Service Shutdowns and Compensation: Cash Refunds or Vouchers?

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Abstract

Government mandated shutdowns in response to the COVID-19 pandemic have led to a vast number of service cancellations and a heated debate on compensation policies. Service providers pushed for vouchers as a means of compensation, whereas customers demand cash refunds or generous vouchers that compensate for postponed service. Regulators, at the same time, insist that customers must be granted the right to be reimbursed in money. To address the debate, this paper develops an analytical framework to study the profitability and efficiency of different compensation policies following shutdown. Surprisingly, we find that the voucher-only policy dominates the hybrid policy that offers customers the choice between a cash-back and a voucher option in terms of profit and welfare if customers have high salvage values for the service. We also find that if the regulator imposes cash-back as an option for compensation, it is optimal for the provider to include a bonus in the voucher. In contrast, the provider will not offer a bonus in the absence of a cash-back option. Overall, our analysis helps to evaluate compensation policies from the perspectives of pre-shutdown efficiency, post-shutdown customer dissatisfaction, and possible compromise between market participants.

Keywords: Voucher, Service Shutdown, Service Cancellation, Compensation Policy.

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1 Introduction

COVID-19 has resulted in government mandated shutdowns of a wide array of services, including international and domestic travel, hospitality, and entertainment. Shutdowns not only left customers on their own devices to get cash refunds or vouchers for cancelled services, but also confronted service providers with existential threats. Under the new normal where uncertainty can reach extreme levels, traditional business models are no longer adequate (McKinsey 2020). At the same time, policy makers are pressured to amend the regulatory framework to best protect the interests of providers and consumers.

In the airline industry, for example, carriers need to figure out how to price their services in the face of shutdown uncertainty and how to compensate customers following a shutdown. Whereas a prompt cash refund to passengers is the default in the United States (U.S. Department of Transportation 2020), European legislation allows providers to offer passengers the choice between a cash refund and a voucher (European Commission 2020).[1] To avoid cash drain, some airlines have been asking to quickly amend the regulations and make vouchers the norm for compensation under shutdown (Passy 2020; Forbes 2021). Others are making vouchers more generous by including a bonus to motivate passengers to give up the cash refund and accept a voucher instead (Forbes 2020). Table 1 gives a snapshot of such bonuses offered by carriers during the COVID-19 pandemic. Consumer organizations, on the other hand, insist on maintaining the current regulations, but welcome the idea of making vouchers more attractive in the event of a shutdown.

This paper presents an analytical framework that helps providers and policy makers address optimal service design under government mandated shutdowns. In particular, we analyze a provider facing heterogeneous customers who differ in their salvage value for a postponed service delivered in the "aftermarket" following a shutdown. The framework is used to address several important research questions emerging from the ongoing cash-back vs. voucher debate: Is it profitable and economically efficient to offer vouchers instead of or along with cash refunds? If yes, how generous should vouchers be? Should the

[1] In practice, providers often try to reimburse customers with a voucher only, which is possible if customers accept the voucher without explicitly asking for a cash refund. Such compensation is consistent with the EU regulations.

Carrier	Bonus
Frontier Airlines, Lufthansa, Singapore Airlines	50 USD/EUR
Aer Lingus, Qatar Airways, TAP Air Portugal	10%
Turkish Airlines	15%
El Al Israel Airlines	25%

Table 1: Bonus offered on top of the purchase price to incentivize the choice of a voucher instead of a cash refund during the COVID-19 pandemic.

regulator insist on cash-back as an option after shutdown? Will customers and society at large be better off with more generous vouchers? And finally, should providers prohibit customers to resell their vouchers in a secondary market?

Traditionally, the notion of voucher has been used in different settings. Sometimes, a voucher refers to a certificate that entitles the customer to receive a replacement service in case of a service failure (e.g., a canceled flight). Other times, it refers to a compensation for the inconvenience that results from service failure (e.g., meal vouchers issued in case of flight delays). In our model, we define a voucher as a bundle that includes a service replacement and a possible bonus under shutdown. In the absence of a bonus, the provider simply offers a *replacement voucher*, which has traditionally been used in practice.

Considering cash-back and voucher options, the provider can choose from four different policies, as shown in Figure 1. Under the *selling as-is policy*, customers receive neither a cash refund nor a voucher following a service shutdown, whereas they are fully refunded under the *cash-back policy*. Alternatively, the provider offers a service replacement and possibly a bonus under the *voucher policy*. Finally, the *hybrid policy* allows customers to choose between the cash-back and voucher options following a shutdown.

We derive three key results from our analysis. First, we show that the cash-back policy leads to the same outcome for the provider, customers, and society at large as the selling as-is policy, our benchmark case. This could come as a surprise to customers who strongly prefer cash-back. We also show that both the cash-back and selling as-is policies lead to

		Voucher	
		No	Yes
Cash-Back	No	**Selling As-Is** (Benchmark)	**Voucher Only** (Favored by Providers)
	Yes	**Cash-Back Only** (Favored by Customers)	**Hybrid: Cash-Back or Voucher** (Considered by Policy Makers)

Notes: Customers favor cash-back following shutdown because it gives them maximum flexibility, and welcome attractive vouchers (e.g., with a bonus). Providers strongly prefer vouchers to prevent cash drain, but customers with low salvage values find vouchers unacceptable. Policy makers insist on cash-back but allow providers to offer vouchers instead if customers agree.

Figure 1: Cash-back vs. voucher options in compensation policies.

a lower profit and welfare compared to the hybrid policy, except if the average salvage value is low relative to the cost of providing the service. Intuitively, a voucher allows customers to use the service replacement, which is economically efficient if the average salvage value is relatively high. In addition, including a bonus gives the provider more flexibility in pricing, which helps extract a higher customer surplus when the service is delivered without disruption.

Second, we show that if the regulator does not require the provider to offer cash-back as an option, the optimal voucher policy simply consists of a service replacement with a zero bonus. The reason is that a bonus increases not only the effective cost of delivering the replacement service but also the number of customers in the aftermarket. Interestingly, under some conditions, the voucher policy dominates all other policies, including the hybrid policy, in terms of both profit and social welfare. Intuitively, the cash-back option of the hybrid policy motivates some customers to give up the replacement service, which is economically inefficient if the customers' average salvage value is high relative to the marginal cost of providing the service. Instead, if the average salvage value is low, the voucher policy is less profitable and less efficient than the benchmark policy. Importantly, if the regulator imposes cash-back as an option (e.g., as mandated by European Commission), this could have the unintended consequence of hurting profit and

Policy	Key Results
Cash-Back Only	Cash-back leads to the same profit and welfare as selling as-is. Both cash-back and selling as-is are less profitable than the hybrid policy and economically inefficient, except if the average salvage value is low compared to the cost of providing the service.
Voucher Only	Offering no bonus is optimal for the provider. Voucher only can be the best policy (dominates all other policies, including the hybrid) or the worst policy in terms of profit and welfare. If the average salvage value is relatively high (but not too high) compared to the cost of providing the service, voucher only improves profit over the hybrid policy but results in a lower welfare.
Hybrid	Offering a bonus is optimal for the provider but can reduce customer surplus. If the average salvage value is relatively low (but not too low) compared to the cost of providing the service, the hybrid policy dominates all other policies in terms of profit and social welfare.

Table 2: Key insights for providers and policy makers.

customer surplus. The policy implication therefore is that imposing a cash-back option is not necessarily optimal from a societal point of view.

Third, if the regulator does require the provider to offer cash-back as an option, the provider faces the choice of simply offering a replacement voucher, or including a bonus to make the voucher more attractive. Intuitively, providers would include zero bonus in the voucher to save cost. However, we show that the provider *always* obtains a higher profit by offering a positive bonus under the hybrid policy. During the COVID-19 pandemic, consumer organizations have called for making vouchers more attractive. Surprisingly, we find that offering a positive bonus along with a service replacement always benefits the provider but can reduce the customer surplus. Table 2 provides an overview of the key findings of our analysis.

Furthermore, in designing the voucher, the provider can decide whether the voucher is non-transferable so that only the customer can use it in case of a shutdown, or transferable so that the voucher can be resold in a secondary market. If cash-back is an option, then a secondary market is unlikely to be viable unless the reselling price is significantly higher than the purchase price. However, in the absence of a cash-back option, customers who realize low salvage values will be interested in reselling their vouchers if possible.[2] We show that the provider should facilitate a reselling market under the voucher policy, because transferable vouchers improve profit and social welfare.

Our study contributes to the literature on service failures and service recovery in several ways. This is the first paper that combines both cash refunds and service replacement in the design of pricing and compensation policies under government mandated shutdown. Prior literature has studied service recovery under at-fault failures and no-fault failures. At-fault service failures occur under poor operations (Parasuraman, Zeithaml and Berry 1985; Zeithaml, Parasuraman and Berry 1990) or strategic quality positioning (Rust and Zahorik 1993; Rust, Zahorik and Keiningham 1995; Rust and Huang 2012; Halbheer et al. 2018; Dukes and Zhu 2019). In contrast, no-fault service failures result from external forces such as accidents, bad weather, or power outages, which lead to monetary compensations as a way for service recovery (Chen, Gerstner and Yang 2009, 2012). Our paper also studies no-fault failures, but differs by focusing on the comparison of alternative compensation policies under shutdown. Whereas it is intuitive that forcing a cash-back option puts stress on the provider's cash flow management, we show that, even in the absence of such concerns, imposing a cash-back option could reduce profit and social welfare.

Second, we formalize the notion of a "bonus voucher," whereby a provider offers a service replacement along with a bonus to motivate customers to opt for the voucher instead of a cash refund. In the absence of a bonus, replacement vouchers are similar to rain-checks offered in retailing when a price-promoted product is out of stock (Hess and Gerstner 1987). Replacement vouchers are also similar to product warranties offered by manufacturers to repair or replace defective products (Cooper and Ross 1985; Mann and Wissink 1990; Halstead, Dröge and Cooper 1993). Product replacement or repairs are

[2]For instance, sites such as `www.roomertravel.com` help buyers salvage non-refundable reservations.

triggered by product defects. In contrast, vouchers in our study are triggered by no-fault service failures. To make service replacement more attractive to customers, we allow for the possibility that the provider can strategically include a bonus in the voucher, which is rarely considered for product replacement or repair. In addition, replacement vouchers share similarities with product recalls. In contrast to rain-check and warranty policies, such recalls affect all customers who bought the product in a certain period of time (Chen, Ganesan and Liu 2009). Whereas under product recalls customers have no choice but to repair or replace the product, under service shutdowns customers sometimes can choose between cash-back and vouchers.

Finally, our analysis identifies different policy regions that provide guidance on how to deal with uncertainty regarding no-fault service outage, which could be the new normal for the years to come. The cash-back policy is similar to money-back guarantees (MBGs) under which dissatisfied customers are given an option to return products for a full refund (Heiman, McWilliams and Zilberman 2001; Heiman et al. 2002; McWilliams 2012). Cash-back and vouchers can also be viewed as service guarantees (Hogreve and Gremler 2009). Such guarantees are problematic under normal service operations (Wirtz and Kum 2004), but they become indispensable in the face of the ongoing pandemic that is fundamentally changing the nature of service operations.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 derives and compares compensation policies, both from the perspective of the provider and society more broadly. Section 4 examines the implications of a reselling market. Finally, Section 5 concludes, discusses policy implications, and offers directions for future research.

2 The Model

We consider a provider who offers a service to potential customers using advance-selling.[3] There is a risk of government-mandated service shutdown resulting in a “no-fault” inability

[3]The term “advance-selling” refers to a selling practice in which the provider offers opportunities for buyers to make purchase commitments before the time of consumption (Xie and Shugan 2001; Shugan and Xie 2005).

to deliver the service at the specified date (e.g., COVID-19). The probability of service shutdown is denoted by $q \in (0,1)$.

The provider designs the service by choosing the price P and the compensation policy under service shutdown using two instruments: cash-back and voucher. Cash-back means that customers are offered a full refund in case of a service shutdown. Instead, a voucher is a certificate that entitles the holder to receive a replacement service (e.g., delayed service) and a bonus $B \geq 0$ to compensate for the inconvenience under service shutdown. Note that if $B = 0$, the provider simply offers a replacement voucher.

The unit cost of providing the service is $C \geq 0$. This constant unit-cost formulation assumes that the provider operates under excess capacity, which is natural under a high likelihood of shutdown. Fixed cost is irrelevant for the analysis, so it is normalized to zero without loss of generality.

There is a unit mass of potential customers who have valuation V for the service, with $V > C$. In the event of a shutdown, customers learn about their salvage value S, drawn independently from a uniform distribution over the interval $[s_l, s_h]$, where $s_l \leq 0$ and $0 < s_h \leq V$. Specifically, this means that a customer can obtain a negative utility from using the service replacement included in the voucher. This captures the reality that some customers will exit the market following service outage due to personal reasons (e.g., scheduling conflicts or health). In contrast, if customers do not use the voucher, they give up the replacement and the bonus, and consequently receive zero utility from the service in the aftermarket. Potential customers know the shutdown probability q, the price P, and the compensation policy when deciding whether to purchase the service or use an outside option, the utility of which is normalized to zero.

3 Compensation Policies

This section studies compensation policies under service shutdown. Using selling as-is as the benchmark, we identify profit-maximizing and economically efficient compensation policies. Our results are illustrated in the context of the current policy debate on how to reimburse travelers following government mandated service shutdown in the airline industry. To facilitate exposition, we have relegated proofs to the Appendix.

3.1 Selling As-Is

Under a selling as-is policy, customers receive neither a cash refund nor a voucher following a shutdown. This policy is reminiscent of Ryanair's famous no-refund strategy, summarized by Michael O'Leary, the airline's CEO, as "You're not getting a refund so fuck off. We don't want to hear your sob stories" (The Economist 2017).

Potential customers purchase from the provider if their expected utility from the service net of price exceeds the utility of the outside option, that is, if

$$(1-q)V - P \geq 0. \tag{1}$$

The provider chooses the price to solve

$$\max_{P} \quad \pi(P) = P - (1-q)C,$$

subject to the participation constraint (1), where $(1-q)C$ is the expected cost of providing the service. Because the expected profit increases in price, the profit-maximizing price is $P^* = (1-q)V$ and the corresponding optimal profit can be derived as

$$\pi^* = (1-q)(V-C).$$

Note that because the provider prices to expected value, the surplus from the customers can be fully extracted. Consequently, social welfare coincides with firm profit.

3.2 Cash-Back Only

Under a cash-back policy, customers are refunded the purchase price following a service outage. In the U.S. for example, the Aviation Enforcement Office mandates that airlines must refund passengers promptly when their scheduled flights are cancelled or significantly delayed—an obligation that remains unchanged during the COVID-19 public health emergency (U.S. Department of Transportation 2020).

If the provider offers cash-back, customers receive utility $(1-q)V$ under normal service delivery and utility qP following shutdown. Therefore, potential customers purchase from the provider if their expected utility from the service net of price is non-negative, that is,

$$(1-q)(V-P) \geq 0. \tag{2}$$

The provider chooses the price to solve

$$\max_{P} \quad \pi(P) = (1-q)(P-C),$$

subject to the participation constraint (2). Clearly, the profit-maximizing price is $P^c = V$ and the corresponding optimal profit can be derived as

$$\pi^c = (1-q)(V-C).$$

Similar to the selling as-is policy, the provider fully extracts the surplus from the customers by pricing to value, and social welfare therefore coincides with firm profit. This leads to our first result.

Lemma 1. *The selling as-is and cash-back policies are equally profitable for the provider and equally efficient from a societal perspective.*

To intuitively understand this result, first note that all potential customers purchase the service under each of the policies. Whereas the expected cost of providing the service are the same across the policies, the pricing power of the provider differs: Under selling as-is, customers do not get a refund in the event of a service outage, which forces the provider to give in on the price. However, the higher pricing power under cash-back is offset because the provider is forced to reimburse its customers in the event of a service outage. Because the expected revenue and cost are the same, both policies are equally profitable for the provider. Finally, as customers receive zero surplus under each of the policies, welfare is the same across the two policies.

3.3 Voucher Only

Under the voucher policy, the provider announces the price P for the service, which might be cancelled with probability q. Following a service outage, the provider offers a bonus B to customers who use the voucher for service replacement. Customers learn their salvage value S and decide whether to use the voucher or exit the market. This policy was used by some carriers in the U.S. to mitigate the impact of the pandemic, which has led to class action lawsuits. For instance, American Airlines faced a lawsuit alleging breach of contract over the airline's refusal to issue refunds to customers whose flights were canceled by COVID-19 (Expert Institute 2020).

A customer chooses to redeem the voucher if the salvage value S plus the bonus B exceeds the utility from not using the service, that is, if $S+B>0$. Note that all customers redeem the voucher if $s_l+B>0$. To focus on the interesting case where some customers exit the market after shutdown, we assume $s_l+B<0$ so that demand for the replacement service is given by

$$D(B)=\frac{s_h+B}{s_h-s_l}. \tag{3}$$

The expected utility of a customer following a service outage can be derived as

$$\int_{s_l}^{-B}\frac{0}{s_h-s_l}dS+\int_{-B}^{s_h}\frac{S+B}{s_h-s_l}dS=\frac{(s_h+B)^2}{2(s_h-s_l)}.$$

Clearly, potential customers purchase from the provider if their overall expected utility from the service net of price is non-negative, that is, if

$$(1-q)V+q\frac{(s_h+B)^2}{2(s_h-s_l)}-P\geq 0. \tag{4}$$

The provider chooses the price and bonus to solve

$$\max_{P,B} \quad \pi(P,B) = (1-q)(P-C) + q[(P-B-C)D(B) + P(1-D(B))], \qquad (5)$$

where the second term captures the profit under shutdown: For the $D(B)$ customers who redeem the voucher, the margin is $P-B-C$, whereas the margin for the $1-D(B)$ customers who exit the market is P because the provider pockets the purchase price without delivering the service. The following result holds.

Lemma 2. *Under a voucher policy, it is optimal for the provider to set the optimal bonus to zero and offer a replacement service only.*

This result is driven by the provider's incentive to minimize its loss following a service shutdown, in which case the unit cost is $B+C$ for every customer who self-selects to redeem the voucher, and zero for customers who give up the service. By reducing the bonus to zero, the provider can minimize both the demand for replacement $D(B)$ and the effective unit cost. As a result, the provider simply offers a replacement service, and only customers with a salvage value in $[0, s_h]$ will redeem the voucher.

The optimal price can be derived as

$$P^r = (1-q)V + \frac{qs_h^2}{2(s_h - s_l)},$$

and the optimal profit therefore is

$$\pi^r = (1-q)(V-C) + \frac{qs_h(s_h - 2C)}{2(s_h - s_l)}.$$

The next result compares the profit and efficiency of the voucher policy with those of the selling as-is and cash-back policies.

Proposition 1. *The voucher-only policy has a profit and welfare advantage over the selling as-is and cash-back policies if $C < \frac{s_h}{2}$. The opposite results hold if $C > \frac{s_h}{2}$.*

This result has two important managerial implications. First, it shows that the salvage values of the customers and the unit cost of the provider jointly determine the optimal policy. If the average salvage value of the customers who take the replacement, $\frac{s_h}{2}$, exceeds the unit cost C, then offering a voucher with replacement only dominates selling as-is and cash-back. Intuitively, the provider can raise the price above the expected price in the benchmark case, $(1-q)V$, because it can exploit the pricing power in the aftermarket. On the other hand, providing the service replacement is costly, so that offering the replacement is optimal only if $C < \frac{s_h}{2}$. In contrast, if $C > \frac{s_h}{2}$, the provider is better off by offering either a selling as-is or cash-back policy, as the cost of providing the service in the aftermarket outweighs the increase in pricing power.

Second, Proposition 1 shows that the policy chosen by the provider also has a welfare advantage. The reason is that the provider can fully extract the customer surplus under each of the policies. Intuitively, if the average salvage value is high relative to the marginal cost, it is economically efficient for the provider to deliver the service in the aftermarket.

In terms of policy implications, Proposition 1 shows that a cash-back policy imposed by a regulatory authority can be to the detriment of the provider and society more broadly. The reason is that it prevents the provider to exploit the aftermarket. Therefore, if $C < \frac{s_h}{2}$, a voucher-only policy is more attractive for the policy maker. This suggests that airlines and society might be better off if carriers are allowed to issue vouchers rather than being forced to offer cash refunds following a shutdown.

3.4 Hybrid: Cash-Back or Voucher

Under a hybrid policy, the provider offers two compensation options: cash-back and voucher. For example, such a rule is compulsory for providers that offer travel by air, rail, ship, bus or coach in the EU (European Commission 2020). Specifically, the EU requires that passengers can choose to be reimbursed in money or in the form of a voucher in the event of cancellations. Moreover, to make the voucher more attractive, TAP Air Portugal and Aer Lingus recently introduced a 10% bonus if customers choose a voucher instead of a cash refund after shutdown (see Table 1). We now study the optimal design of the hybrid policy.

Following a service outage, customers learn their salvage value S and decide whether to redeem the voucher or receive the purchase price P by giving up the service. A customer chooses to redeem the voucher if the salvage value S plus the bonus B exceeds the utility from the cash-back option, that is, if $S+B>P$. Demand for the replacement service is therefore given by

$$D(P,B)=\frac{s_h-P+B}{s_h-s_l}. \tag{6}$$

Note that a higher bonus B expands demand in the aftermarket, whereas the opposite holds for a higher price P. The expected utility of customers following a service outage, as derived below, is higher than P:

$$\int_{s_l}^{P-B}\frac{P}{s_h-s_l}dS+\int_{P-B}^{s_h}\frac{S+B}{s_h-s_l}dS=\frac{(P-B)^2-2s_lP+2Bs_h+s_h^2}{2\,(s_h-s_l)}\geq P, \tag{7}$$

where the inequality follows from the fact that customers can always recover the purchase price under the hybrid policy.

Under the hybrid policy, there are two participation constraints for potential customers. The first constraint requires that customers receive a non-negative surplus in the absence of an outage, i.e.,

$$V-P\geq 0. \tag{8}$$

The second constraint requires that the overall expected utility from purchase is non-negative, that is,

$$(1-q)(V-P)+q\left[\frac{(P-B)^2-2s_lP+2Bs_h+s_h^2}{2\,(s_h-s_l)}-P\right]\geq 0,$$

which is implied by (7) and (8).

The provider chooses the price and bonus to solve

$$\max_{P,B}\quad \pi(P,B)=(1-q)(P-C)+q[(P-B-C)D(P,B)+(P-P)(1-D(P,B))],$$

subject to the participation constraint (8). The optimal purchase price and bonus can be derived as

$$P^h = V$$

and

$$B^h = V - \frac{s_h + C}{2},$$

and the optimal profit therefore is

$$\pi^h = (1-q)(V-C) + \frac{q(s_h - C)^2}{4(s_h - s_l)}.$$

Note that customers receive a positive surplus under the hybrid policy as

$$CS^h = \frac{q(s_h - C)^2}{8(s_h - s_l)}.$$

In contrast to the voucher policy, it is optimal for the provider to always include a positive bonus in the voucher if the cash-back option is offered (because $V > s_h$ and $V > C$). This is consistent with the positive bonuses offered by carriers during the COVID-19 pandemic. The following result holds.

Proposition 2. *Suppose that $s_h > C$ to ensure existence of the aftermarket. Then, the hybrid policy should include a bonus in the voucher and is more profitable than the selling as-is and cash-back policies. Compared to the voucher policy, the hybrid policy has a profit advantage if $s_h < (1+\sqrt{2})C$, whereas the opposite holds if $s_h > (1+\sqrt{2})C$.*

Figure 2 illustrates this result by showing how the profit-maximizing choice of compensation policy is determined by the interplay between the willingness to pay in the aftermarket captured by s_h and the unit cost C to provide the service. To intuitively understand Proposition 2, first note that the hybrid policy boils down to the cash-back policy if $s_h < C$, in which case there is no demand for the replacement service because all customers self-select the cash-back option following a service outage. Instead, if $s_h > C$ but is not "too large," the hybrid policy dominates the benchmark policy because it allows the provider to increase its pricing power due to the possibility of earning a margin on

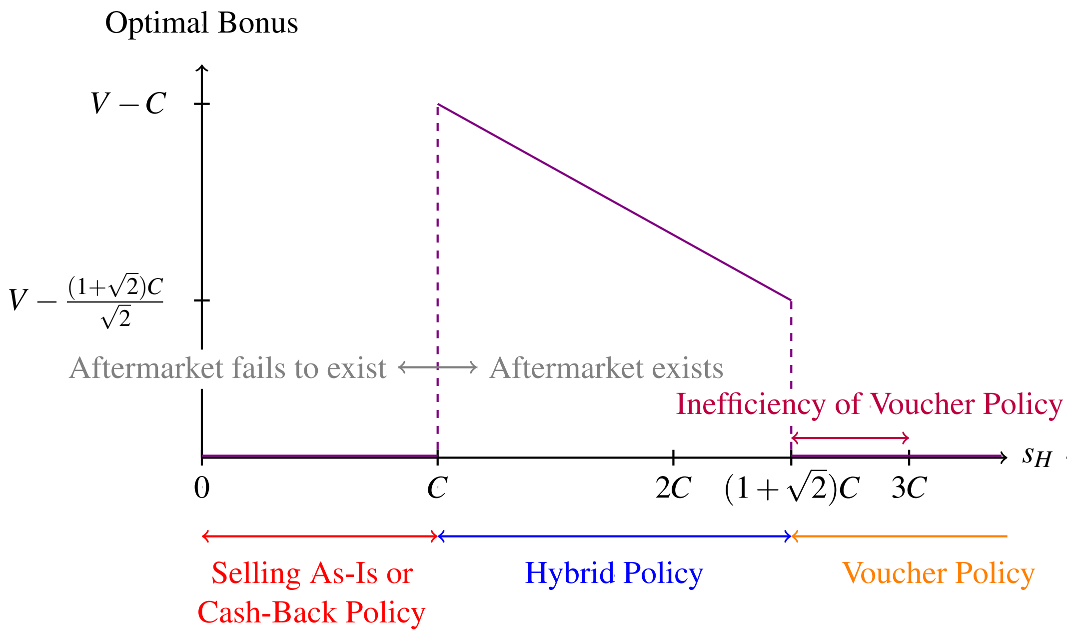

Figure 2: The optimal bonus as well as the profit-maximizing and economically efficient compensation policy as a function of the maximum salvage value s_H. The inefficiency arises because the provider chooses the voucher policy even though the hybrid policy would be optimal from a social point of view.

those customers who self-select the replacement option. In addition, by offering a bonus, the provider can enhance demand in the aftermarket, which reduces the cash payments to customers following a service outage, and thereby further increases profit. However, if $s_h > (1+\sqrt{2})C$, the demand in the aftermarket is large even though the optimal bonus is small. Therefore, switching to a voucher-only policy with a zero bonus boosts profit because it allows the provider to earn a higher margin in the aftermarket and at the same time eliminate payments to customers who demand cash refund following shutdown.

Proposition 2 also helps shed light on the debate about how to reshape the EU rules on reimbursement policies due to the Covid-19 pandemic (European Parliament 2020). For example, airlines have been asking to amend the existing hybrid policy and let them (rather than the customers) choose the means of reimbursement in the event of a service shutdown. Our result shows that there is indeed a profit motive for providers to lobby for a voucher-only policy. Across the Atlantic, American Airlines were involved in a class action lawsuit because they refused to issue cash refunds to customers whose flights were canceled by COVID-19 (Expert Institute 2020). Moreover, Figure 2 illustrates that the optimal bonus should be reduced if the (effective) average salvage value increases, and eventually drops to zero. This provides an explanation why the bonuses vary across different airlines. Intuitively, the salvage value of customers is likely to be higher for larger airlines with many routes, which allows them to offer a smaller or zero bonus. The next result summarizes the welfare implications of different compensation policies.

Proposition 3. *If $s_h > C$, then the hybrid policy has a welfare advantage over the benchmark policy, whereas the selling as-is and cash-back policies are more efficient if $s_h < C$. Compared to the voucher policy, the hybrid policy increases welfare if $s_h < 3C$, whereas the opposite holds if $s_h > 3C$.*

Figure 2 illustrates this result and shows that the profit-maximizing policy is generally consistent with welfare maximization—except for the case where $(1+\sqrt{2})C < s_h < 3C$. In this case, the provider offers the voucher policy even though the hybrid policy would be socially optimal. Said differently, suppressing the cash-back option is to the detriment of the customers. Intuitively, under a hybrid policy, the provider leaves some money on the table and does not fully extract the surplus of the customers. However, by switching to the voucher policy, the provider can extract the full surplus of the customers and thereby boost profit.

Proposition 3 helps to further shed light on the ongoing policy debate about the voucher policy in the EU (European Parliament 2020): For intermediate values of s_h, the lobbying efforts of the providers (and transport ministers) to suppress the cash-back option, which limits their cash drain during the shutdown, can be detrimental to the society. For industries with high values of s_h, switching to the voucher policy is not only more profitable but also improves social welfare.

If cash-back must be offered as an option under the current regulation, providers often simply offer a replacement voucher. At the same time, consumer organizations demand that providers make vouchers more attractive by including a bonus credit in the voucher.

Our next result illustrates that vouchers with a bonus help providers to improve profit in the presence of the cash back option.

Proposition 4. *Under the hybrid policy, both the bonus and profit advantage over the voucher policy are higher if the unit cost is low, or if potential customers either have a high valuation or a low average salvage value for the service. However, offering a bonus could reduce customer surplus.*

Proposition 4 shows that providers benefit from including a bonus in the voucher. Intuitively, a bonus increases the pricing power of the provider under the hybrid policy, which translates into a higher margin and profit if the service is delivered regularly. On the other hand, the positive bonus induces more customers to take the voucher option and consume the replacement service if a shutdown occurs, which further enhances the profit advantage. Surprisingly, note that offering a bonus does not necessarily improve the surplus of customers, so what consumer organizations advocate could hurt customers. This is because more attractive voucher unavoidably leads to higher prices.

The comparisons so far focused on *pre-shutdown efficiency*, which aggregates the expected profit and customer surplus. Another important measure to evaluate the compensation policies is *customer post-shutdown dissatisfaction*, which captures whether customers are worse-off following a shutdown compared to pre-shutdown expectations for a given policy. This measure is applied in the literature on satisfaction guarantees (Fruchter and Gerstner 1999; Hogreve and Gremler 2009). Empirical evidence shows that post-purchase satisfaction is given more weight than pre-purchase satisfaction in determining overall satisfaction and purchase intention (Posselt and Gerstner 2005). Table 3 summarizes the number of dissatisfied customers and the total customer dissatisfaction under each compensation policy. Importantly, note that the cash-back and hybrid policies prevent customer dissatisfaction, whereas selling as-is creates the highest dissatisfaction. Under a voucher-only policy, customers with low salvage values are frustrated under shutdown and could create negative word-of-mouth in the marketplace.

Policy	Dissatisfied Customers	Customer Dissatisfaction
Selling As-Is	1	$-(1-q)V$
Cash-Back Only	0	0
Voucher Only	$-\frac{s_l}{s_h-s_l}$	$\frac{s_l}{s_h-s_l}\left[(1-q)V+\frac{qs_h^2}{2(s_h-s_l)}\right]$
Hybrid: Cash-Back or Voucher	0	0

Table 3: Customer post-shutdown dissatisfaction. Customer dissatisfaction measures the total dissatisfaction across the number of dissatisfied customers (note that $s_L \leq 0$).

4 Reselling Market

In the analysis so far, we have assumed that the voucher is non-transferable. That is, only the customer can use the replacement service and the bonus following a shutdown. In this section, we analyze the implications when the voucher is transferable, i.e., when the voucher can be resold in an external resale market at a price below P. As a result, customers with the cash-back option will not actively engage in the reselling activity under the hybrid policy. Thus, we now consider the voucher policy only.

Following a service outage, customers learn their salvage value S and decide whether to use the service replacement or sell the voucher in an external market. In the resale market, a customer obtains $r+B$ when reselling the voucher because the resell price directly depends on the amount of bonus B as well as a fair market price r for the service.[4] A customer exerts the replacement option if $S+B > r+B$. Following a service outage, the demand of customers who have initially purchased is given by

$$D(r) = \frac{s_h - r}{s_h - s_l}.$$

[4]We assume that the resale market is "covered" in the sense that all customers can resell the voucher if they choose to. Alternatively, one could consider a situation where only a fraction of customers are able to trade in the resale market.

The expected utility of these customers following a service outage can be derived as

$$\int_{s_l}^{r} \frac{r}{s_h - s_l} dS + \int_{r}^{s_h} \frac{S}{s_h - s_l} dS + B = \frac{r^2 - 2s_l r + s_h^2}{2(s_h - s_l)} + B.$$

Hence, the participation constraints read

$$(1-q)(V-P) \geq 0 \tag{9}$$

and

$$(1-q)V + q\left[\frac{r^2 - 2s_l r + s_h^2}{2(s_h - s_l)} + B\right] - P \geq 0. \tag{10}$$

The provider chooses the price and bonus to solve

$$\max_{P,B} \quad \pi(P,B) = P - C - qB = (1-q)V + q\left[\frac{r^2 - 2s_l r + s_h^2}{2(s_h - s_l)} + B\right] - C - qB,$$

subject to participation constraints (9) and (10). Note that the profit function does not depend on B, which implies that there exist alternative optimal solutions with different bonus levels leading to the same profit. For simplicity, we therefore set $B = 0$ for the rest of analysis.

Note that, in contrast to replacement only, all customers are active in the aftermarket due to the existence of the resale market. Because the profit increases in P, the provider sets the optimal price as $\min\{V, (1-q)V + q\frac{r^2-2s_l r+s_h^2}{2(s_h-s_l)}\}$. Assuming that $\frac{r^2-2s_l r+s_h^2}{2(s_h-s_l)} \leq V$, the optimal price and profits are

$$P^m = (1-q)V + q\frac{r^2 - 2s_l r + s_h^2}{2(s_h - s_l)}$$

and

$$\pi^m = (1-q)V + q\frac{r^2 - 2s_l r + s_h^2}{2(s_h - s_l)} - C.$$

Observe that the optimal price and profit increase in r. Therefore, if r is sufficiently large, the optimal price is $P^m = V$ and the optimal profit is $\pi^m = V - C$. The following result holds.

Proposition 5. *Introducing a reselling market increases profitability and welfare under the voucher policy if and only if the reselling price satisfies* $r > s_l + \sqrt{s_l (s_l - 2C)}$.

The intuition is as follows: The presence of the reselling market increases both the pricing power of the firm and the number of customers in the aftermarket. Specifically, with a reselling market, the service is delivered to all customers, whereas only a fraction of the customers are served in the absence of a reselling market. This increases the total cost of providing the service. Therefore, if $r > s_l + \sqrt{s_l (s_l - 2C)}$, the benefit of the higher pricing power outweighs the cost of providing the service to a larger number of customers for both the provider and society more broadly. Finally, note that the number of customers who trade in the reselling market is given by $1 - D(r)$. This implies that there must be at least $1 - D(r)$ customers in the reselling market that have a willingness to pay that exceeds r for the reselling market to be covered.

5 Discussion

This paper developed an analytical framework to understand and evaluate compensation policies under government mandated shutdowns. Using selling as-is as the benchmark policy, we examined the profit and welfare implications of three compensation policies: cash-back (favored by customers), voucher (favored by providers), and a hybrid that includes both cash-back and voucher options. The results show that the cash-back policy leads to the same profit and welfare as the benchmark policy. Both these policies are less profitable than the hybrid policy and economically inefficient, except when the average salvage value is low relative to the cost of providing the service.

Conventional wisdom suggests that, to save cost, providers would simply offer a service replacement following a shutdown. We confirm this intuition and show that it is indeed optimal not to include any bonus under the voucher policy. Surprisingly, such a replacement-only policy can dominate all other policies, including the hybrid policy, in terms of profit and social welfare. From the legal point of view, customers have the right to be reimbursed if they do not consume the replacement service. However, imposing the cash-back option can reduce both profit and social welfare, or maximize social welfare but result in lower profit. On the other hand, if the regulator requires the provider to offer cash-back as an option, our analysis suggests that including a positive bonus always improves profit but can reduce customer surplus.

Overall, the provider might want to choose between the voucher policy and the hybrid policy. For potential customers, pushing for the cash-back option is always beneficial, but asking for a positive bonus could backfire. Regulators in turn might evaluate compensation policies based on the following criteria:

(a) **Pre-shutdown efficiency** - whether the compensation policy maximizes expected welfare consisting of expected profit and customer surplus. This measure, which reflects the traditional concept of Pareto efficiency, does not take into account the distribution of surplus between the provider and the customers. Therefore, this measure does not capture the notion of distributional fairness (Pazner and Schmeidler 1974).

(b) **Post-shutdown dissatisfaction** - whether customers are worse off following a shutdown compared to pre-shutdown expectations. This measure is applied in the literature on satisfaction guarantees (Fruchter and Gerstner 1999; Hogreve and Gremler 2009). Empirical evidence shows that post-purchase satisfaction is given more weight than pre-purchase satisfaction in determining overall satisfaction and purchase intention (Posselt and Gerstner 2005). Dissatisfied customers could also pressure consumer organizations to lobby for tighter government regulation.

(c) **Compromise** - whether the compensation policy offers a compromise between the opposing positions of the provider and customers. Regulators typically prefer policies that offer compromise over policies that antagonize customers or providers (Gutmann and Thompson 2010).

Figure 3 summarizes the implications of these three criteria on policy design. It suggests that regulators are likely to favor the hybrid policy because it eliminates customer

Selling As-Is	Voucher Only
Pre-shutdown efficiency – No unless the average salvage value of customers is sufficiently low. **Post-shutdown dissatisfaction – Yes** because customers lose their purchase price. **Compromise – Yes** because customers pay a low price and the provider does not offer a service replacement.	**Pre-shutdown efficiency – Yes** if the average salvage value of customers is sufficiently high. **Post-shutdown dissatisfaction – Yes** because customers with a low salvage value lose their purchase price. **Compromise – No** because customers cannot get a cash refund.
Cash-Back Only	**Hybrid: Cash-Back or Voucher**
Pre-shutdown efficiency – No unless the average salvage value of customers is sufficiently low. **Post-shutdown dissatisfaction – No** because customers can recover the purchase price. **Compromise – No** because providers cannot offer vouchers.	**Pre-shutdown efficiency – Yes** if the average salvage value is high but not too high. **Post-shutdown dissatisfaction – No** because customers can choose to be fully refunded. **Compromise – Yes** because the provider can offer vouchers and customers can get a cash refund.

Figure 3: Evaluation of compensation policies based on efficiency, satisfaction, and compromise.

post-shutdown dissatisfaction and offers some level of compromise between the provider and customers. However, regulators need to recognize that by imposing the cash-back option, the hybrid policy could lower social welfare. On the other hand, allowing a voucher-only policy may lead to higher social welfare, but the additional surplus will be entirely appropriated by the provider. Regulators could therefore consider mechanisms that motivate providers to share the additional surplus with the customers.

Future research could study empirically how the distribution of salvage values affect the design of the optimal compensation policy. Also, experimental research could explore post-shutdown (dis)satisfaction, which is critical for understanding the implications of service shutdown on customer lifetime value. Another interesting direction for future research is to examine how customers react to a bonus voucher. More broadly, a voucher is a written promise (a bond) for a specific service to be delivered on a future date. Therefore, its value to customers depends on the likelihood that the provider delivers its promise and how "liquid" the voucher is. The voucher is likely to be more valuable if its terms are flexible (e.g., a longer expiration date, a reselling option) or the provider is large and financially stable. It would be interesting to investigate what drives the liquidity and flexibility of vouchers, and the impact of these features on profit and welfare.

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Appendix

Proof of Lemma 1. The policies are equally profitable for the provider as $\pi^* - \pi^c = 0$. Because $CS^* = CS^c = 0$ and $W = \pi + CS$, it follows that $W^* - W^c = 0$, which implies that the two policies are equally efficient. □

Proof of Lemma 2. Substituting demand $D(B)$ in Equation (3) into the profit function (5) yields

$$\pi(P,B) = (1-q)(P-C) + q\left[(P-B-C)\frac{s_h+B}{s_h-s_l} + P\left(-\frac{s_l+B}{s_h-s_l}\right)\right].$$

Taking the partial derivative with respect to B yields

$$\frac{\partial \pi(P,B)}{\partial B} = q\left[-\frac{s_h+B}{s_h-s_l} + (P-B-C)\frac{1}{s_h-s_l} + P\left(-\frac{1}{s_h-s_l}\right)\right] < 0,$$

which implies that the optimal bonus is equal to zero. □

Proof of Proposition 1. With $B=0$ from Proposition 2, the participation constraint (4) boils down to

$$(1-q)V + q\frac{s_h^2}{2(s_h-s_l)} - P \geq 0. \tag{A.1}$$

Under replacement only, the provider solves

$$\begin{aligned}\max_P \quad \pi(P) &= (1-q)(P-C) + q\left[(P-C)\frac{s_h}{s_h-s_l} + P\left(-\frac{s_l}{s_h-s_l}\right)\right]\\ &= P-(1-q)C - qC\frac{s_h}{s_h-s_l}\end{aligned}$$

subject to the participation constraint (A.1). Because the profit increases in P, the provider has an incentive to raise P until (A.1) holds with equality. Therefore, the optimal price is

$$P^r = (1-q)V + \frac{qs_h^2}{2(s_h-s_l)}.$$

Note that because $\frac{s_h^2}{2(s_h-s_l)} \leq \frac{V}{2}$, we have $P^r \leq \frac{(2-q)V}{2} \leq V$. By substitution, the optimal profit can be derived as

$$\pi^r = (1-q)(V-C) + \frac{qs_h(s_h-2C)}{2(s_h-s_l)}.$$

Because the provider fully extracts the surplus of the customers, $CS^r = 0$ and hence $W^r = \pi^r$. Finally, comparing the profit under a voucher-only policy to the benchmark case yields

$$\pi^r - \pi^* = \frac{qs_h(s_h - 2C)}{2(s_h - s_l)}.$$

Clearly, $\pi^r - \pi^* > 0$ if and only if $C < \frac{s_h}{2}$. □

Proof of Proposition 2. Under a hybrid policy, the profit function can be written as

$$\max_{P,B} \quad \pi(P,B) = (1-q)(P-C) + q(P-B-C)\frac{s_h - P + B}{s_h - s_l}.$$

Taking the partial derivative with respect to B yields and setting to zero yields

$$B = P - \frac{s_h + C}{2}. \tag{A.2}$$

Substituting this expression for B back into the profit function yields

$$\pi(P) = (1-q)(P-C) + \frac{q(s_h - C)^2}{4(s_h - s_l)}.$$

Since $\frac{d\pi(P)}{dP} = 1 - q \geq 0$, the optimal price satisfies $P^h = V$, so that

$$B^h = V - \frac{s_h + C}{2}$$

and

$$\pi^h = (1-q)(V-C) + \frac{q(s_h - C)^2}{4(s_h - s_l)}.$$

Because the demand for the replacement service in Equation (6) evaluated at the optimal price and bonus is given by

$$D(P^h, B^h) = \frac{s_h - C}{2(s_h - s_l)},$$

demand in the aftermarket is positive if and only if $s_h > C$.

Next, comparing the profit under a hybrid policy to the benchmark case yields

$$\pi^h - \pi^* = \frac{q(s_h - C)^2}{4(s_h - s_l)}.$$

Clearly, $\pi^h > \pi^* = \pi^c$ for $s_h > C$. In terms of profitability, the hybrid option dominates the replacement option if $\pi^h - \pi^r > 0$. This profit comparison boils down to

$$\frac{q(s_h - C)^2}{4(s_h - s_l)} - \frac{qs_h(s_h - 2C)}{2(s_h - s_l)} > 0,$$

which holds if and only if $s_h < (1+\sqrt{2})C$. □

Proof of Proposition 3. The customer surplus under a hybrid policy can be derived as

$$CS^h = (1-q)(V - P^h) + \frac{q(s_h - P^h + B^h)^2}{2(s_h - s_l)} = \frac{q(s_h - C)^2}{8(s_h - s_l)}$$

and welfare is given by

$$W^h = (1-q)(V - C) + \frac{3q(s_h - C)^2}{8(s_h - s_l)}.$$

Clearly, $W^h > W^* = W^c$ for $s_h > C$. The hybrid policy dominates voucher policy if $W^h > W^r$. Specifically, this condition boils down to

$$\frac{3q(s_h - C)^2}{8(s_h - s_l)} > \frac{qs_h(s_h - 2C)}{2(s_h - s_l)},$$

which holds if and only if $s_h < 3C$. □

Proof of Proposition 4. We refer to the hybrid policy with zero bonus as the *hybrid-zero policy*. In this case, $B = 0$, and the provider solves

$$\max_P \quad \pi^h(P, 0) = (1-q)(P - C) + q(P - C)\frac{s_h - P}{s_h - s_l}$$

subject to the constraint (8). Taking the partial derivative with respect to P and setting it to zero yields

$$P^0 = \frac{(1-q)(s_h - s_l)}{2q} + \frac{s_h + C}{2}.$$

The optimal price is $P = \min\{V, \frac{(1-q)(s_h-s_l)}{2q} + \frac{s_h+C}{2}\}$. Hence, the profit is

$$\begin{array}{ll} \pi^h\left(P^0,0\right) = \frac{q}{s_h-s_l}\left(\frac{(1-q)(s_h-s_l)}{2q} + \frac{s_h-C}{2}\right)^2 & \text{if } P^0 \leq V \\ \pi^h\left(V,0\right) = (1-q)(V-C) + q(V-C)\frac{s_h-V}{s_h-s_l} & \text{if } P^0 > V. \end{array}$$

The hybrid–zero policy can be viewed as the hybrid policy with an additional constraint $B = 0$. Therefore, it is always dominated by the hybrid policy.

Under the hybrid policy, given price P^0, the optimal bonus is $B = \frac{(1-q)(s_h-s_l)}{2q}$ by Equation (A.2). Note that this feasible solution of the hybrid policy dominates the interior solution of the hybrid–zero policy because $\pi^h\left(P^0, \frac{(1-q)(s_h-s_l)}{2q}\right) \geq \pi^h\left(P^0,0\right)$ if and only if

$$\left(\frac{s_h - C}{2}\right)^2 \geq \left(\frac{s_h - C}{2}\right)^2 - \left(\frac{(1-q)(s_h - s_l)}{2q}\right)^2,$$

which always holds. By the property of optimization problems, we have $\pi^h(P^h,B^h) \geq \pi^h(P^0, \frac{(1-q)(s_h-s_l)}{2q})$ and $\pi^h\left(P^0,0\right) \geq \pi^h\left(V,0\right)$, so we have $\pi^h(P^h,B^h) \geq \pi^h\left(P^0,0\right) \geq \pi^h\left(V,0\right)$.

When the hybrid-zero policy reaches the interior solution, the profit difference can be written as

$$\begin{aligned} & \pi^h(P^h,B^h) - \pi^h\left(P^0,0\right) \\ = & \left[\pi^h(P^h,B^h) - \pi^h(P^0, \frac{(1-q)(s_h - s_l)}{2q})\right] + \left[\pi^h(P^0, \frac{(1-q)(s_h - s_l)}{2q}) - \pi^h\left(P^0,0\right)\right] \\ = & (1-q)(V-P^0) + \frac{(1-q)^2(s_h - s_l)}{4q} \\ = & (1-q)\left[V - \frac{(1-q)(s_h - s_l)}{4q} - \frac{s_h + C}{2}\right] > 0. \end{aligned}$$

Instead, when the hybrid-zero policy reaches the boundary solution, the profit difference can be written as

$$\begin{aligned} & \pi^h(P^h, B^h) - \pi^h(V, 0) \\ = \; & \frac{q(s_h - C)^2}{4(s_h - s_l)} - q(V - C)\frac{s_h - V}{s_h - s_l} > 0 \\ = \; & \frac{q}{s_h - s_l}\left[\frac{(s_h - C)^2}{4} + (V - C)(V - s_h)\right] > 0 \end{aligned}$$

which is guaranteed, because $C < s_h < V$.

Next we address the sensitivity analysis. Recall that the optimal bonus is $B^h = V - \frac{s_h + C}{2}$. So we know the optimal bonus increases with V and decreases with C and s_h.

Let $\Delta\pi = \pi^h(P^h, B^h) - \pi^h\left(P^0, 0\right)$ when $P^0 \leq V$ and $\Delta\pi = \pi^h(P^h, B^h) - \pi^h(V, 0)$ when $P^0 > V$. The sensitivity analysis leads to the results below:

1. $\frac{\partial \Delta\pi}{\partial V} > 0$. $\Delta\pi$ first increases quadratically (under boundary solution) and then linearly (under interior solution) with V.

2. $\frac{\partial \Delta\pi}{\partial C} < 0$. $\Delta\pi$ first decreases linearly (under interior solution) and then quadratically (under boundary solution) with C.

3. $\frac{\partial \Delta\pi}{\partial s_h} < 0$. $\Delta\pi$ first decreases linearly (under interior solution) and then non-linearly (under boundary solution) with s_h because

$$\frac{\partial \Delta\pi}{\partial s_h} = -\frac{q}{(s_h - s_l)^2}\left[\frac{(s_h - C)^2}{4} + (V - C)(V - s_h)\right] + \frac{q}{s_h - s_l}\left[\frac{s_h + C - 2V}{2}\right] < 0.$$

So the profit difference increases with V and decreases with C and s_h. That is, the profit difference changes in the same direction as the optimal bonus. The higher the optimal bonus, the higher the profit loss from offering a zero bonus under the hybrid policy.

Finally, we show that offering a bonus could reduce customer surplus. When the interior solution arises under the hybrid-zero policy, customer surplus is

$$\begin{aligned} CS^0 &= (1-q)V + q\frac{(P^0)^2 - 2s_l P^0 + s_h^2}{2(s_h - s_l)} - P^0 \\ &= (1-q)\left(V - P^0\right) + q\frac{(P^0 - s_h)^2}{2(s_h - s_l)}, \end{aligned}$$

which is higher than the customer surplus under the hybrid policy if and only if

$$\begin{aligned} CS^0 &> CS^h \\ (1-q)\left(V - P^0\right) + q\frac{(P^0 - s_h)^2}{2(s_h - s_l)} &> \frac{q(s_h - C)^2}{8(s_h - s_l)} \\ V &> \frac{3(1-q)(s_h - s_l)}{8q} + \frac{3s_h + C}{4}. \end{aligned}$$

For the interior solution to be valid, we need $P^0 = \frac{(1-q)(s_h - s_l)}{2q} + \frac{s_h + C}{2} \leq V$.

$$\begin{aligned} \frac{3(1-q)(s_h - s_l)}{8q} + \frac{3s_h + C}{4} &\geq \frac{(1-q)(s_h - s_l)}{2q} + \frac{s_h + C}{2} \\ s_h - C &\geq \frac{(1-q)(s_h - s_l)}{2q}. \end{aligned}$$

Therefore, when $V > \max\left(\frac{3(1-q)(s_h - s_l)}{8q} + \frac{3s_h + C}{4}, \frac{(1-q)(s_h - s_l)}{2q} + \frac{s_h + C}{2}\right)$, the interior solution arises under the hybrid-zero policy and results in a higher customer surplus than under the hybrid policy. In this case, customers are worse off if they push for more attractive vouchers. □

Proof of Proposition 5. Note that $\frac{r^2 - 2s_l r + s_h^2}{2(s_h - s_l)} \leq V$ is satisfied if and only if

$$r \leq s_l + \sqrt{2(s_h - s_l)V - (s_h^2 - s_l^2)}.$$

Also, $s_l + \sqrt{s_l(s_l - 2C)} \leq s_l + \sqrt{2(s_h - s_l)V - (s_h^2 - s_l^2)}$ if and only if $2(V - C)s_l \leq 2s_h V - s_h^2$, which always holds because $V \geq s_h$, $V > C$ and $s_l \leq 0$.

If $s_l + \sqrt{s_l\,(s_l - 2C)} \leq r \leq s_l + \sqrt{2(s_h - s_l)V - (s_h^2 - s_l^2)}$, the profit difference can be expressed as

$$\pi^m - \pi^r = \frac{q\left(r^2 - 2s_l r + 2s_l C\right)}{2(s_h - s_l)},$$

which is positive if $f(r) := r^2 - 2s_l r + 2s_l C > 0$, that is, if $r > s_l + \sqrt{s_l\,(s_l - 2C)}$. Note that $f(r)$ takes its minimum at $r = s_l < 0$, and that $f'(r) > 0$ for $> s_l$. Finally, $s_l\,(s_l - 2C) > 0$ for all C as $s_l \leq 0$ and $C \geq 0$.

Otherwise, if $r \leq s_l + \sqrt{2(s_h - s_l)V - (s_h^2 - s_l^2)}$, $\pi^m = V - C$ is the highest the provider can achieve by selling to the market. Therefore, the profit is higher under the reselling market.

Because the surplus of the customers who initially purchased is fully extracted under the interior solution, $CS^m = 0$. Assuming further that customers in the reselling market obtain zero surplus, $W^m = \pi^m$. For the boundary solution, $P^m = V$ and the customer surplus is given by

$$CS^m = q\left[\frac{r^2 - 2s_l r + s_h^2}{2(s_h - s_l)} - V\right] > 0.$$

Together with a higher profit in this case, the social welfare is again higher than the voucher policy. Note that assuming a positive consumer surplus in the outside market would increase welfare. Hence, the zero surplus assumption for consumers in the reselling market is a conservative assumption. □