

# A variant design of Tuned Mass Damper Inerter for reducing displacements of base isolated structures

Chiara Masnata<sup>1</sup>, Alberto Di Matteo<sup>1</sup>, Miriam Chillemi<sup>2</sup>, Antonina Pirrotta<sup>1</sup>

<sup>1</sup>Dipartimento di Ingegneria, Università degli Studi di Palermo, viale delle Scienze, 90128 Palermo, Italy

<sup>2</sup>Unit of Applied Mechanics, University of Innsbruck, Technikerstr. 13, 6020 Innsbruck, Austria

email: chiara.masnata@unipa.it, alberto.dimatteo@unipa.it, miriam.chillemi@uibk.ac.at, antonina.pirrotta@unipa.it

**ABSTRACT:** In this study an innovative passive control strategy, referred to as New Tuned Mass Damper Inerter (New TMDI), coupled with a base isolation system (BI), is presented with the aim of reducing displacements in base isolated structures subject to seismic actions.

The proposed New TMDI employs the synergetic benefits of a recently developed non-traditional Tuned Mass Damper (known as New TMD) and an inerter device to achieve higher control performances in isolated structures. Specifically, the New TMD is a secondary mass system connected to the BI system by a spring and to the ground by a dashpot. In addition, in the New TMDI configuration, the inerter device is placed in parallel with the damper.

The New TMDI optimal parameters are determined by performing a simplified approach which provides simple closed-form formulae as a quick tool to design the examined device. The reliability of the proposed approach is assessed by a comparison with a more accurate and computationally complex numerical optimization procedure. Further, the performance of a base-isolated multi-degree-of-freedom structure equipped with a New TMDI is investigated taking into account different recorded ground motions as base excitation. Results from time history analyses assess the mitigation effect due to the optimal New TMDI on the response of base-isolated structures, also in comparison with those of a conventional Tuned Mass Damper Inerter (TMDI). On this base, the proposed New TMDI might be considered as preventive conservation strategy for applications also in historic buildings, where general constraints require limited space for the base displacements.

**KEY WORDS:** Base-isolation system; Tuned Mass Damper; Inerter; Optimal design.

## 1 INTRODUCTION

Nowadays, seismic base-isolation [1] is one of the most effective and widely used seismic protection technique to mitigate damage in buildings, bridges, industrial facilities and, in general, relatively stiffness structures, prone to the earthquake excitation [2] [3]. Base-isolation system introduces a layer of low lateral stiffness between the structure and the foundation, this leads to a decoupling of the building structure from the ground motion so the superstructure essentially behaves as a rigid body. In this manner, the majority of the displacement occurs within the base isolation sub-system, while displacements and accelerations of the main structure are greatly reduced. This effect is possible thanks to the so-called isolators that are devices supplied concurrently by low lateral and high vertical stiffness. On this base, recent studies concerning smart structure strategies have been dedicated also to the development of innovative materials for the manufacturing of novel seismic isolation devices [4].

The effectiveness of base-isolation in reducing structural forces is closely tied to the lengthening of the natural period of the structure. It is shifted to the velocity-sensitive region of the spectrum with a much smaller pseudo-acceleration. On the other hand, the deformation increases but this deformation is concentrated in the isolation sub-system and it leads to only a small deformation in the structure [5]. Isolation systems can also be used as a retrofit approach for existing buildings that are brittle and weak: for instance, unreinforced masonry buildings or reinforced concrete buildings of early design, not

including the type of detailing of the reinforcement necessary to ensure ductile performance. It is therefore an attractive seismic improvement technique for monumental buildings of historical or architectural merit whose aspect and character should be preserved [6].

Although these beneficial features have led to base isolation strategies being extensively employed, some detrimental effects have to be taken into account. Specifically, laminated rubber bearings are prone to undergo large and undesirable displacements due to their limited lateral stiffness [1]. This could be a problem in case of densely populated urban areas, in fact, adjacent buildings could be structurally compromised if the controlled system presents significant displacements. In this regard, one possibility for the reduction of this undesirable displacement demand could be the implementation of damped isolators, even providing additional damping, for instance, resorting to external dampers. It is worth noting, however, that this approach leads to an increase of inter-storey drifts of the main structure which means a thwarting of the beneficial effect of the base-isolation in converting the superstructure behaviour as that of a rigid body [7].

Therefore, research efforts in the area of smart structural technologies have been also focused in developing a possible technology to enhance the dynamic performance of isolated structures considering a combined control mechanisms that avoid the use of active control systems which may cause undesirable effects [8].

As an alternative and effective strategy, some researchers began to consider using passive vibration control devices for

the seismic response reduction of base-isolated structures. In this respect, the majority of these studies have analysed the control performances of an hybrid control strategy in which a classical Tuned Mass Damper (TMD) is attached to the base isolation sub-system [9] [10]. TMD is a simple device consisting of a damped spring-mass system attached to a vibrating main system to mitigate any undesirable vibrations. By connecting this auxiliary mass to the base-isolation subsystem it is possible to obtain better control performance than providing supplemental damping to the isolation layer [11] [12]. In addition, the application of the TMD does not alter base-isolation benefits in terms of inter-storey displacement since the small inter-storey drifts typical of base-isolated structures are preserved. Finally, this device allows the displacement demand of the base-isolation sub-system to be reduced [13].

On the other hand, TMD is more effective at reducing the structural response for low damping. In addition, traditional TMD requires large masses to be effective and would need a large stroke to mitigate the structural responses in resonance circumstances, hence large spaces should be designed to accommodate the device. To overcome this possible drawback, some variants of TMDs have been proposed. In this regard, a novel configuration in which a so-called inerter [14] is in conjunction with the TMD has been recently proposed in order to enhance the effectiveness of the TMD without simultaneously amplifying the relevant mass ratio. The inerter is a mechanical device that ideally produces a force proportional to the relative acceleration between its two terminals.

Several kind of inerters have been proposed [15] [16] [17] but in essence they act as an apparent mass (also called inertance) that can be orders of magnitude higher than its physical mass. When the inerter is placed in parallel with the spring and the damper of a classical TMD, it consists of a hybrid device generally referred to as Tuned Mass Damper Inerter (TMDI). Another possibility, to enhance the performance of a TMD attached to the base-isolation sub-system, could be an innovative non-conventional form of the TMD, referred to as “non-traditional TMD” or New TMD [18]. In this case a spring is placed between the TMD mass and the basement, while the damper is located between the ground and the TMD mass rather than in parallel with the spring (like in the classical TMD).

In this regard, comparison with traditional TMD controlled based isolated structure has proved the improved control performance of the non-traditional design, for different types of earthquake excitations, especially in terms of TMD stroke [18]. Inspired by the successful outcomes deriving from the integration of an inerter element in a traditional TMD and from the attractive possibility to further reduce the auxiliary mass displacement by considering the non-traditional configuration, in this paper innovative structure strategies comprising non-traditional design of the TMDI, hereinafter referred to as New TMDI, is proposed to reduce in a more efficient way the displacement demand at the isolation floor of isolated buildings. Consider the proposed layout as New TMD connected with an inerter device installed in parallel with the damper, both located between the TMD mass and the ground. Adopting this non-traditional TMDI, it could be possible to

solve issues related to the proximity between historic buildings mentioned so far, offering higher control performance over an ordinary design. In this regard, in this paper, after the introduction of the analytical model of the New TMDI system an optimum design method is proposed. Specifically, this method has been performed by means of a recently developed procedure [19] [20], here extended to the case of the New TMDI attached to the base of an isolated structure subjected to a Gaussian white noise process. Finally, to show the validity of the proposed procedure and the efficiency of the New TMDI coupled with base-isolation sub-system, numerical simulations are carried out on a 5-storey building subjected to recorded ground excitations.

## 2 PROBLEM FORMULATION

Consider a base isolated (BI) structure shown in Figure 1 subjected to a horizontal ground acceleration  $\ddot{x}_g(t)$ . Let the main structure have  $n$  degrees of freedom (DOF); thus, the whole BI system has  $n+1$  DOF. Denote as  $m_b$ ,  $K_b$ ,  $C_b$ , respectively the mass, the stiffness and damping coefficient of the base isolation sub-system, assumed to behave linearly. Further, let  $M_i$  be the mass associated to the  $i$ th degree of freedom of the  $n$ -DOF main structure ( $i = 1, \dots, n$ ), whereas  $C_{ij}$  and  $K_{ij}$  are the generic elements of the corresponding damping and stiffness matrices.

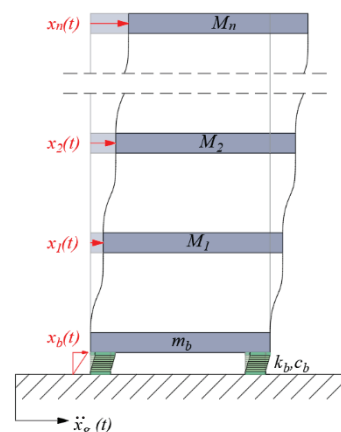


Figure 1. MDOF base isolated shear-type frame

In this way, the total mass of the system can be formulated as  $M_{tot} = m_b + \sum_{i=1}^n M_i$ . The structural response of the BI structure is finally given by the system of  $n+1$  equation of Equation 1.

$$\begin{cases} M_{tot} \ddot{x}_b(t) + \sum_{i=1}^n M_i \ddot{x}_i(t) + C_b \dot{x}_b(t) + K_b x_b(t) = -M_{tot} \ddot{x}_g(t) \\ M_i \ddot{x}_b(t) + M_i \ddot{x}_i(t) + \sum_{j=1}^n C_{ij} \dot{x}_j(t) + \sum_{j=1}^n K_{ij} x_j(t) = -M_i \ddot{x}_g(t) \end{cases} \quad (1)$$

where ( $i=1, \dots, n$ ),  $x_b(t)$  is the displacement of the base isolation sub-system relative to the ground,  $x_i(t)$  is the displacement of the  $i$ -th DOF of the main structure relative to  $x_b(t)$ , as shown in Figure 1, and a dot over a variable stands for derivation with respect to time.

Figure 2 shows the differences between the traditional TMDI (Figure 2 (a)) and non-traditional TMDI (Figure 2 (b)),

attached to the degree of freedom of the base isolation sub-system, where  $m_d$ ,  $k_d$ ,  $c_d$ , denote the mass, the stiffness and the damping of the considered device, respectively, and  $b$  the inertance.

As can be seen, the only difference is that the damper of the New TMDI is not connected to the base isolation sub-system but directly to the ground, while the inerter is connected, in both cases, to the moving mass of the device on one terminal and to the ground on the opposite one. In this way, the force generated by the inerter device is directly proportional to the relative acceleration of the device and to the inertance parameter  $b$ .

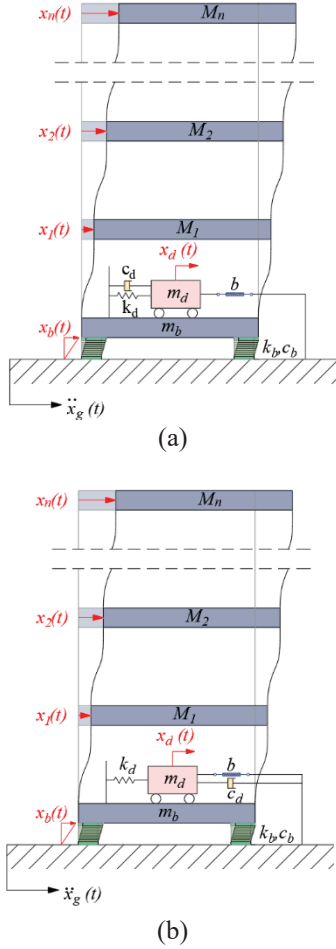


Figure 2. Base isolated structures: a) MDOF base-isolated TMDI-controlled shear-type frame; b) MDOF base-isolated New TMDI-controlled shear-type frame.

The equations of motion which govern the behaviour of a base isolated system controlled by the traditional TMDI and of a base isolated system controlled by the New TMDI are given in Equation 2 and Equation 3, respectively,

$$\begin{cases} (M_{tot} + m_d + b)\ddot{x}_b(t) + \sum_{i=1}^n M_i \ddot{x}_i(t) + (m_d + b)\ddot{x}_d(t) + \\ + C_b \dot{x}_b(t) + K_b x_b(t) = -(M_{tot} + m_d)\ddot{x}_g(t) \\ (m_d + b)(\ddot{x}_b(t) + \ddot{x}_d(t)) + c_d \dot{x}_d(t) + k_d x_d(t) = -m_d \ddot{x}_g(t) \\ M_i \ddot{x}_b(t) + M_i \ddot{x}_i(t) + \sum_{j=1}^n C_{ij} \dot{x}_j(t) + \sum_{j=1}^n K_{ij} x_j(t) = -M_i \ddot{x}_g(t) \end{cases} \quad (2)$$

and

$$\begin{cases} M_{tot} \ddot{x}_b(t) + \sum_{i=1}^n M_i \ddot{x}_i(t) + C_b \dot{x}_b(t) + K_b x_b(t) - k_d x_d(t) = -M_{tot} \ddot{x}_g(t) \\ (m_d + b)(\ddot{x}_b(t) + \ddot{x}_d(t)) + c_d \dot{x}_d(t) + c_d \dot{x}_b(t) + k_d x_d(t) = -m_d \ddot{x}_g(t) \\ M_i \ddot{x}_b(t) + M_i \ddot{x}_i(t) + \sum_{j=1}^n C_{ij} \dot{x}_j(t) + \sum_{j=1}^n K_{ij} x_j(t) = -M_i \ddot{x}_g(t) \end{cases} \quad (3)$$

where  $x_d(t)$  is the displacement of the considered device relative to the base. Note that if the parameter  $b$  is set equal to zero in Equation 2 and Equation 3, the conventional cases of the BI+TMD and BI+New TMD are obtained, respectively.

Clearly, Equation 1 and Equation 3 greatly simplify if an SDOF main structure is considered. In this regard, Equation 1 for the BI structure can be rewritten as

$$\begin{cases} \ddot{x}_b(t) + \mu_b \ddot{x}_1(t) + 2\omega_b \zeta_b \dot{x}_b(t) + \omega_b^2 x_b(t) = -\ddot{x}_g(t) \\ \ddot{x}_b(t) + \ddot{x}_1(t) + 2\omega_1 \zeta_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = -\ddot{x}_g(t) \end{cases} \quad (4)$$

where  $\mu_b = M_1/M_{tot}$ ,  $\omega_b = (k_b/M_{tot})^{1/2}$ , and  $\zeta_b = C_b/(2\omega_b M_{tot})$  are the mass ratio, natural frequency and damping ratio of the base isolation sub-system, respectively. Further,  $\omega_1 = (K_1/M_1)^{1/2}$  and  $\zeta_1 = C_1/(2\omega_1 M_1)$  are the natural frequency and damping ratio of the SDOF main structure. Analogously, focusing on the BI+New TMDI system with a SDOF main structure (Figure 3 (a)), Equation 3 is particularized as

$$\begin{cases} \ddot{x}_b(t) + \mu_b \ddot{x}_1(t) + 2\zeta_b \omega_b \dot{x}_b(t) + \omega_b^2 x_b(t) - \omega_d^2 (\mu_d + \beta)x_d(t) = -\ddot{x}_g(t) \\ \ddot{x}_b(t) + \ddot{x}_d(t) + 2\zeta_d \omega_d (\dot{x}_d(t) + \dot{x}_b(t)) + \omega_d^2 x_d(t) = -\frac{\mu_d}{(\mu_d + \beta)} \ddot{x}_g(t) \\ \ddot{x}_b(t) + \ddot{x}_1(t) + 2\zeta_1 \omega_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = -\ddot{x}_g(t) \end{cases} \quad (5)$$

where  $\beta = b/M_{tot}$  is the inertance ratio,  $\mu_d = m_d/M_{tot}$  the New TMDI mass ratio, while  $\omega_d = [k_d/(m_d + b)]^{1/2}$  and  $\zeta_d = c_d/[2\omega_d(m_d + b)]$  are the natural frequency and damping ratio of the New TMDI, respectively.

Although to derive the so far mentioned equation, a specific type of inerter has not been considered, from a practical standpoint, the current trend emerging from the literature suggests the use of ball screw inerters, which might be arranged on a par with dampers connected to the base isolation system [21]. It is worth stressing some technical issues could arise since stand-alone inerter devices, due to geometric limitations and alignment requirements, would be very difficult to design. To tackle this kind of drawback, a configuration based on the use of V braces as extenders or frames [22], to transfer the displacement and acceleration of the TMD mass to the end terminal of the inerter, both allocated in the basement, might represent a possible solution.

### 3 OPTIMAL DESIGN PARAMETERS OF NEW TMDI SYSTEM

As can be seen in Equation 5, the New TMDI is characterized by the following parameters: the inertance ratio  $\beta$ , the mass

ratio  $\mu_d$ , the natural frequency  $\omega_d$ , and the damping ratio  $\zeta_d$ . Clearly, the achievement of the best TMDI system performances in controlling base-isolation displacements would require these four parameters to be conveniently chosen.

However, taking into account that  $\beta$  and  $\mu_d$  are usually determined on the basis of structural constraints, only  $\zeta_d$  and  $\omega_d$ , or better to say the frequency ratio  $\nu = \omega_d/\omega_b$ , are required to be appropriately determined through an optimization procedure.

In this regard, since the system is assumed to be subjected to a zero-mean stationary Gaussian white noise process due to the intrinsic random nature of the seismic excitation. The white noise process is characterized by the one-sided Power Spectral Density (PSD)  $G_0$  and this allows the optimal parameters to be found by minimizing the steady-state displacement response variance of the base isolation sub-system  $\sigma_{x_b}^2$  expressed as

$$\sigma_{x_b}^2 = \int_0^{\infty} |H_b(\omega)|^2 G_0 d\omega \quad (6)$$

where  $H_b(\omega) = X_b(\omega)/\ddot{X}_g(\omega)$  denotes the base isolation sub-system displacement transfer function. Specifically, Fourier transforming the system in Equation 5, followed by some algebra, yields

$$H_b(\omega) = \frac{1 + \frac{\omega_d^2 \mu_d}{c(\omega)} + \frac{\omega^2 \mu_b}{a(\omega)}}{-b(\omega) - \frac{\omega_d^2 (\mu_d + \beta)(-\omega^2 + 2i\omega\zeta_d\omega_d)}{c(\omega)} + \frac{\omega^4 \mu_b}{a(\omega)}} \quad (7)$$

where

$$\begin{aligned} a(\omega) &= -\omega^2 + 2i\omega\zeta_1\omega_1 + \omega_1^2 \\ b(\omega) &= -\omega^2 + 2i\omega\zeta_b\omega_b + \omega_b^2 \\ c(\omega) &= -\omega^2 + 2i\omega\zeta_d\omega_d + \omega_d^2 \end{aligned} \quad (8)$$

Considering the complexity of solving the integral involved in Equation 6, the evaluation of the optimal parameters ( $\nu$ ,  $\zeta_d$ ) would require a rather cumbersome numerical optimization procedure, which should be implemented for each specific values of the input parameters to minimize the response variance  $\sigma_{x_b}^2$ .

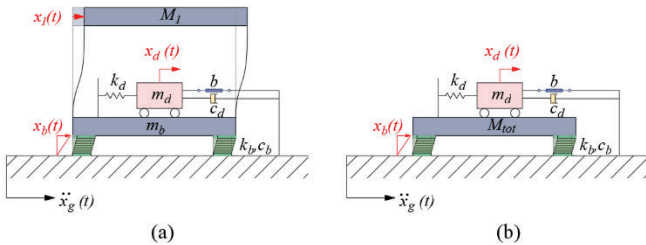


Figure 3. Simplified models: a) A SDOF base-isolated New TMDI-controlled shear-type frame; b) Base isolated rigid structure equipped with New TMDI.

On the other hand, further simplification can be achieved considering that in base-isolated buildings the main structure

essentially behaves like a single rigid body [18] [23], and the base isolation sub-system displacements can be order of magnitude higher than those of the main structure. Thus, it is feasible to simplify the system in Equation 5, assuming that the entire base isolated structure can be modelled as an SDOF system, as shown in Figure 3. Hence, the original system in Equation 5 can be recast as

$$\begin{cases} \ddot{X}_b(t) + \ddot{X}_d(t) + 2\zeta_b\omega_b\dot{X}_b(t) + \omega_b^2 X_b(t) - \omega_d^2(\mu_d + \beta)X_d(t) = -\ddot{X}_g(t) \\ \ddot{X}_b(t) + \ddot{X}_d(t) + 2\zeta_d\omega_d(\dot{X}_d(t) + \dot{X}_b(t)) + \omega_d^2 X_d(t) = -\frac{\mu_d}{\mu_d + \beta}\ddot{X}_g(t) \end{cases} \quad (9)$$

where capital letters are used since  $\ddot{X}_g(t)$  is a white noise process and hence the structural responses are also stochastic processes. Following the approach in [19] and [20], the base isolation sub-system response displacement variance of the simplified system in Equation 9 can be given in the form

$$\sigma_{x_b}^2 = \frac{\pi G_0}{4\omega_b^3 z_{x_b}} \quad (10)$$

where  $z_{x_b}$  is a function of both the design parameters  $\nu$  and  $\zeta_d$ , and can be defined as  $z_{x_b} = N_Z/D_Z$ , with

$$\begin{aligned} N_Z &= 4\zeta_b^3\zeta_d^2\nu^2 + \zeta_d^2(\beta + \mu_d)\nu^5 + \zeta_b\zeta_d\{2(-1 + \beta + 2\zeta_d^2 + \mu_d)\nu^2 + \\ &+ [1 + (\beta + \mu_d)(\beta + 4\zeta_d^2 + \mu_d)]\nu^4 + 1\} + \zeta_b^2\{(\beta + \mu_d)\nu^3 + \\ &+ 4\zeta_d^2[\nu + (1 + \beta + \mu_d)\nu^3]\} \end{aligned} \quad (11)$$

and

$$\begin{aligned} D_Z &= \zeta_d + \zeta_b(4\zeta_d^2 + \mu_d^2)\nu + \zeta_d(-2 + \beta + 4\zeta_d^2 - \mu_d + 4\zeta_b^2(1 + \mu_d)^2)\nu^2 + \\ &+ \zeta_b(1 + \mu_d)^2(\beta + 4\zeta_d^2 + \mu_d)\nu^3 + \zeta_d(1 + \mu_d)^2\nu^4 \end{aligned} \quad (12)$$

In this regard, taking into account Equation (11) and Equation (12), the optimal values of  $\nu$  and  $\zeta_d$  that minimize  $\sigma_{x_b}^2$  can be equivalently obtained from the minimum of the function  $\phi(\zeta_d, \nu) = 1/z_{x_b}$  which does not depend on  $G_0$  and on the natural frequency of the base isolation sub-system  $\omega_b$ .

Therefore, an explicit expression for the minimum of  $\phi(\zeta_d, \nu)$  could be determined considering that the derivatives of this function with respect to the design parameters must be equal to zero.

However, a more wieldy procedure consists in minimizing  $\phi(\zeta_d, \nu)$  by using well-known numerical minimization algorithms already implemented in most commercial software (for instance FindMinimum in Mathematica or fminsearch in MATLAB environment) allows to achieve these computationally complex expressions. In this manner, the optimal design parameter values  $\nu_{opt}$  and  $\zeta_{d,opt}$  can be directly found. As can be seen in Equation 11 and Equation 12, the function  $z_{x_b}$  depends also on the damping ratio of the base

isolation sub-system  $\zeta_b$ . Hence, the proposed optimization procedure can take into account also the effect of this parameter, and the corresponding optimal values  $\nu_{opt}$  and  $\zeta_{d,opt}$  must be found numerically.

However, approximate solutions of the optimal parameters can be achieved by considering  $\zeta_b$  tending to zero, as it usually happens in optimization procedures for passive vibration control systems [24]. Moreover, as suggested in [18], further simplification can be obtained assuming that for base-isolated structures the stiffness of the attached New TMDI could be set approximately equal to the stiffness of base isolation bearings; thus, the approximate optimal frequency ratio can be given as

$$\tilde{\nu}_{opt} = (\mu_d + \beta)^{-\frac{1}{2}} \quad (13)$$

Therefore, using the just proposed equation, the minimum of the function  $\phi(\zeta_d, \nu)$  yields an analytical approximate optimal solution of the optimal damping ratio of the New TMDI

$$\tilde{\zeta}_{d,opt} = \frac{\sqrt{1 - 2\beta + 2\beta^2 + 2\beta\mu_d + \mu_d^2}}{2\sqrt{\beta + \mu_d}} \quad (14)$$

Clearly, in this manner optimal design parameters can be easily evaluated.

#### 4 ANALYSIS OF THE CONTROL PERFORMANCE

The previous optimal design procedure uses, for sake of simplicity, a stationary white noise as base acceleration. Therefore, to further assess the validity of the proposed analytical solution for the optimal design parameters, in this section, the control performances of the BI system equipped with the New TMDI is investigated in the time domain using a set of 44 real recorded ground motions with different features taken from the FEMA P-695-FF set described in [25].

In order to properly design the New TMDI device to be connected to the base-isolated structure, the one-sided PSD  $G_{\ddot{x}_g}(\omega)$  of the input has been evaluated at  $\omega = \omega_b$ , thus determining the corresponding value  $G_0 = G_{\ddot{x}_g}(\omega_b) = 0.002$ . The numerical simulation takes into account the base-isolated five-storey planar frame building ( $n=5$ ) analysed in [26]. Specifically, the mass of each floor of the main structure is equal to  $M_i = 3.5 \cdot 10^3$  kg ( $i=1, \dots, 4$ ), the storey stiffness is  $K_i = 35 \cdot 10^6$  N/m, and the dashpot damping coefficient is  $C_i = 35 \cdot 10^3$  Ns/m. The natural frequencies of the structure are  $\omega_i$  [rad/s] = [28.46, 83.08, 130.97, 168.25, 191.90], and the system is assumed to be a classically damped structure with damping ratio of each mode  $\zeta_i = 0.0142$ . The base isolation sub-system has a mass  $m_b = 3.5 \cdot 10^3$  kg, a natural frequency  $\omega_b = \pi$  rad/s and a damping ratio  $\zeta_b = 0.02$ . Finally, assuming a mass ratio  $\mu_d = 5\%$  and inertance ratio  $\beta = 0.3$ , the optimal design parameters of the New TMDI device  $\tilde{\nu}_{opt} = 1.69$  and  $\tilde{\zeta}_{d,opt} = 0.66$  have been determined using the aforementioned optimization procedure.

In this way, the evaluation of the equations of motion (Equation 1 and Equation 2) can be retrieved by direct

numerical integration, in order to find out the response of the base isolated structure, with and without New TMDI, subjected to the FEMA P-695-FF 44 records. Specifically, for each of the FEMA P-695-FF 44 records, the displacement relative to the ground has been determined for both the simple base-isolated structure and the base-isolated structure equipped with the New TMDI. In this regard, comparison of the corresponding profiles of the median, 16th and 84th percentiles are shown in Figure 4 for the base-isolated structure with (red lines) and without (green lines) New TMDI. A similar comparison is also reported in Figure 5 in terms of total accelerations and inter-storey drift ratios, considering in these cases also the structural response of the uncontrolled system (UC) fixed at the base (grey lines).

As can be seen, although the optimized New TMDI device is able to effectively reduce the relative displacement demand (Figure 4), this reduction may lead to slightly higher inter-storey drift ratios and total peak accelerations compared to the base-isolated structure without device (Figure 5). This effect is probably caused by the employed optimization criterion that is related only to the minimization of the base displacement variance. Further, note that this behaviour is similar to those arising when highly-damped isolators are used [7] [27].

Nevertheless, as can be seen in Figure 5, the benefits in terms of total accelerations and inter-storey drift ratios of the BI with New TMDI system clearly are proven by the reduction with respect to the corresponding ones of the UC system fixed at the base. Thus, classical beneficial features of base isolation are still kept when employing the New TMDI.

In this respect, Figure 6 shows the comparison between the response time histories of the base-isolated benchmark structure with New TMDI (red dash-dot line), with traditional TMDI (black solid line) and without any devices (green dashed line), considering two records of the FEMA P-695-FF 44 with different characteristics, namely the Imperial Valley and the Duzce earthquakes. As far as the TMDI is concerned, its optimal parameters have been obtained considering the same mass ratio and inertance as the New TMDI and the procedure described in [19]. In particular, the optimal design parameters of the TMDI device are  $\tilde{\nu}_{opt} = 0.798$  and  $\tilde{\zeta}_{d,opt} = 0.258$ .

As can be seen in Figure 6, both devices are effective in controlling the base isolation displacement. However the proposed device achieves the better performance, compared with the TMDI one, for both the Imperial Valley and the Duzce earthquakes. Despite this, both systems show the characteristic feature of passive control devices which generally have minimal effects in the first few seconds of the excitation [28]. On the other hand, as previously mentioned, lower control performances are achieved in terms of storey drift of the last floor. In this regard, a comparison including also the uncontrolled (UC) system (grey dotted lines) is represented in Figure 7, for both Imperial Valley record (Figure 7 (a)), and the Duzce ground motion (Figure 7 (b)). Again, as expected on the basis of the results shown in Figure 5, although the New TMDI slightly alters the benefits of the simple BI system in terms of storey drifts, pertinent response is still much lower than the uncontrolled system (grey dotted line).

Finally, as can be seen in Figure 8, the New TMDI design yields smaller displacements of the mass compared to the

traditional TMDI for the two considered inputs. Notably, this aspect can be particularly advantageous in practical cases where the space designed to host the device is limited.

On this base, it can be argued that, the analytical solution of Equation 13 and Equation 14 leads to optimal design parameters that work satisfactorily also for real earthquake records despite they have been retrieved assuming a white noise base excitation.

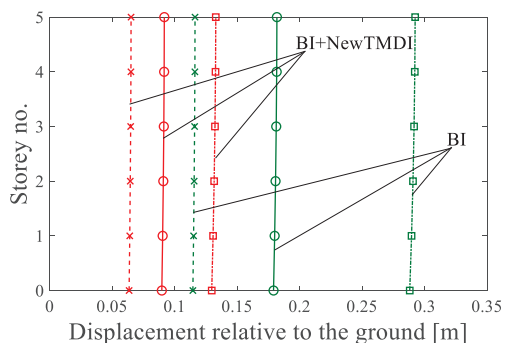


Figure 4. Response profiles of the peak floor displacement relative to the ground for the hybrid-controlled structure (BI with New TMDI – red lines) and base-isolated structure (BI – green lines) subjected to the 44 FEMA P-695-FF records: circles - median; crosses - 16th percentile; squares - 84th percentiles.

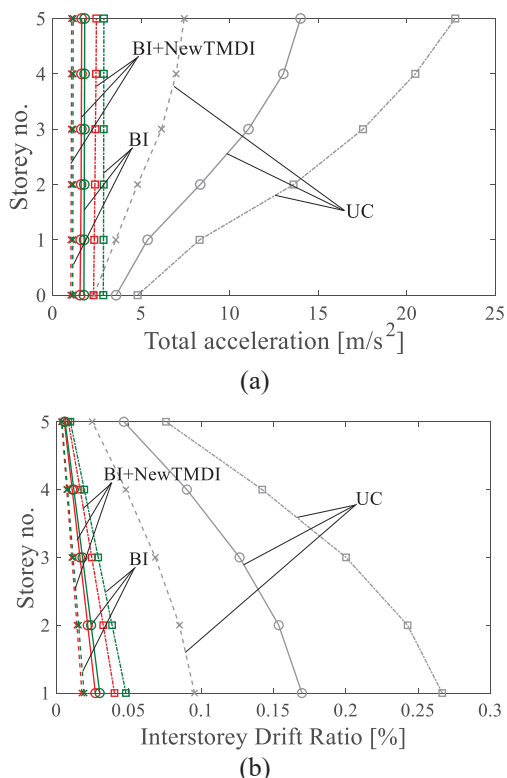


Figure 5. Response profiles for the uncontrolled structure (UC – grey lines), hybrid-controlled structure (BI with New TMDI – red lines) and base-isolated structure (BI – green lines) subjected to the 44 FEMA P-695-FF records: a) peak floor total acceleration; b) peak floor inter-storey drift ratio. Circles - median; crosses - 16th percentile; squares - 84th percentiles.

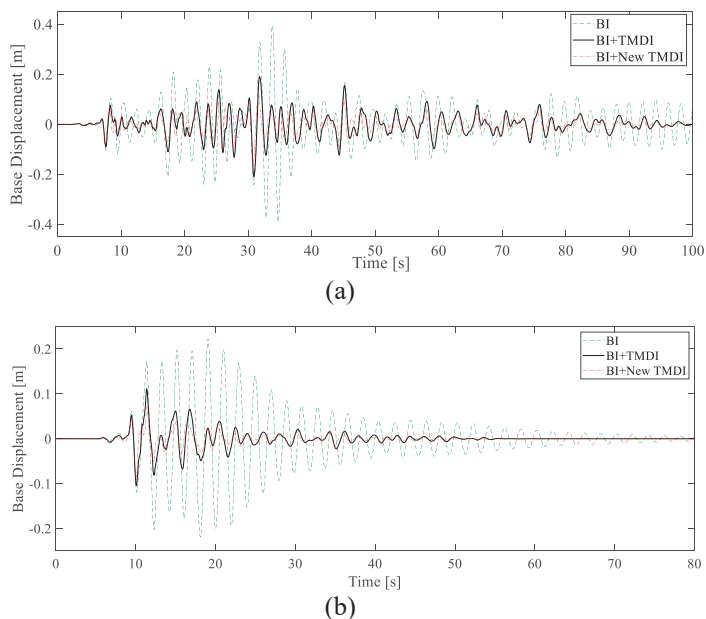


Figure 6. Base isolation displacements relative to the ground: a) Response to the Imperial Valley earthquake; b) Response to the Duzce earthquake. BI with New TMDI - red dash-dot lines, BI with TMDI - black solid lines; BI without devices - green dashed lines.

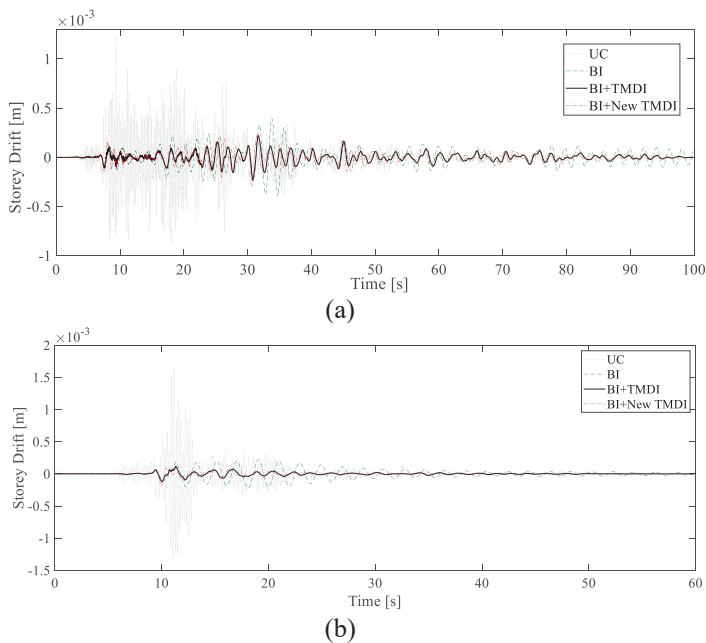


Figure 7. Storey drift: a) Response to the Imperial Valley earthquake; b) Response to the Duzce earthquake; BI with New TMDI - red dash-dot lines, BI with TMDI - black solid lines; BI without devices - green dashed lines; Uncontrolled structure – grey dotted lines.

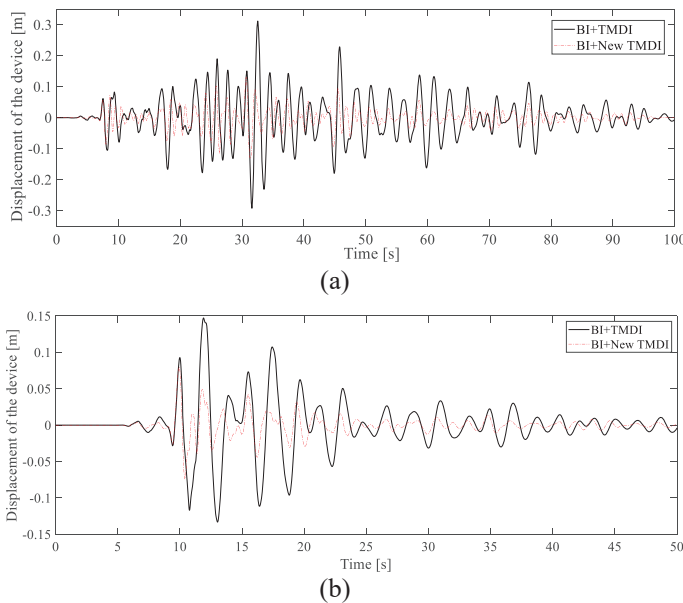


Figure 8. Displacement of the device relative to the base: a) Response to the Imperial Valley earthquake; b) Response to the Duzce earthquake; BI with New TMDI - red dash-dot line, BI with TMDI - black solid line.

## 5 CONCLUDING REMARKS

In this paper, a smart control vibration strategy has been proposed consisting of a non-traditional layout of tuned mass damper inerter, referred to as New TMDI, for base isolated systems to further reduce base displacements with respect to the classical design. This novel design includes also a damping element, the inerter, which is directly connected to the ground.

Optimal parameters of the proposed system have been determined by means of an optimization procedure based on the minimization of the base displacement variance of the base isolation sub-system. Further, corresponding approximate closed-form expressions have been achieved based on some assumptions related to the base excitation and isolation damping.

A numerical investigation has been carried out using a set of real earthquakes records with different characteristics. From these analytical analysis emerged that New TMDI can effectively further reduce displacements of the base isolation sub-system compared to the system controlled by a classical TMDI with the same mass ratio. Although this reduction leads to a minor increase of storey drifts and total accelerations of the main structure, these are in any case significantly lower than those of the uncontrolled structure fixed at the base.

On the base of the preceding observations, considering the excellent outcomes in reducing base-isolation degree of freedom displacement, it can be argued that the proposed New TMDI represents a smart strategy not only for new constructions but also for existing buildings, with a focus on historic or monumental one, for which conventional seismic retrofit approaches often cannot be used.

## ACKNOWLEDGMENTS

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 847476. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

## REFERENCES

- [1] J. Kelly, *Base isolation: linear theory and design*, J. Earthq. Spectra 6, p. 223–244, 1990.
- [2] P. Tsopelas, M. Constantinou, Y. Kim and S. Okamoto, *Experimental study of FPS system in bridge seismic isolation*, Earthq Eng Struct Dyn, no. 25(1), p. 65–78, 1996.
- [3] P. Castaldo, B. Palazzo and P. Della Vecchia, *Seismic reliability of base-isolated structures with friction pendulum bearings*, Eng Struct, no. 95, pp. 80–93, 2015.
- [4] A. Amendola, G. Carpentieri, L. Feo and F. Fraternali, *Bending dominated response of layered mechanical metamaterials alternating pentamode lattices and confinement plates*, Compos. Struct., no. 157, p. 71–77, 2016.
- [5] A. Chopra, *Dynamics of Structures*, Fourth Edition ed., University of California at Berkeley: Pearson Education, 2012.
- [6] R. Jangid and A. Vasant, *Base Isolation for Seismic Retrofitting of Structures*, Practice Periodical on Structural Design and Construction, pp. 175–185, 2008.
- [7] J. Kelly, *The role of damping in seismic isolation*, Earthq. Eng. Struct. Dyn., no. 28, p. 3–20, 1999.
- [8] C. Bilello, M. Di Paola and A. Pirrotta, *Time Delay Induced Effects on Control of Non-linear Systems under Random Excitation*, Meccanica, no. 37, p. 207–220, 2002.
- [9] H. Tsai, *The effect of tuned-mass damper on the seismic response of base-isolated structures*, Int. J. Solids Struct., no. 32, p. 1195–1210, 1995.
- [10] J. Yang, A. Danielians and S. Liu, *Aseismic hybrid control systems for building structures*, J. Eng. Mech, no. 117(4), p. 836–853, 1991.
- [11] B. Palazzo and L. Betti, *Combined control strategies: base isolation and tuned mass damping*, ISET J. Earthq. Technol., no. 36, p. 121–137, 1999.
- [12] B. Palazzo and L. Betti, *Aspects of passive control of structural vibrations*, Meccanica, no. 32, p. 529–544, 1997.
- [13] Y. Arfiadi and N. H. Muhammad, *Hybrid base isolation-passive mass damper systems*, Computing in Civil and Building Engineering, p. 279–286, 2000.
- [14] M. Smith, *Synthesis of mechanical networks: the inerter*, IEEE Trans on Automatic Control, no. 47(10), p. 1648–1662, 2002.
- [15] C. Papageorgiou, N. Houghton and M. Smith, *Experimental testing and analysis of inerter devices*, Dyn. Syst. Meas. Control., no. 131, p. 11001, 2009.
- [16] D. De Domenico, P. Deastra, G. Ricciardi, N. Sims and D. Wagg, *Novel fluid inerter based tuned mass dampers for optimised structural control of baseisolated buildings*, J. Franklin Inst., 2018.
- [17] X. Liu, J. Jiang, B. Titurus and A. Harrison, “Model identification methodology for fluid-based inerters,” *Mech. Syst. Signal. Proc.*, no. 106, p. 479–494, 2018.
- [18] P. Xiang and A. Nishitani, *Optimum design for more effective tuned mass damper system and its application to base-isolated buildings*, Struct. Control Health Monit., no. 21, p. 98–114, 2014.
- [19] A. Di Matteo, C. Masnata and A. Pirrotta, *Simplified analytical solution for the optimal design of Tuned Mass Damper Inerter for base isolated structures*, Mech. Syst. Signal Process., no. 134C, p. 106337, 2019.
- [20] A. Di Matteo, T. Furtmüller, C. Adam and A. Pirrotta, *Optimal design of tuned liquid column dampers for seismic response control of base-isolated structures*, Acta Mech., no. 229, p. 437–454, 2018.
- [21] F. Qian, Y. Luo, H. Sun, W. Tai and L. Zuo, *Optimal tuned inerter dampers for performance enhancement of vibration isolation*, Engineering Structures, no. 198, 2019.

- [22] R. M. Hessabi and O. Mercan, *Investigations of the application of gyro-mass dampers with various types of supplemental dampers for vibration control of building structures*, Engineering Structures, no. 126, pp. 174-186, 2016.
- [23] J. Love, M. Tait and H. Toopchi-Nezhad, *A hybrid structural control system using a tuned liquid damper to reduce the wind induced motion of a base isolated structure*, Engineering Structures, no. 33, pp. 738-746, 2011.
- [24] L. Marian and A. Giaralis, *Optimal design of a novel tuned mass-damper-inerter (TMDI) passive vibration control configuration for stochastically support-excited structural systems*, Probabilistic Engineering Mechanics, no. 38, pp. 156-164, 2014.
- [25] FEMA P-695: *Quantification of Building Seismic Performance Factors*, Washington, D.C.: Technical Representative, Federal Emergency Agency, 2009.
- [26] C. Adam, A. Di Matteo, T. Furtmuller and A. Pirrotta, *Earthquake Excited Base-Isolated Structures Protected by Tuned Liquid Column Dampers: Design Approach and Experimental Verification*, Procedia Engineering, no. 199, p. 1574-1579, 2017.
- [27] T. Taniguchi, A. Der Kiureghian and M. Melkumyan, *Effect of tuned mass damper on displacement demand of base-isolated structures*, Engineering Structures, no. 30, pp. 3478-3488, 2008.
- [28] L. Chuan, M. Liang and Y. Wang, *Vibration suppression using two terminal flywheel*, Journal of Vibration and Control, no. 18, pp. 1096-1105, 2011.