



Research papers

Prediction of river discharges at confluences based on Entropy theory and surface-velocity measurements

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ABSTRACT

Hydrodynamic features of the confluence zone of large rivers are complicated because of their three-dimensional flow structure. The confluence between the Rio Negro and the Rio Solimões, characterised by black and white waters, respectively, ranks among the largest river junctions on Earth. An Entropy-based investigation was carried out to assess the discharge and analyse the 2D structure of velocity distribution for large river flows relying on monitoring of near-surface velocity only. The estimated flow data were compared with in-situ ADCP data gathered across some transects of the Negro and Solimões rivers during both low and relatively high flow conditions. Results are illustrated through some transects at the confluence zone, upstream and downstream of the zone and in the Careiro Channel. Comparisons highlight that the Entropy-based flow velocity in terms of depth-averaged velocity, cross-sectional mean flow velocity and vertical velocity distributions, starting from the measured surface velocity, is in agreement with those determined by the ADCP measurements, with an error in mean flow velocity and discharge lower than 15%. The research highlights the potential of the Entropy method to estimate the discharge and velocity in very large rivers just relying on near-surface velocities monitoring.

1. Introduction

River confluences are key features of every riverine system and represent interfaces with complex hydrodynamics and typical bed morphologies; in fact, they are characterized by a highly complex three-dimensional flow structure with enhanced flow strength and secondary currents (Kenworthy and Rhoads, 1995; Biron and Lane, 2008). Mixing processes and hydrodynamic characteristics of flow at confluences have been investigated based either on field data (Rhoads and Sukhodolov, 2001; Ramón et al., 2013; Ramón et al., 2016; Lewis and Rhoads, 2015; Herrero et al., 2018; Gualtieri et al., 2017; Gualtieri et al., 2019; Lewis et al., 2020; Yuan et al., 2021) or through laboratory experiments (Bryan and Kuhn, 2002; Parsons et al., 2007; Ismail et al., 2016) or by numerical simulations (White and Helfrich, 2013; Lyubimova et al., 2014; Zhong-Chao and Ze-Yi, 2011; Constantinescu et al., 2011; Constantinescu et al., 2016; Ahadiyan et al., 2018).

Regarding the confluence of Negro and Solimões Rivers, past studies (Laraque et al., 2009; Gualtieri et al., 2018; Ianniruberto et al., 2018; Gualtieri et al., 2019) have demonstrated that a more dense and sediment-laden flow from the Solimões is entering the confluence with

the Negro giving origin to complex mixing processes that can be explained as a combination of the four hydrologic and morphologic processes: difference in (1) velocity, (2) density between the rivers, (3) bed friction, including form drag, and (4) change in channel width along the confluence due to the adjustments of hydrodynamic and morphodynamic processes. Immediately downstream of the junction, the effects of differences in velocity and density on mixing are comparable, while further downstream, beyond the so-called Confluence Hydrodynamic Zone (CHZ) (concept introduced by Kenworthy and Rhoads, 1995), the latter is more important and the role of bed friction is significant (Gualtieri et al., 2019). Due to the difference in sediment load, mixing processes can be tracked by acoustic backscatter intensity observed with ADCP, suggesting that mixing interface at the Negro/Solimões confluence is tilted due to density difference. Furthermore, it is found that the location of the mixing interface is closely related to the discharge ratios between the tributaries, confirming the role of this parameter on mixing at large and small confluences (Lane et al., 2008; Lyubimova et al., 2014; Lewis and Rhoads, 2015). As most of the time the discharge in the Solimões is much larger than that in the Negro, the mixing interface is usually closer to the Negro bank. Based on the previous studies, it is

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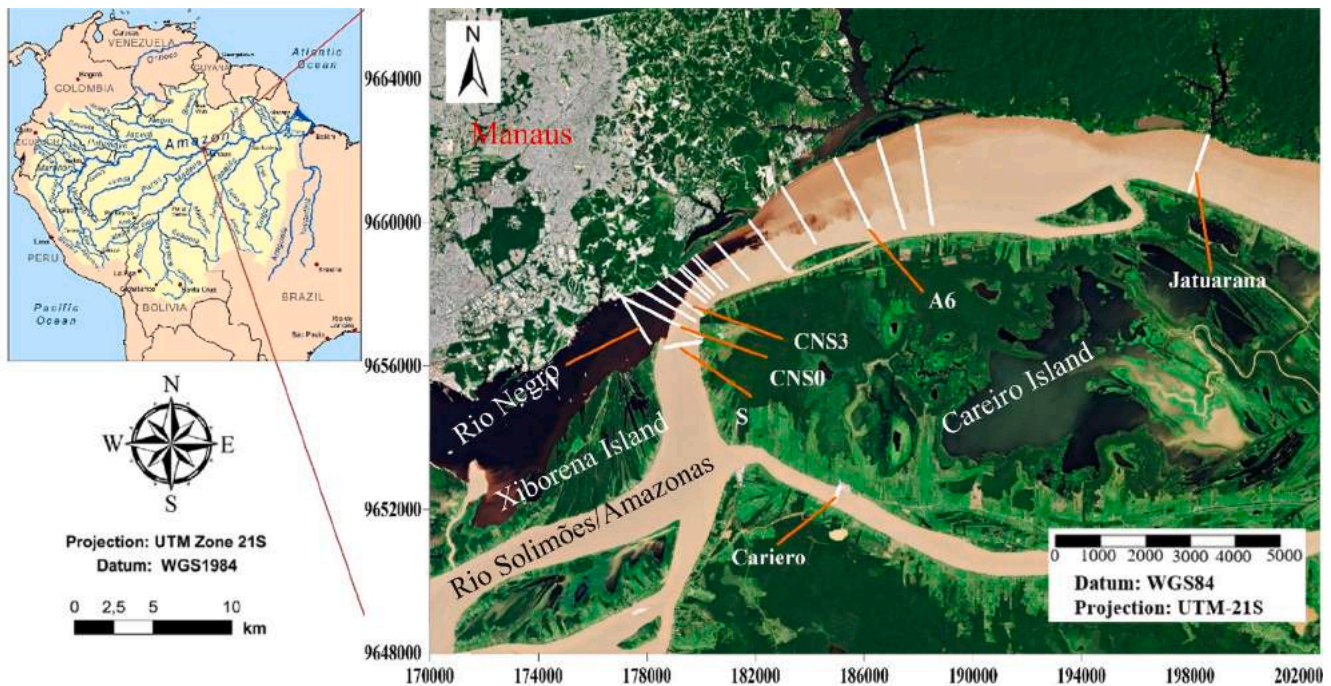


Fig. 1. The Basin of Amazon river and the map of the Rio Negro Rio Solimões confluence (European Space Agency) with the selected transects.

quite evident that the confluence of large rivers is a challenging hydrodynamic environment due to the complex factors that determine the flow characteristics, so that robust datasets in terms of streamflow measurements are necessary for estimation of discharge, mainly during high flow conditions, when in-situ measurements are more difficult due to the stronger currents.

In general, measuring the flow characteristics in rivers is a key factor for understanding the hydrologic cycle, flood forecasting, water resources management and climate change issues. For instance, discharge is mentioned as one of the Essential Climate Variables by the Global Climate Observing System (GCOS) (GCOS, 2011). However, measurements can be complicated especially during relatively high flow conditions because of difficulties related to boat manoeuvrability and operation of ADCP along cross-sections due to flow turbulence. Furthermore, conventional monitoring procedures are costly, time-consuming and pose safety challenges for hydrographers, requiring more efficient methodologies. In this context, Moramarco et al. (2017), Moramarco et al., (2019) developed an alternative non-contact approach based on the Entropy theory to estimate flow characteristics, starting from measurements of the free-surface velocity.

The Entropy theory has been widely used by researchers to estimate the velocity distribution and other relevant parameters in open channels flow (Chiu, 1989; Chiu and Said, 1995; Sterling and Knight, 2002; Chiu et al., 2005; Moramarco and Singh, 2010; Termini and Moramarco, 2017; Singh et al., 2017; Ebtehaj et al., 2018). Recently Termini and Moramarco (2020) have also verified that the feedback effect of cross-sectional flow on the longitudinal velocity distribution, occurring in curved channel reaches, could be qualitatively identified by applying the entropy concept and by analyzing the spatial distribution of the entropic parameter along with the location of dip that represents the location where the maximum flow velocity occurs below the water surface.

The present work aims to answer the question whether using only near-surface velocity measurements, the Entropy-based velocity distribution model can be used to predict the velocity field at confluences well enough to accurately estimate flow discharge, despite the complex hydrodynamic settings. The analysis could be also of support to identifying the complex dynamic of the flow just through the monitoring of surface velocity across the river. The estimation was conducted using the “near-

surface” velocity (first depth bin below blank depth) based on ADCP measurements across the Rio Negro and Rio Solimões confluence of the Amazon River. The proposed methodology could be of considerable interest to estimate discharge using surface velocities collected by either in situ measurements or by satellite observations such as those proposed by Tarpanelli et al. (2013), Termini and Di Leonardo (2018), Moramarco et al. (2019) and Termini (2021).

The first part of the paper describes the available ADCP field data collected across the Rio Negro and Rio Solimões river system, as well as the statement of the mathematical model. Next, the Entropy-based depth-averaged velocity maps ($U_{\text{dep-ave}}$ (m/s)) for different simulated scenarios are presented and compared with those obtained by using the measured field data. Finally, some significant observed and calculated vertical profiles are shown and discussed along with statistical analysis. [Supplementary Material](#) is also provided for some other transects.

2. Field site and data collection

The Amazon River has a total length of 6400 km, being the largest river and drainage system on the Earth in terms of flow volume and basin area. The annual average discharge is 206,000 m³/s with a catchment area of 7 million km² (for Rio Negro, the basin area is 700,000 km²) (Callede et al., 2010). The confluence between the Negro and Solimões rivers, one of the largest river junctions on the Earth, is located near Manaus, in Northern Brazil, where these rivers merge to form the Amazon River approximately 1600 km upstream from its mouth at the Atlantic Ocean. The river slope for the central part of the Amazon River is on average 2 cm/km (Leopoldo et al., 1987). This confluence is also known as the “Meeting of Waters” (Portuguese: Encontro das Águas) for the meeting of the black and white waters of the Negro and Solimões rivers (Fig. 1). As part of the CLIM-Amazon Project, a joint European and Brazilian Research Project funded by the EU about climate and sedimentary processes of the Amazon River Basin, two field studies were conducted in both low (28–31 October 2014, FS – CNS1 campaign) and relatively high flow conditions (29–30 April 2015, FS – CNS2 campaign) at the confluence zone. During the field campaigns, a Teledyne RDI 600 kHz Rio Grande Acoustic Doppler Velocity Profiling (ADCP) was used to collect the flow velocity components along different cross-sections to investigate key features of hydrodynamics, mixing and

Table 1
Length and maximum depth of selected transects.

Transect name	Location	Transect length (m)	Maximum measurement depth (m)
CNS0	Upper part of the confluence zone	4870	53.1
CNS3	Middle of the confluence zone	2270	61.1
A6	End of the confluence zone	4280	33.1
N	Rio Negro	2836	40.1
S	Rio Solimões	1986	52.1
Jatuarana	Further downstream of the confluence zone	2725	54.4
Careiro	Rio Careiro	780	31.4

sediment transport and morphodynamic in the confluence area (Fig. 1) (Gualtieri et al., 2019). These campaigns enabled identification of the characteristic hydrodynamics features typically observed in confluence studies such as stagnation zone, velocity deflection and re-alignment zone, separation region with recirculation, maximum velocity and flow recovery region (Best, 1987). Moreover, it was observed that some features may adjust position and extension when discharge changed from low flow to relatively high flow conditions. Data based on measurements using similar equipment and repeated transects conducted by the Geological Survey of Brazil in 2017 for Careiro channel and Jatuarana were used to further test the methodology. To analyse the depth-averaged velocity, the resultant velocity sampled in all transects (16 and 19 transects for high and low flow conditions, respectively) was considered. In the present paper, to investigate the vertical and cross-sectional distributions of flow velocity, six transects were selected: two transects (CNS3 and A6) in the middle and downstream of the confluence zone, one transect (N) in Rio Negro, one transect (S) in Rio Solimões, one transect in Jatuarana away from the confluence zone, and one transect at Careiro across the Careiro channel (Table 1). The average ratio of channel widths (Rio Negro/Rio Solimões) upstream of the confluence zone is 0.92. The discharge ratios (Rio Negro/Rio Solimões) for low and relatively high flow conditions are 0.39 and 0.32, respectively, while the momentum flux ratios were 0.10 and 0.07.

$$U_{maxv}(x_i) = \frac{U_{surf}(x_i, D(x_i))}{\frac{1}{M} \ln[1 + (e^M - 1)\delta(x_i)e^{1-\delta(x_i)}]} \quad (4)$$

Table 2
The magnitudes of $\Phi(M)$ and M for the selected transects. Observed mean and max velocity refers to ADCP measurements.

Date	Flow condition	Transect	Number of passes	Observed mean velocity (m/s)	Observed maximum velocity (m/s)	Discharge (m ³ /s)	$\Phi(M)$	M
28.10.2014	Low	CNS0	single	1.08	2.61	86,088	0.414	-1.05
31.10.2014	Low		single	1.01	2.52	86,193	0.402	-1.20
30.04.2015	High	CNS3	single	1.62	3.53	142,550	0.459	-0.49
28.10.2014	Low		single	1.08	2.53	89,393	0.424	-0.92
31.10.2014	Low		single	1.24	2.87	84,908	0.432	-0.83
30.04.2015	High	A6	single	1.33	2.49	140,892	0.533	0.40
30.10.2014	Low		single	1.20	2.04	89,267	0.588	1.08
29.04.2015	High	S	single	1.78	3.20	90,578	0.556	0.68
30.04.2015	High		single	1.74	3.47	105,064	0.501	0.01
31.10.2014	Low		single	1.42	2.43	62,137	0.587	1.06
29.04.2015	High	N	single	0.37	0.72	27,503	0.522	0.27
30.04.2015	High		single	0.48	0.78	33,433	0.609	1.35
31.10.2014	Low		single	0.39	0.75	23,561	0.521	0.26
24.05.2017	High	Jatuarana	three	1.51	2.71	149,251	0.557	0.70
28.07.2017	High		three	1.55	2.85	148,851	0.546	0.56
08.02.2017	Low		three	1.36	2.35	120,119	0.579	0.97
24.05.2017	High	Careiro	three	1.25	2.17	23,828	0.577	0.94
27.07.2017	High		three	1.16	2.12	20,347	0.548	0.58
07.02.2017	Low		three	0.99	1.91	15,719	0.521	0.26

3. Theoretical methodology

Estimation of the velocity distribution U based on the Entropy probability density function was first developed by Chiu (1989) and then simplified by Moramarco et al. (2004). According to this approach, the Entropy-based velocity profile along the verticals can be determined as:

$$U(x_i, y) = \frac{U_{maxv}(x_i)}{M} \ln \left[1 + (e^M - 1) \frac{y}{D(x_i) - h(x_i)} \exp \left(1 - \frac{y}{D(x_i) - h(x_i)} \right) \right] \quad i = 1 \dots N_v$$

where U is the time-averaged velocity, $U_{maxv}(x_i)$ is the maximum value of U along the i^{th} vertical, x_i is the distance of the i^{th} sampled vertical from the left bank, $h(x_i)$ is the dip, i.e., the depth of $U_{maxv}(x_i)$ below the water surface, $D(x_i)$ the flow depth, y is the distance of the velocity point from the bed, and N_v is the number of verticals sampled across the river section. M is the entropic parameter, characteristic of the cross-section that can be estimated based on mean and maximum flow velocity, U_m and U_{max} , respectively, using the linear entropic relation (Chiu, 1989):

$$U_m = \left(\frac{e^M}{e^M - 1} - \frac{1}{M} \right) U_{max} = \phi(M) U_{max}$$

U_{max} is assumed as the maximum value of $U_{maxv}(x_i)$ sampled in the cross-sectional flow area.

For a gauged river site, $\Phi(M)$ is constant for all flow conditions, while for ungauged sites $\Phi(M)$ can be estimated as (Moramarco and Singh, 2010):

$$\phi(M) = \frac{\frac{1}{n} R^{1/6} / \sqrt{g}}{\frac{1}{k} \left[\ln \left(\frac{y_{max}}{y_o} \right) + \frac{h}{y_{max}} \ln \left(\frac{h}{D} \right) \right]} \quad (3)$$

where y_{max} is the location of U_{max} from the bottom and y_o is the datum where the velocity is equal to zero, k is the von Karman constant, R is the hydraulic radius and D is the maximum flow depth.

Whether at river site only surface velocities, $U_{surf}(x_i, D(x_i))$ are available, then $U_{max}(x_i)$ can be estimated as (Fulton and Ostrowski, 2008):

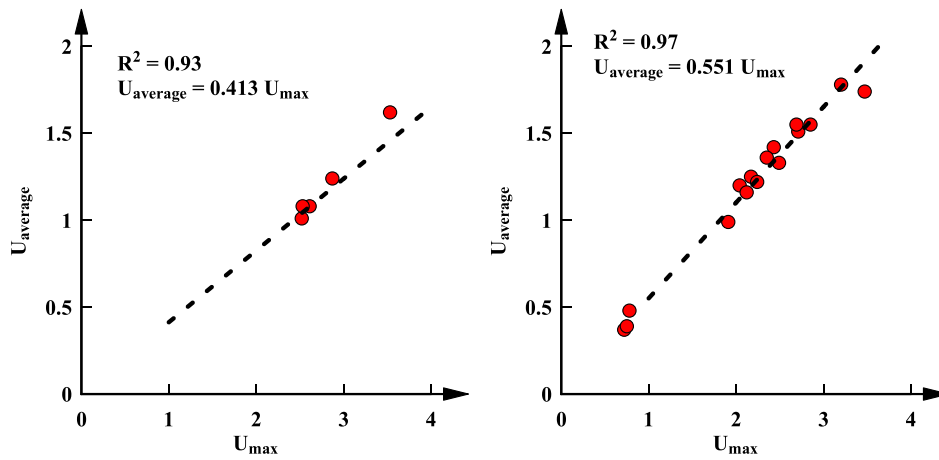
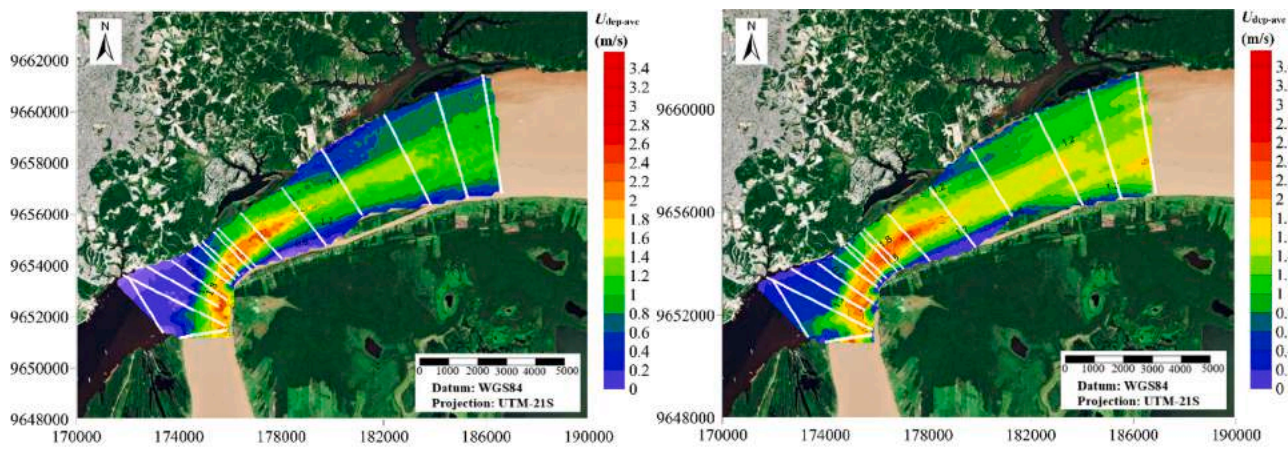
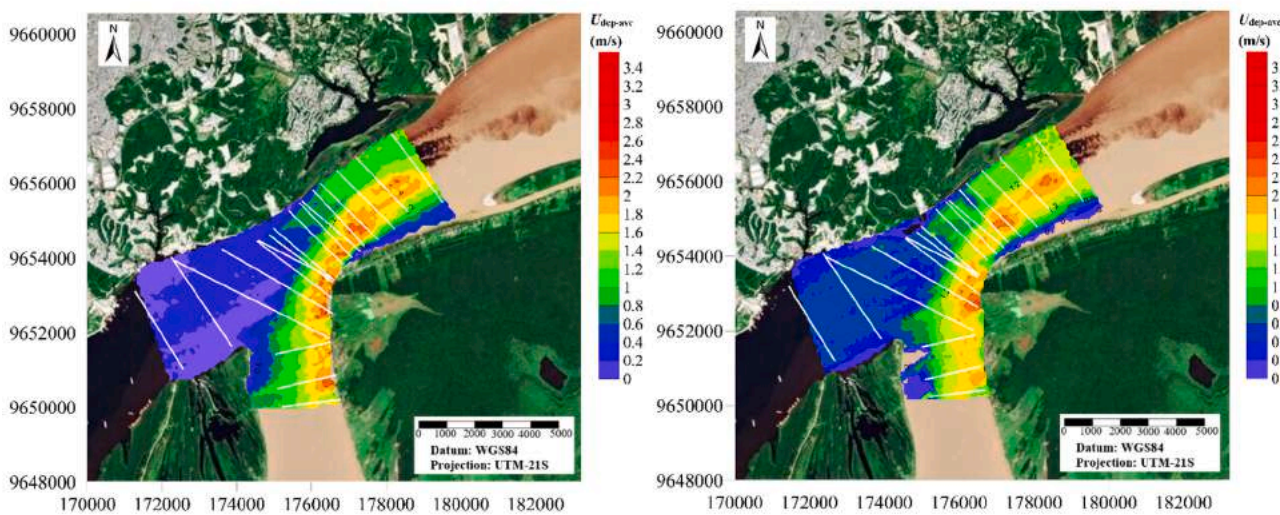


Fig. 2. The correlation between maximum and mean flow velocity.



a) Relatively high flow condition (left ADCP by Gualtieri et al. (2017,2018), right calculated data)



b) Low flow condition (left ADCP by Gualtieri et al. (2017,2018), right calculated data)

Fig. 3. Depth-averaged velocity distribution, confluence of Rio Negro and Rio Solimões based upon all velocity points at the surface. (a) Relatively high flow condition (left ADCP by Gualtieri et al. (2017), Gualtieri et al. (2018), right calculated data). (b) Low flow condition (left ADCP by Gualtieri et al. (2017), Gualtieri et al. (2018), right calculated data)

where $\delta(x_i) = \frac{D(x_i)}{D(x_i) - h(x_i)}$. Specifically, if $h(x_i) = 0$, it follows $\delta(x_i) = 1$ and, hence, $U_{maxv}(x_i) = U_{surf}(x_i, D(x_i))$. In this context, to estimate $\delta(x_i)$ the iterative procedure proposed by Moramarco et al. (2017) can be applied. The procedure holds for sites where $\Phi(M)$ is known. It consists of assigning an initial dip, $h(x_{i,p=1})$, where the maximum surface velocity occurs (p is the iteration). Herein, based on this initial value of dip, a laboratory distribution law of dip proposed by Yang et al. (2004) was applied across the river, and the $U_{maxv}(x_{i,p=1})$ was assessed by Eq. (4) for all verticals. $U_{max(p=1)}$ was identified as the maximum of $U_{maxv}(x_{i,p=1})$. Therefore, once $U_{maxv}(x_{i,p=1})$ is replaced in Eq.(1) it enables estimation of the depth-averaged velocities in the cross-sectional flow area, and for the first iteration the mean flow velocity, $U_{m(p=1)}$, can be evaluated by the velocity-area method. As a consequence, $\Phi(M_{com,p=1})$ can be computed by Eq.(2), using $U_{m(p=1)}$ and $U_{max(p=1)}$. The iteration continues until the error of $\Phi(M_{com,p})$ - $\Phi(M)$ reaches a magnitude less than 0.01. For more details, the reader can refer to Moramarco et al. (2017).

4. Results and discussions

4.1. Entropic parameter

For the confluence of Rio Negro and Rio Solimões, a value of M was obtained based on the observed data for both high and low flow conditions. Table 2 presents the magnitude of $\Phi(M)$ and M for the selected transects.

For the ADCP measurements campaigns, Fig. 2 highlights the correlation between maximum and mean velocity magnitudes, as a description of $\Phi(M)$, where $\Phi(M) = U_m/U_{max}$ (Eq. (2)). Transects CNS0 and CNS3 are located at the beginning and middle of the confluence zone, respectively, and for this reason they are considered to belong to the same group, as shown in Fig. 2a. Inspecting Fig. 2 and considering Table 2, one can infer that for CNS0 and CNS3 $\Phi(M)$ values are, on average, quite similar and equal to 0.413. This value is lower than the $\Phi(M)$ for transects N, S, A6, Careiro and Jatuarana which on average yielded $\Phi(M) = 0.551$. The difference can be ascribed to the mixing of two flows by a combination of different hydrologic and morphologic processes (Gualtieri et al., 2019). The number of measurements in terms of pairs (U_m , U_{max}) is not sufficient to establish a strong linear entropic relationship, however, the data provide information on flow dynamics. Indeed, a high $\Phi(M)$ value reflects diminished effects of banks and secondary currents on flow (Termini and Moramarco, 2020).

4.2. Comparison of predicted versus observed depth-averaged velocity and discharge

The Entropy theory was applied to estimate the velocity distribution for both high and low flow conditions at the confluence of Rio Negro and Rio Solimões. During the two campaigns of October 2014 (relatively low flow) and May 2015 (relatively high flow), the average velocity for Rio Solimões was 1.52 and 1.32 m/s, and for Rio Negro 0.4 and 0.39 m/s, for high and low flow conditions, respectively. Adopting the observed M at each transect, the patterns of the depth-averaged velocity during high (A) and low (B) flow conditions were estimated by using the ADCP measured data and the Entropy-based 2D model leveraging only the near-surface velocities provided by ADCP. The pattern of the Entropy-based depth-averaged velocities distribution was computed by minimizing the error on $\Phi(M)$ by varying the dip location, as proposed by Moramarco et al. (2017). Based on the data of all transects (16 and 19 transects for high and low flow conditions, respectively), the comparison is shown in Fig. 3 and as it can be seen the patterns (observed and computed), are quite similar. For both measured and calculated data, the spatial spacing of 0.001 degrees was applied for interpolating the data with Longitude range (degree): $-59.93441773 : -59.78454208$ and Latitude range (degree): $-3.150481462 : -3.048341513$ based on the Kriging tool available in Surfer® (Golden Software, LLC). Fig. 3 left

Table 3

Relatively high flow conditions: error percentage for discharge at selected transects.

Transect	Discharge (m ³ /s)		Discharge error (%)
	ADCP	Entropy	
CNS3	142,550	129,238	9.34
A6	140,892	126,716	10.06
S	104,209	90,578	13.08
N	31,091	27,503	11.54
Jatuarana	149,780	149,251	0.01
Careiro	23,995	23,828	0.01

(for the ADCP data), conducted based on the same dataset, is reported here to show the good agreement obtained by applying the Entropy approach, which correctly reproduces the same distribution of depth-averaged velocity for the five hydrodynamic zones in the confluence area.

According to Gualtieri et al. (2019): a) for the relatively high flow condition, the last two transects at the downstream part are located outside the CHZ, i.e. in the so-called recovery zone; b) the discharge and velocity rates for the Rio Solimões were always greater than those for the Rio Negro during both field studies. The difference in average velocity between the Negro and the Solimões at the junction corner was comparatively larger in relatively high flow conditions, resulting in different rates of momentum transfer during the field studies (Gualtieri et al., 2018). In terms of depth-averaged velocity, the normalized averaged error computed for all transects (i.e. 16 and 19 transects for high and low flow conditions, respectively) was on average 8.4% for both flow conditions, while in terms of discharge, it was 6.9% and 6.3% for high and low flow conditions, respectively. Table 3 presents the magnitudes of discharge together with the error percentage through some selected transects for relatively high flow conditions.

In terms of hydrodynamic characteristics, both the observed and calculated depth-averaged velocity data are in agreement with the descriptive model of hydrodynamic features at the confluence zone suggested by Best (1987), Best (1988). For both cases, five zones including stagnation zone at the upstream of CHZ, velocity deflection, separation region with recirculation, and maximum velocity zone in the CHZ and flow recovery in the downstream of the CHZ are detected in the same locations of the confluence zone. However, ADCP data available for this study, for a few cross-sections referred to one single transect, while repeated transects were collected over a few days. This condition limited the analysis of the confluence hydrodynamics features. However, the focus of the work is not to verify the robustness of ADCP measurements for the discharge estimation as a function of passes, but mainly to show how the Entropy model is able to assess discharge and simulate velocity profiles starting from the near-surface velocity observed by ADCP. Therefore, as regards Rio Negro and Solimões we have assumed the ADCP-based spatial velocity distribution to be unbiased for two reasons. First, Oberg and Mueller (2007) found that the uncertainty associated with a discharge measurement is better characterized by the duration of the measurement than the number of transects included in the measurement. In other words, the uncertainty of the measured discharge is more dependent on the exposure time of the instrument, than on the number of passes collected. Indeed, using one pass they found the percentage error from mean discharge to be less than 10%, and it decreases with the increase of exposure time. Notably, for an exposure time greater than 1000 sec, the uncertainty did not significantly change with the number of passes. Although our fluvial context is different from that investigated by Oberg and Mueller (2007), for Rio Negro and Solimões the exposure time, i.e. total time spent sampling the flow, is over to 1800s. Moreover, Despax et al. (2019) showed that exploring more cross-sections is more efficient to reduce the uncertainty than staying at the same cross-section and increasing the number of transects. In our case, we have some transects carried out on the same

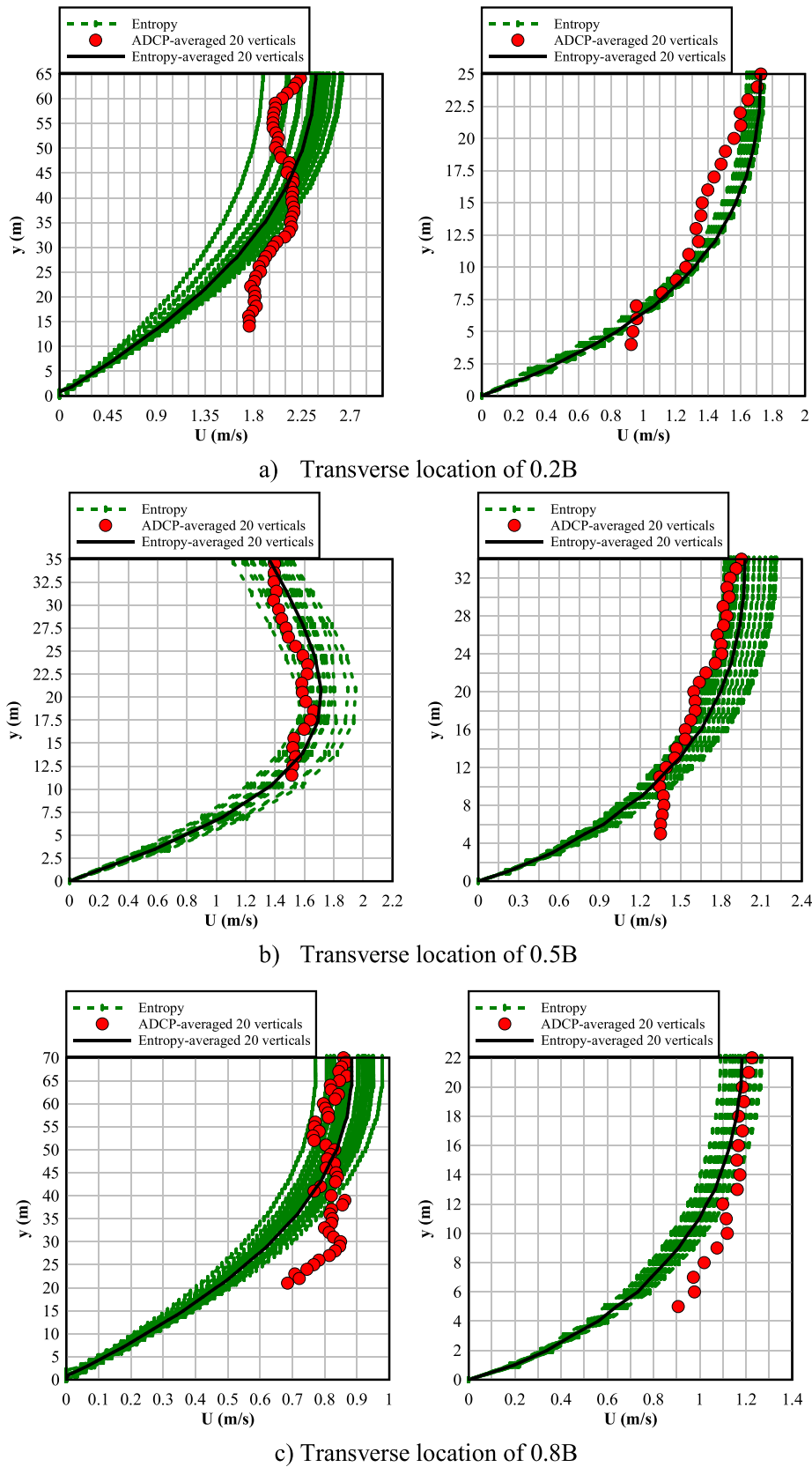


Fig. 4. Comparison of vertical distribution of the velocity magnitude at transverse locations of 0.2B, 0.5B and 0.8B, transects CNS3 (left) and A6 (right) between ADCP velocity data averaged for 21 ensembles and entropy-based velocity profiles.

Table 4

Coefficient of determination (R^2), and Root Mean Square Error (RMSE) in estimating the entropy-based velocity profiles at transverse locations of 0.2B, 0.5B and 0.8B, for all selected transects.

Transect	R^2 (for averaged 21 verticals)			RMSE (for averaged 21 verticals)		
	0.2B	0.5B	0.8B	0.2B	0.5B	0.8B
CNS3	0.62	0.81	0.58	0.228	0.116	0.126
A6	0.88	0.81	0.92	0.152	0.185	0.124
Jatuarana	0.94	0.90	0.96	0.107	0.226	0.068
Careiro	0.92	0.85	0.77	0.126	0.120	0.307
S	0.86	0.98	0.94	0.318	0.115	0.050
N	0.88	0.77	0.94	0.055	0.049	0.058

day that provides similar values of discharge, as can be inferred by Table 2, e.g. for measurements carried out on 28th October 2014 for CNS0 and CNS3 sites and on 30th April 2015 for CNS3 and A6. For the location of sites see Fig. 1. The second reason refers to Jatuarana and

Careiro river sites (see Fig. 1), where three passes are available on the same day and the comparison between the discharge “observed” by one pass with the averaged discharge from multiple passes, did not indicate any significant deviation (see supplementary material).

4.3. Comparison of predicted versus observed streamwise velocities

To compare observed and calculated velocity profiles for the same transect, velocity magnitudes were interpolated at regular intervals of 1 m. The velocity data were interpolated by Dplot® (Hydesoft Computing, LLC) over the entire cross-section. Notably, there are two options for interpolation in Dplot: Quadratic and Planar interpolation. The quadratic interpolation, used in this study, weights the result by the inverse of the square of the distance to the nearest control points. Quadratic interpolation, in general, produces a smoother surface than what is typically displayed with randomly-spaced points. In many cases, the resulting grid is a more accurate representation of the input than the

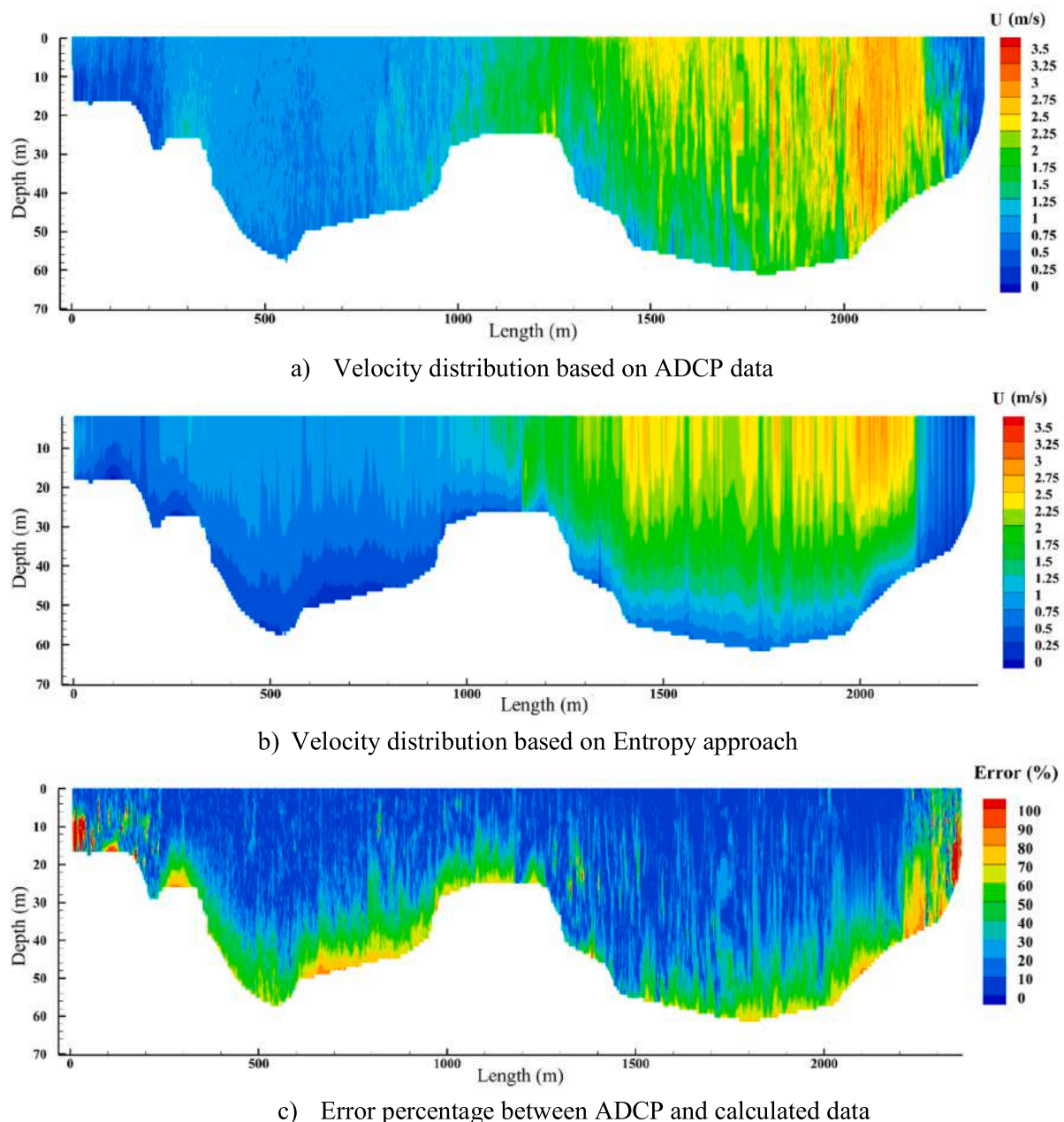


Fig. 5. Cross-sectional distribution of velocity for transect CNS3 in relatively high flow condition.

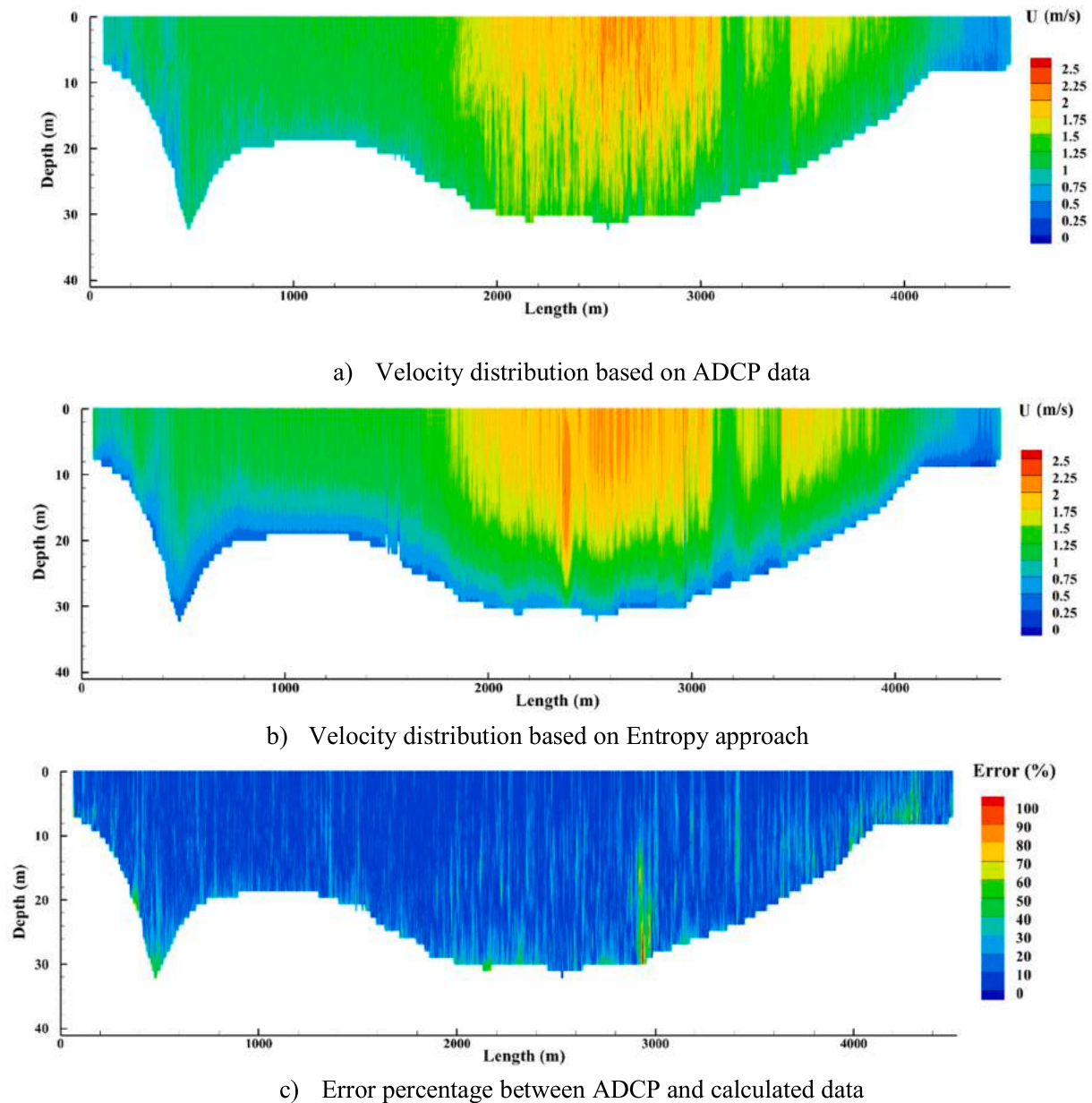


Fig. 6. Cross-sectional distribution of velocity for transect A6 in relatively high flow condition.

triangular-mesh plot. Herein, the results for transects CNS3 and A6 are presented and discussed, whereas the output graphs and evaluations for transects S through the Solimoes river and transect N through the Negro river are presented in the [Supplementary Material \(SM\)](#) file.

Transect CNS3 and A6 are located at the core and end of the confluence zone with a length of 2270 and 4280 m, respectively. To reduce the bias of the ADCP velocity distribution profiles collected from moving boats, different methods of smoothing were applied by researchers ([Muste et al., 2004a; Muste et al., 2004b, Petrie and Diplas, 2016](#)). Herein, with the aim to smooth the potential disturbances of measures and partially compensate for the lack of repeated passes, the velocity magnitude was averaged across 20 m (20 pixels 1 m wide) corresponding to a window of 21 verticals, that is 10 verticals to the left and right side of each selected vertical. For each transect, three verticals were selected at 0.2B, 0.5B and 0.8B, where B (m) is the channel width corresponding to the given transect ([Fig. 4](#)). For transect CNS3 in $x = 0.2B$ and 0.8B, and for transect A6 in $x = 0.5B$, the measured velocities close to the bed show higher values compared to computed velocities for

different reasons that deserve a dedicated analysis beyond the scope of this work. However, some possible causes could be suggested: a) post-processing on ADCP raw data in terms of smoothing and identification of river bottom ([Muste et al., 2004a; Muste et al., 2004b](#)), b) secondary currents related to the density difference between the tributaries close to the bed within the CHZ ([Bahmanpouri et al., 2017; Gualtieri et al., 2019; Gualtieri et al., 2020](#)), and c) the dip formulation adopted in the entropy-based model which is based on laboratory data ([Moramarco et al., 2004](#)). Flow dynamics at the confluence is also likely affected by the bed morphology ([Ianniruberto et al., 2018](#)). [Table 4](#) presents the statistical analysis for these verticals, indicating the relatively high determination coefficients and low RMSE. For Jatuarana and Careiro sites, the analysis refers to the velocity distribution averaged along the three passes.

[Figs. 5 and 6](#) compare the cross-sectional velocity distribution for the ADCP measurements and the velocities computed by Eq. (1) for transects CNS3 and A6, respectively. Note that the calculated velocity is based on considering only the near-surface velocity measurements and was

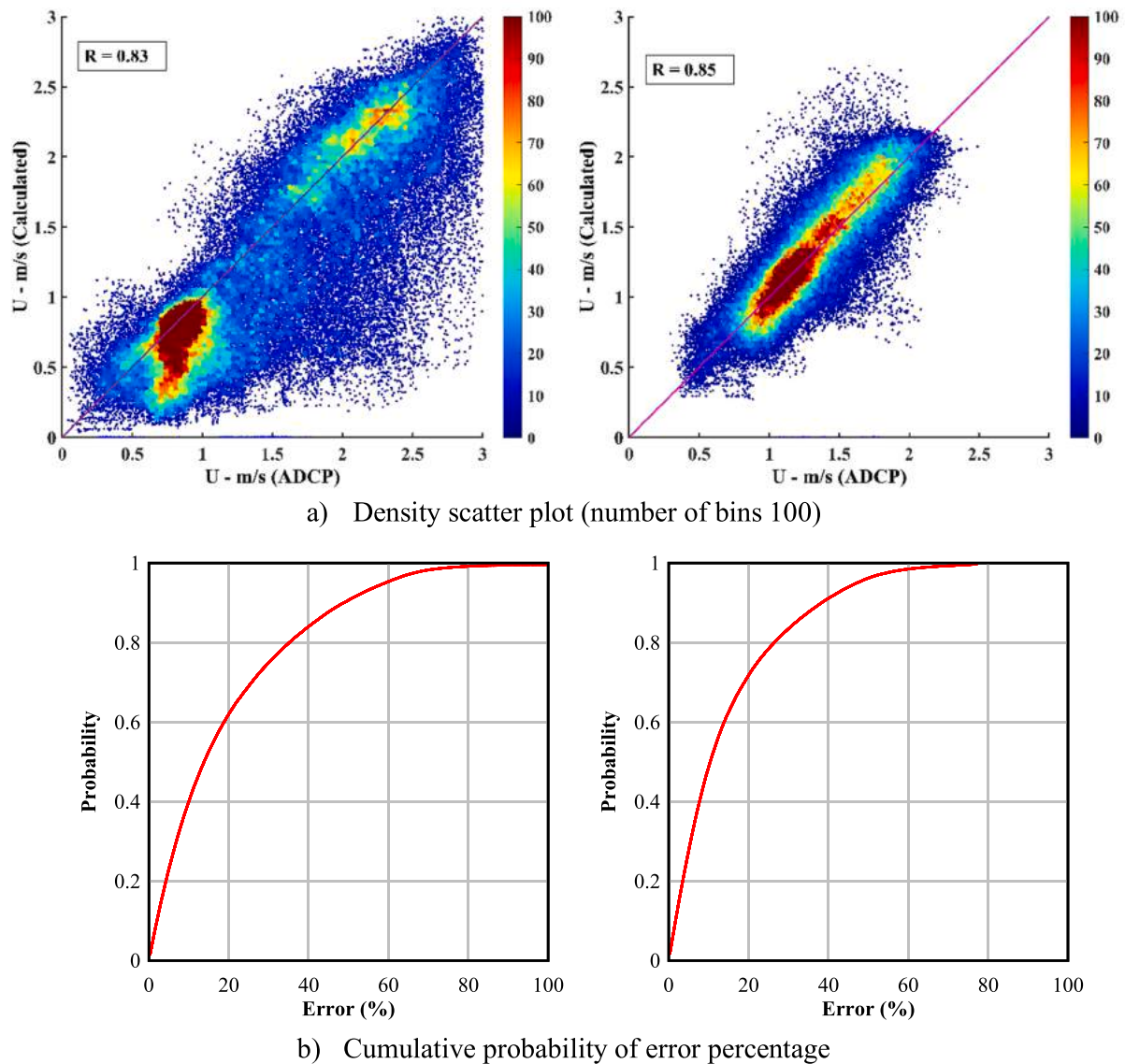


Fig. 7. Error analysis for transect CNS3 (left) and A6 (right), number of cells: 93,983 (CNS3) and 84,793 (A6). The legend refers to the density of points.

interpolated with the cell size of 1 m as for ADCP measurements. According to Figs. 5 and 6, the largest percentage error between the ADCP and calculated data is observed near the bed, where velocities and volumetric water flux tend to be at a minimum; so that there is no significant effect on estimating the depth-averaged velocity and consequently on the total discharge.

For sake of clarity, density scatterplots comparing observed and predicted velocities for high flow are shown in Fig. 7. A high percentage of data close to the centre suggests good results with $R = 0.83$ and 0.85 for transects CNS3 and A6, respectively (Fig. 7a). Further, the cumulative probability of percentage normalized error suggests that 75% of the total data (70487 out of 93983) have an error rate less than 30% for transect CNS3 and that 80% of the data (68000 out of 84793) have an error rate of less than 22% for transect A6 (Fig. 7b). As a consequence, for relatively high flow the computed discharge for CNS3 and A6 transects was found not to exceed 12%, as shown in Table 3.

Fig. 8 presents the density scatter plot of the ADCP velocity data versus the estimated velocity data at ensembles (averaged for 21 verticals) with the transverse location of 0.2B, 0.5B and 0.8B for all transects. Table 4 reports the statistical analysis for selected transects. As can be seen, regardless of transect location, the coefficient of determination is

larger than 0.6 and the Root Mean Square Error is less than 0.32.

5. Transects Jatuarana, Careiro, S and N – Supplementary Material

The results for transects Jatuarana, Careiro, S and N have been presented in the Supplementary Material (SM) file. It is worth noting that for Jatuarana and Careiro sites, three ADCP passes were carried out along the transect on the same day. For them, we have compared the discharge estimated at each pass and, as can be inferred from Table S1, no significant deviation observed between the discharge averaged on three passes and that referring to a single pass. Moreover, as can be seen from Figs. S1 and S4 results referring to shapes of the averaged entropy-based velocity profiles are satisfactory for Jatuarana and Careiro, respectively, considering both single and multi passes.

Cross-sectional velocity distribution graphs demonstrate well-estimated results by the Entropy approach (Figs. S2, S5, S8, S11). The statistical analysis for the vertical velocity ensembles based on averaged 21 windows highlights the correlation of coefficient of R larger than 0.95, 0.88, 0.93 and 0.83 for transects Jatuarana, Careiro, S and N, respectively (Figs. S1, S4, S7, S10). Overall, the results for these four

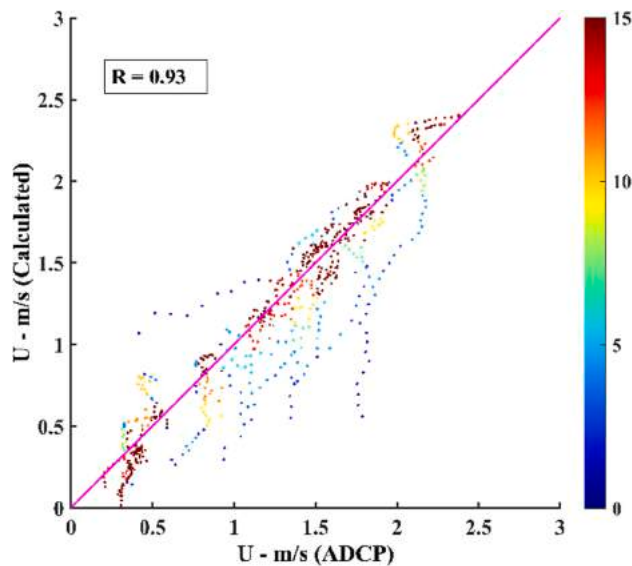


Fig. 8. Density scatter plot for the averaged velocity along verticals (20 m) at the transverse location of 0.2B, 0.5B and 0.8B, all selected 6 transects, number of cells: 648 (number of bins 15).

transects show the correlation of coefficient of $R = 0.78, 0.79, 0.81$ and 0.76 , respectively (Figs. S3a, S6a, S9a, S12a). The cumulative probability of error percentage reveals a high rate of accuracy for large rivers with transect lengths and depths up to approximately 5 km and 65 m, respectively. For example, for transects Jatuarana and Careiro which are located out of the confluence zone, 70% of data have an error percentage of less than 19% and 20%, respectively (Figs. S3b, S6b). Even for these transects, the difference in velocity ensembles for the three transects should be ascribed, on one hand, to raw data and topography and, on the other hand, to the complex dynamic of flow.

6. Discussion and conclusions

The cross-sectional velocity distribution along with the discharge assessment is investigated using only near-surface velocities observed at the confluence of Rio Negro and Rio Solimões network system in the Amazon River using ADCP measurements. Herein, an Entropy-based model, as a first attempt, is applied for this type of river system to predict 2D cross-sectional flow velocity distribution, relying on near-surface velocity measurements recorded by ADCP.

Comparisons between measured and predicted velocity profiles show that the method provides results that might be of potential benefit in simplifying monitoring procedures in large rivers, mainly for high flow conditions. Vertical and cross-sectional distributions of flow velocity at different parts of the confluence zone, as well as upstream and downstream of the CHZ and Careiro channel, are in accordance with the observed ADCP data with the error percentage less than 15%, suggesting acceptable results with the purpose of estimating discharge. The large magnitudes of errors on cross-sectional velocity distribution are mainly detected at the near-bed area which, considering the low values of velocity at the bottom, has no notable impact on the total discharge calculation. The difference between the observed and estimated ensembles, especially near the bed, could be associated with several factors such as the post-processing on ADCP raw data, the secondary currents related to the density currents flow close to the bed from the Solimões side to the Negro side of confluence, and the robustness of laboratory dip formulation used in the Entropy model. All these aspects are already objects of a work in progress.

The Entropy-based cross-sectional velocity distributions are compared against the ADCP data for different transects. It should be noted that in transects located at the confluence area there are larger

values of velocity at the right-side, which leads to transferring the momentum from the right side to the left side of the confluence zone (Rhoads et al., 2009), and this can be inferred by visual inspection of the computed cross-sectional velocity distribution. For transects at Rio Solimões, Rio Negro, Jatuarana (where the flow completely recovered) and also in Careiro, the entropy-based velocities may deviate significantly from the ones observed at channel bottom. However, this result is somehow expected and in accordance with Moramarco et al. (2011), who found for gauged sites along the Po River similar deviations in terms of velocity observed near the river bed. Nevertheless, considering that in the lower portion of the flow area the velocity profiles tend to zero, the impact of this discrepancy in the velocity computation has a limited influence on the estimation of cross-sectional mean flow velocity and discharge. Indeed, for all investigated transects, more than 70% of values of velocity cells have an error lower than 30% and the error in estimated discharge did not exceed 15% (see Table 3).

For ensembles at the transverse distance of 0.2B, 0.5B and 0.8B of the six selected transects, the statistical analysis shows a high-level accuracy in computing the entropy-based velocity profiles, as reported in Table 4. Increasing the number of verticals (ensembles) to 40, a negligible impact on the entropy-based model accuracy is found. As can be inferred from Table 4, for all transects the coefficient of determination, R^2 , ranges between 0.62 (transect CNS3) – 0.98 (transect S), while the Root Mean Square Error does not exceed 0.32. Therefore, results suggest a reasonable level of accuracy for the Entropy approach to estimate the velocity magnitude, especially outside the confluence zone. It is worth noting, the analysis for the Jatuarana and Careiro sites, where three passes were available on the same day, shows that the number of passes does not significantly influence the velocity distribution at the three investigated distances of 0.2B, 0.5B and 0.8B, as can be inferred from Figs. S1 and S4.

Finally, it is worth noting that remote sensing methodologies (e.g. satellite observation-based methods) are getting more popular and commonly used to estimate flow discharge instead of costly and time-consuming devices such as ADCP. Moramarco et al. (2019) demonstrated that discharge rate based on satellite observations led to an error percentage, on average, of about 30% by coupling MODIS and Altimeter. In addition, Fulton et al. (2020) reported that estimating the discharge rate based on radar data (QCam) led to an error percentage of around 20%. Therefore, satellite observations as input for the entropy model would be an interesting subject for future studies, particularly for inland waters. However, using ADCP data as input for Entropy theory provides lower errors for discharge estimate, lower than 15%, supporting to invest in time-consuming, costly, and labour-intensive measurements.

Overall, the findings of the present research are of particular interest for the hydrodynamics of very large rivers as Rio Negro and Rio Solimões/Amazon.

CRedit authorship contribution statement

Farhad Bahmanpouri: Conceptualization, Validation, Investigation, Methodology, Formal analysis, Writing – original draft. **Silvia Barbetta:** Investigation, Visualization, Formal analysis, Writing – review & editing. **Carlo Gualtieri:** Investigation, Validation, Visualization, Formal analysis, Writing – review & editing. **Marco Ianniruberto:** Investigation, Visualization, Formal analysis, Writing – review & editing. **Naziano Filizola:** Visualization, Formal analysis, Writing – review & editing. **Donatella Termini:** Investigation, Visualization, Formal analysis, Methodology, Writing – review & editing. **Tommaso Moramarco:** Conceptualization, Supervision, Investigation, Methodology, Formal analysis, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jhydrol.2021.127404>.

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