Drewitz, Alexander; **Prévost, Alexis**; **Rodriguez, Pierre-François**
Critical exponents for a percolation model on transient graphs. (English) Zbl 07662556
Invent. Math. 232, No. 1, 229-299 (2023)

The authors study the bond percolation model obtained by considering the clusters of a weighted graph G (transient for the random walk on G) induced by the excursion sets of the Gaussian free field ϕ on the cable system $\tilde{G}$ associated to G.

They give two theorems describing the near-critical regime of the phase transition for the corresponding percolation model and derive various associated critical exponents, all of them functions of two parameters, ν and α, describing resp. the decay of correlations and the volume growth of G.

The proofs make use of continuity and strong Markov properties and of potential theory.

Reviewer: Valeria Ricci (Roma)

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
82B43 Percolation
60G15 Gaussian processes

Cited in **5** Documents

Keywords:

phase transition; percolation models; critical exponents

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