s\psi_{wf}(\theta_s - \theta_i)}{i_p(i_p - K_s)}$	A proposed explicit modification for the infiltration model (Mein & Larson,) is presented for the case when steady rainfall rate is less than the initial infiltration capacity of the soil until ponding time ($0 \le t \le t_p$). After ponding ($t > t_p$), the cumulative infiltration will follow the classical Green-Ampt model; the designated equation and that suggested by Li et al. (1976) are similar, but the equation here imposes an additional term of $0.15t^*$. This additional term accounts for the remaining components of the first order approximation of the power series expansion of the natural log in GA equation.
Bautista et al.	2014	$I\,(t_i) = I(t_{i-1}) + \Delta Z_2(t_i) + \Delta E(t_i)$ $\Delta Z_2(t_i)$ is defined in Bautista Eq.s (5) to (9). $\Delta E(t_i)$ is defined in Bautista Eq.(11).	This article discussed a methodology, derived from the two-dimensional Richards equation, for estimating two-dimensional furrow infiltration for time-variable surface flow depth h_0(t). The method was originally derived assuming a constant pressure head at the infiltrating surface by Warrick et al. (2007).
Lassabatere et al.	2014	$I = w_f\, I_f + (1 - w_f)I_m$ $\frac{2\Delta K_m^2}{S_{0,m}^2} t = \frac{1}{1-\beta_m}\,[\frac{2\Delta K_m}{S_{0,m}^2}(I_m - K_{i,m}t) - \ln(\frac{e^{\beta_m \frac{2\Delta K_m}{S_{0,m}^2}(I_m - K_{i,m}t)} + \beta_m - 1}{\beta_m})]$ and $\frac{2\Delta K_f^2}{S_{0,f}^2} t = \frac{1}{1-\beta_f}\,[\frac{2\Delta K_f}{S_{0,f}^2}(I_f - K_{i,f}t) - \ln(\frac{e^{\beta_f \frac{2\Delta K_f}{S_{0,f}^2}(I_f - K_{i,f}t)} + \beta_f - 1}{\beta_f})]$ $I_{3D} = w_f\,(I_{1D,f} + \frac{\gamma_f S_{0,f}^2}{r\Delta\theta_f} t) + (1 - w_f)(I_{1D,m} + \frac{\gamma_m S_{0,m}^2}{r\Delta\theta_m} t)$ Where $\Delta K = K_s - K_i$ and $\Delta\theta = \theta_s - \theta_i$	Lassabatere et al. assumed that infiltration into the dual-permeability medium may be derived from infiltration into individual single-permeability domains, i.e., the matrix and the fast-flow region, while assuming that there is no water transfer between the two pore regions. As such, infiltration fluxes into the dual-permeability medium are simply a linear combination of fluxes into individual regions, with the proportionality coefficients corresponding to the fractions of surface occupied by each region. Lassabatere et al. further developed a three-dimensional water infiltration equation from a circular disk source into dual-permeability media assuming surface pressure head $h_0 \le 0$.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Vatankhah	2015	$I = (\psi_{wf} + h_0)(\theta_s - \theta_i)(t^* + 2.693ln[1 + 0.527\sqrt{t^*}])$ Where $t^* = \frac{K_s}{(\psi_{wf}+h_0)(\theta_s-\theta_i)}\ t$	Vatankhah developed an explicit expression by further modifying the Almedeij and Esen (2014) model using a nonlinear technique to reduce the maximum absolute relative error; use Eq. (5) in the paper for any any rainfall rate whether greater or less than the initial infiltration capacity and without requiring the determination of the ponding time, t_p(T).
Bautista et al.	2016	$I^i_{2D} = Z^i_k + E^i_k$ Computational Method 1: Use Equations (10) and (11) for Z^i_1 and E^i_1. Computational Method 2: Use Equations (14)-(16) for Z^i_2 Use Equations (18)-(20), and (21) for E^i_2 Computational Method 3: Use Equation (23) for Z^i_3 Use Equations (24) and (25) for E^i_3 (constant γ) Use Equations (25) and (26) for E^i_3 (depth dependent γ)	Bautista et al. proposed a methodology for estimating furrow infiltration under time-variable ponding depth. Bautista et al. (2014a) previously proposed a modification to the infiltration equation developed by Warrick et al. (2007) to account for time-variable ponding depth h_0 (t). This study further examined this problem, proposed an alternative formulation, and examined the problem of calibrating the parameter γ under variable depth conditions.
Nie et al.	2017a	$I = 0.5K_s t + \sqrt{2K_s(h_0 + \psi_{wf})(\theta_s - \theta_i)t[1 + \frac{K_s t}{8(h_0+\psi_{wf})(\theta_s-\theta_i)}]}$	Nie et al. present an approximate explicit solution to the Green-Ampt (GA) infiltration model for estimating the wetting front depth of 1D infiltration based on Valiantzas (2010).
Selker and Assouline	2017	$i = K_s + \frac{\beta_0 K_s + \sqrt{\frac{(\theta_s-\theta_i)K_s\psi_{wf}}{2t}}}{1+\beta_0\frac{K_s t}{(\theta_s-\theta_i)\psi_{wf}}+\sqrt{\frac{2K_s t}{(\theta_s-\theta_i)\psi_{wf}}}} +$ $0.1461(\theta_s - \theta_i)0.212[K_s t)0.269(h_0 + \psi_{wf})0.269]0.788$	Selker and Assouline presented a simple explicit solution for ponded infiltration into soils using the Green and Ampt approach by describing early infiltration behavior in terms of the sum of gravitational flow and the exact solution for capillary imbibition. β_0 can be approximated as 2/3 (Brutsaert, 1977; Selker & Assouline, 2017; Stewart, 2019).
Stewart and Abou Najm	2018a	$I = S_0 t^{0.5} + aK_s t + \frac{S_0{}^2 ab}{(\theta_s-\theta_i)(d+\frac{r}{2})}t \quad 0 \le t \le t_c$ $I = fK_s\ t + f(1 - a)K_s t_c \quad t > t_c$ Where $t_c = \frac{(\theta_s-\theta_i)(h_0+\lambda_c)}{4bK_s f^2(1-a)^2};\ f = \frac{h_0+\lambda_c}{d+r/2} + 1$	Based on two-term Philip type solutions for short and long times, the proposed model accounts for different ring sizes and depths of insertion, initial water content and matric pressure head, transient and steady-state infiltration behaviors, and non-zero water supply pressures. For real soils, $0.4 < a < 0.5$ (Stewart & Abou Najm, 2018a); a was determined through curve-fitting to be approximately equal to 0.91 (Wu & Pan, 1997). In most cases, $a = 0.45$ is recommended. b varies between 1/2 and $\pi/4$ depending on the shape of the soil water diffusivity function (White & Sully, 1987). $b = 0.17$ (Wu & Pan, 1997). A value of $b = 0.55$ is often assumed (Haverkamp et al., 1994; Reynolds & Elrick, 1990; White & Sully, 1987). λ_c(L) is defined as being equal to the matric flux potential, φ(L^2 T^{-1}), scaled by K_s (L T^{-1}): $\lambda_c = \varphi/K_s$. According to Iovino et al. (2021), a could be fixed in advance.

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Stewart	2018	$I = I_m + I_f$ $I_m = q_{0,m}\, t$ $\quad t \leq t_{p,m}$ $I_f = \min(q_{0,f} t, K_f t, \frac{V_f}{A_f})$ $t > t_{p,m}$ $I_m = q_{0,m}\, t_{p,m} + K_m(t - t_{p,m}) +$ $(\theta_s - \theta_i)\psi_{wf,m} \ln\left[\frac{1+\beta_0 \frac{K_m t}{(\theta_s-\theta_i)\psi_{wf,m}} + \sqrt{\frac{2K_m t}{(\theta_s-\theta_i)\psi_{wf,m}}}}{1+\beta_0 \frac{K_m t_{p,m}}{(\theta_s-\theta_i)\psi_{wf,m}} + \sqrt{\frac{2K_m t_{p,m}}{(\theta_s-\theta_i)\psi_{wf,m}}}}\right]$ $I_f = \min(q_{0,f} t_{p,m} + q_0(t - t_{p,m}) - (I_m - I_{p,m}), K_{crack} t, \frac{V_f}{A_f})$ Where $q_{0,m} = (1 - \frac{\Upsilon \emptyset_f}{1-\emptyset_{sub}})\, q_0$ and $q_{0,f} = (\frac{\Upsilon \emptyset_f}{1-\emptyset_{sub}})\, q_0$	A multidomain infiltration model was proposed based on Green and Ampt (1911) to estimate infiltration in shrink-swell soils mediated by the matrix, which may include small-scale interaggregate shrinkage cracks, from those associated with inter-block cracks that surround the matrix. The two domains are related to the total crack porosity via a proportionality factor Υ ($0 \leq \Upsilon \leq 1$). To apply the Green–Ampt model in a multidomain formulation, Stewart divided the rainfall (precipitation) rate, q_0 (L T^{-1}), between the soil matrix and border crack domains.
Rahmati et al.	2019	$I = S_0 t^{0.5} + \frac{2-\beta}{3} K_s t + \frac{1}{9}(\beta^2 - \beta + 1)\frac{K_s^2}{S_0} t^{\frac{3}{2}}$	Rahmati et al. applied the Taylor series (Philip 1957a) up to third order in powers of 0.5 to the quasi-exact implicit analytical expansion presented by Haverkamp et al. (1994) to introduce a three-term infiltration model. β can take the value of 0.6 over the whole range of θ (Haverkamp et al., 1990). It ranges from 0.3 for sand and 1.7 for silty soils (Lassabatère et al., 2009; Rahmati et al., 2019).
Stewart	2019	$I_f = q_0\, t - I_m$ $\quad t < t_{p,f}$ (1) $t < t_{p,m}$ (2) $I_m = (1 - w_f)\, q_0 t$ $I_f = w_f\, q_0 t$ $I_m = (1 - w_f)\, K_{s,m} t\left[(\frac{q_0}{K_{s,m}} - 1)\frac{t_{p,m}}{t} + 1 + \frac{1}{\tau} ln\left(\frac{1+\beta_0\, \tau + \sqrt{2\tau}}{1}\right.\right.$ $\left.\left. + \beta_0\, \tau_{p,m} + \sqrt{2\tau_{p,m}}\right)\right]$ $\quad t \geq t_{p,m}$ (4) $I_f = K_{s,m}\, t\left[(\frac{q_0}{K_{s,m}})\frac{t_{p,f}}{t} + w_f \frac{K_{s,f}}{K_{s,m}}(1 - \frac{t_{p,f}}{t}) - I_{mp,f}\right]$ $\quad t \geq t_{p,f}$ (5) Where $I_{mp,f} =$ $(1 - w_f)\, K_{s,m} t\left[(\frac{q_0}{K_{s,m}} - 1)\tau_{p,m} + \tau_{p,f} + \frac{1}{\tau} ln\left(\frac{1+\beta_0\, \tau_{p,f} + \sqrt{2\tau_{p,f}}}{1+\beta_0\, \tau_{p,m} + \sqrt{2\tau_{p,m}}}\right)\right]$; $\tau = \frac{K_{s,m} t}{\Delta\theta_m \psi_{wf,m}}$; $\tau_{p,m} = \frac{K_{s,m} t_{p,m}}{\Delta\theta_m \psi_{wf,m}}$; $\tau_{p,f} = \frac{K_{s,m} t_{p,f}}{\Delta\theta_m \psi_{wf,m}}$	Stewart applied the Green–Ampt infiltration model within a dual-domain framework to distinguish water movement through the soil matrix vs. through macropores. For all time prior to ponding in the macropore domain ($t < t_{p,f}$), the cumulative infiltration as preferential flow is determined as the cumulative precipitation minus cumulative infiltration into the soil matrix (1). Before the soil matrix experiences ponding ($t < t_{p,m}$), both domains absorb rainfall in proportion to their surface-connected areas (2). After the soil matrix ponds ($t \geq t_{p,m}$), its cumulative infiltration is determined by Equation (4). By assuming that water is moving through the macropores as plug flow under a unit hydraulic gradient, cumulative infiltration into the preferential flow domain after ponding ($t \geq t_{p,f}$) is found by Equation (5). β_0 (Called α in Stewart, 2019) can be approximated as 2/3 (Brutsaert, 1977; Selker & Assouline, 2017; Stewart, 2019).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Baiamonte	2020	$I = I^* K_s t_s$ Where (1): $I^* = 2\rho^2 \ln(\frac{\rho-\sqrt{\Theta_i}}{\rho-\sqrt{\Theta}}) - 2\rho(\sqrt{\Theta} - \sqrt{\Theta_i}) - (\Theta - \Theta_0)$; $0 < \tau \le T_s$ $I^* = 2\rho^2 \ln(\frac{\rho-\sqrt{\Theta_i}}{\rho-\sqrt{\Theta_s}}) - 2\rho(\sqrt{\Theta_s} - \sqrt{\Theta_i}) - (\Theta - \Theta_0)$; $\tau > T_s$ (2): I^*, respectively. $\rho = q_0 / K_s$; $\Theta = (\theta - \theta_i)/\Delta\theta$; $\tau = t/t_c$	An exact analytical solution of Richards' equation under gravity-driven infiltration and constant rainfall intensity is derived. The solution is presented under Torricelli's law, which simulates the soil hydraulic conductivity function and describes the emptying or filling process of a nonlinear water reservoir.
Su et al.	2020	$i = -K_e(\partial\psi/\partial z + 1)$ $h \le 0$ (1) $i = -\partial\psi/\partial t - K_e(\partial\psi/\partial z + 1)$ $0 < h \le h_{\max}$ (2) Where $K_e = K_{sh}\, S_e^{0.5}[1 - (1 - S_e^{1/m_1})^{m_1}]^2$; $K_{sh} = K_s\, 10^{m'(\theta_{sh}-\theta_s)}$; $\theta_{sh} = 2 - \emptyset_i - \frac{1}{\rho_b(a_3+\alpha_3\omega)} - \frac{A''+B''\ln(\rho_{bw}z)}{\rho_b}$; $S_e = [\frac{1}{1+(\alpha\psi)^{n_{SSC}}}]^{m_{SSC}}$ and $m_{SSC} = 1 - 1/n_{\mathrm{SSC}}$	A modified Richards model (RMSD) was developed to simulate soil water movement into deformable soils subject to the following upper boundary conditions: (1) when rainfall intensity does not exceed the infiltration capacity of the soil, and (2) when rainfall intensity exceeds the infiltration capacity of the soil. Su et al. introduced the unsaturated hydraulic conductivity of deformable soils, K_e (L T^{-1}) and related it to physical properties using Lambe and Whitman (1979) and van Genuchten (1980, 1991) models. The model accounts for changes in saturated hydraulic conductivity (Ksh) and saturated water content (θsh) with depth. RMSD: $\frac{\partial\theta}{\partial t} + \frac{\theta_i}{1+\emptyset_i}\frac{\partial\emptyset}{\partial t} = \frac{\partial}{\partial z}[K_e(\psi)\frac{\partial\psi}{\partial z}] + \frac{\partial K_e(\psi)}{\partial z} + W_r$ Where $W_r = -S_r \Delta x \Delta y \Delta z \Delta t$ S_r (T^{-1}) is the water absorption strength of plant roots.
Poulovassilis and Argyrokastritis	2020	$I = S_0\, t^{0.5}(1 - e^{-c_0(\frac{K_s}{S})t^{0.5}}) + K_s t$	A new two-term analytical equation, which takes the form of Philip's two-term equation, was derived for estimating vertical cumulative infiltration occurring in homogeneous porous media under zero ponding, including a factor a_t (L), characteristic of each porous body, valid for all t.
Abou Najm et al.	2021	$i_{WR} = i(1 - e^{-\alpha_{wr}t})$	Abou Najm et al. (2021) proposed a simple correction term $(1 - e^{-\alpha_{wr}t})$ that can be applied to any infiltration model to simulate infiltration behaviors of water-repellent soils. The correction term starts with a value of zero at the beginning of the infiltration experiment (t = 0) and asymptotically approaches 1 as time increases, thus simulating decreasing soil water repellency through time. To demonstrate the effectiveness of this method, Abou Najm et al. (2021) used a simple two-term infiltration model similar to two-term Philip type solution (Check Equation 9 in the paper).

(Continues)

TABLE A2 (Continued)

Model	Year	Equation(s)	Applied concepts
Di Prima et al.	2021	$I = S_0\sqrt{t} - \frac{S_0\sqrt{\pi}}{2\sqrt{\alpha_{wr}}}\mathrm{erf}(\sqrt{\alpha_{wr}t} + [A(1-B){S_0}^2 + Bi_\infty]t - \frac{[A(1-B){S_0}^2+Bi_\infty](1-e^{-\alpha_{wr}t})}{\alpha_{wr}}$ small t $I = (AS^2 + K_s)\,t + C\frac{S^2}{K_s}$ large t Where $A = \frac{\gamma}{r\Delta\theta}$; $B = \frac{2-\beta}{3}\,[1-(\frac{\theta_i}{\theta_s})^n] + (\frac{\theta_i}{\theta_s})^n$; $C = \frac{1}{2[1-(\frac{\theta_i}{\theta_s})^n](1-\beta)}\,ln(\frac{1}{\beta})$ and $i_\infty = A{S_0}^2 + K_s$	Di Prima et al. presented an adaptation of the BEST method, namely BEST-WR, to characterize soils at any stage of water-repellency. They modified the Haverkamp explicit transient infiltration model, included in BEST for modeling infiltration data, by embedding the scaling factor $(1-e^{-\alpha_{wr}t})$ introduced by Abou Najm et al. (2021) that describes the rate of attenuation of infiltration rate due to water repellency. Di Prima et al. suggest B = 0.467 and C = 0.639 for most initially dry soils. The general equations of B and C presented here are defined by Lassabatere et al. (2006).
All parameters are algebraically positive values, unless otherwise specified.			

Note: Subscripts: m, f, and h denote the matrix, the fast-flow, and the interface between these two regions; p, w, and d denote the ponding, wetting, and draining stage; i, s, r, and ∞ denote the initial, saturated, relative, and final phase, respectively; 0 denotes the soil surface ($z = 0$).