

Entropic model application to identify cross-sectional flow effect on velocity distribution in a large amplitude meandering channel

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ABSTRACT

Literature shows the application of the entropy concept in open channels flow. No existing research has been addressed to the application of the entropy concept for evaluating the effect of secondary flow which especially develops in curved channels. The purpose of this paper is to explore the possibility to evaluate the effect of the secondary flow on the stream-wise velocity distribution starting only from the knowledge of the surface velocity and the entropic concept application. The analysis is conducted with the aid of data collected in a large-amplitude meandering laboratory flume for two different values of the width-to-depth ratio. Results show that the location of the maximum velocity varies not only along the bend but also along the water depth, depending on the relative entity of the cross-circulation and the convective flow acceleration. It has been also verified that the entropic parameter varies along the bend as a function of the width-to-depth ratio. The application of the entropic model to the examined cases has allowed the verification that not only the shape of the velocity profile but also the bed shear stress evolution along the bend could be estimated by the entropic model starting from the knowledge of the surface velocity only.

1. Introduction

As it is well known, in a river bend secondary flow develops by the combination of the longitudinal flow with the cross-sectional motion, which is induced by the channel's curvature (Yalin, 1992) and is due to the interaction of the forces associated with centrifugal acceleration, transverse water surface slope and bed shear stress.

Secondary flow exerts an important role in several processes involving in a natural stream, such as the dispersion of nutrients and/or of chemical substances as well as the trapping or transport of sediments. In particular, the cross-sectional motion contributes both to shift the core of the maximum velocity (among others Leschziner and Rodi, 1979; Johannesson and Parker, 1989; Wilson et al., 2003) and of the boundary shear stress along the bend, thus modifying and reshaping the channel's bed and platform (see as an example de Vriend and Gendolf, 1983; Whiting and Dietrich, 1993; Knaapen et al., 2001; Blanckaert and Graf, 2004; Roca et al., 2009; Termini and Piraino, 2011), and to enhance the mass mixing and the energy losses, thus affecting the channel's conveying capacity (Camporeale et al., 2005; Nepf, 2012; Gurnell, 2014; De Serio et al., 2018).

The estimation of the rate of mass and momentum transport requires the knowledge of the velocity distribution along the channel. Thus, the identification of the feedback between the cross-sectional motion and the vertical profiles of the longitudinal velocity along the bend is fundamental in simulating dispersion and mixing processes.

In meandering bends, which are characterized by a continuously changing curvature, the cross-sectional motion varies from section to

section. Thus, unlike straight channels, the identification of the evolution of the cross-sectional motion along the bend is especially important to estimate the variation of the turbulent activity of flow and the distribution of the downstream flow velocity along the bend itself.

According to Yalin (1992), in meandering bends the cross-sectional motion can be considered as composed by the curvature-induced cross-circulation, which is characterized by zero depth-average, and the convective component, which is due to the changing channel's curvature and produces a net flux of mass in the transversal direction (see Fig. 1). Literature (among others Johannesson and Parker, 1989; Whiting and Dietrich, 1993; da Silva et al., 2006; Termini, 2009; Termini and Piraino, 2011) indicates that the role of the aforementioned components cannot be analyzed independently of the width-to-depth ratio and the channel's sinuosity. Da Silva et al. (2006) focused on the case of large width-to-depth ratio and analysed the influence of channel's sinuosity on the flow pattern in flat-bed sine-generated laboratory flumes characterized by different values of the deflection angle at the crossover, θ_0 (i.e. 30° , 50° , 70° , 90° , 110°). As a result, they verified that the channel's sinuosity, especially affects the location of the convectively accelerated/decelerated flow along the channel. For a given channel's sinuosity, the width-to-depth ratio is the most important parameter affecting the entity of the two aforementioned cross-sectional motion components, being the cross-circulation more significant for "small" width-to-depth ratios than for "large" width-to-depth ratios (see also in Whiting and Dietrich, 1993; Lanzoni et al., 2006; Blackheart et al., 2008; Termini, 2015).

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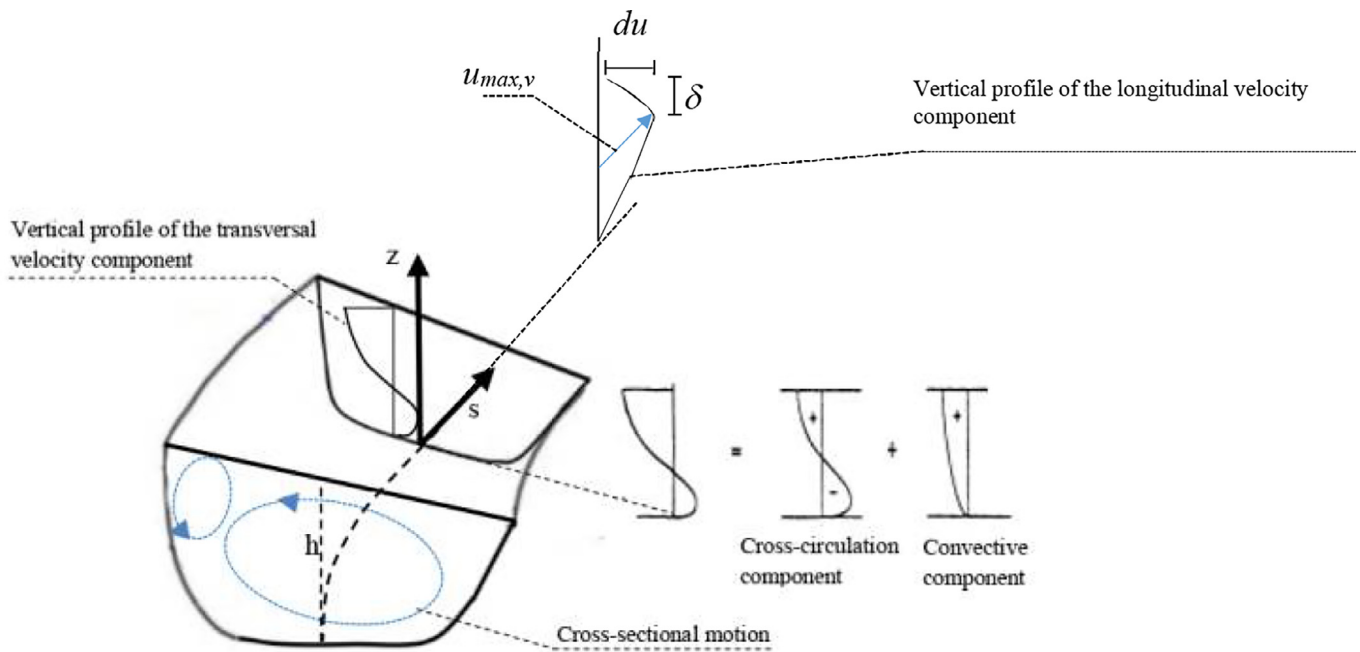


Fig. 1. Definition sketch ($u_{max,v}$ = maximum longitudinal velocity; $du = u_{max,v} - u_{s,v}$).

It should be also considered that several studies conducted in straight channels (among others Steffler et al., 1985; Cardoso et al., 1989; Nezu and Nakagawa, 1993; Yang et al., 2004; Troiani et al., 2004; Yang and Lee, 2007; Najafabadi et al., 2018) have related the transport of momentum due to the secondary flow to the so-called “dip-phenomenon”, for which the position of the maximum longitudinal velocity occurs below the water surface (as schematized in Fig. 1). Moramarco et al. (2017) investigated the spatial distribution of the velocity-dip in sites along straight natural channels by leveraging the entropy theory. They identified the distance of the maximum longitudinal velocity from the free surface (velocity-dip δ in Fig. 1 and showed how turn the surface velocity into depth-averaged velocity. Termini and Moramarco (2018) analyzed the u -profiles measured only at the apex and inflection sections of a large amplitude meander wave. From their analysis, it appeared clear that the dip-phenomenon occurs in all the considered sections but with different entity. This result suggested that the variation of the velocity-dip along the channel could be strongly related to the evolution of the cross-sectional motion along the bend. Termini and Moramarco (2018) also verified that the velocity-dip δ varies as a function of the width-to-depth ratio W/h and assumes, in average, a value of around 53% of the local flow depth for “small” width-to-depth ratio and of around 34% for “large” width-to-depth ratio. This suggests that the velocity-dip is evident for small width-to-depth ratios but it is also present for large width-to-depth ratio. This is consistent with previous literature findings obtained in straight channels (among others Sarma et al., 1983; Tachie et al., 2003).

Successively, Termini (2019) identified an equation which relates the main controlling parameters of the flow processes (i.e. the local channel’s curvature and the width-to-depth ratio) to the local velocity-dip. Specifically, she demonstrated that, for a given value of the width-to-depth ratio, the velocity-dip δ varies along the meander wave as a function both of the normalized distance from the outer bank, ξ/W (with ξ =coordinate in the transversal direction – see Fig. 2), and of the local channel’s curvature which could be parameterized by the local deflection angle θ . She also proposed a relation which directly relates the velocity-dip δ to the parameters θ and W/h .

From the aforementioned considerations, it is clear that although literature researches have allowed significant progress in under-

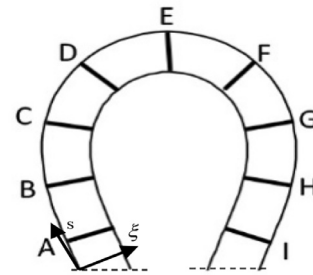


Fig. 2. Measurement sections considered for the present analysis.

standing meander flow dynamics, the mechanism of the velocity-dip formation and the relation with secondary motion is still poorly understood.

In this context, the main aim of the present work is to explore the link between the velocity-dip and the secondary motion and the possibility to explain it by leveraging the entropy theory, starting from the monitoring of surface velocity. This allows us to answer to the question if it is possible to evaluate the feedback effect of cross-sectional flow on the longitudinal velocity and shear stress distributions in curved channels starting from the monitoring of surface velocities only. Such a possibility could be of great interest for hydrological practices considering that some fundamental insights could be inferred along natural channels in terms of flow erosion and shear stress distribution just sampling the surface velocity.

This study has been inspired by the recent findings obtained by Moramarco et al. (2017) and Termini and Moramarco (2018) and is motivated by the fact that, although considerable attention has been given to the application of entropy concept to estimate either the velocity distribution in open channels (Moramarco and Singh, 2010; Termini and Moramarco, 2017) or other specific parameters (Sterling and Knight, 2002; Atieh et al., 2015a; 2015b; Atieh et al., 2017; Sheikh and Bonakdari, 2015; Kumbhakar and Ghoshal, 2016a; 2016b; Kumbhakar et al., 2016; Atieh et al., 2017; Kumbhakar et al., 2017; Kumbhakar and Ghoshal, 2016; Kundu, 2017; Singh et al., 2017; Ebtehaj et al., 2018; Khozani and Bonakdari, 2018;

Kazemian-Kale-Kale et al., 2018), no existing research addresses the application of the entropy concept to estimate secondary flow effect especially in curved open channels.

Thus, the present work is part of a broader research project aimed at examining the meander dynamics and the applicability of the entropic concept to predict the effect of secondary currents on the primary velocity distribution and the flow discharge just using surface velocity observations. Specifically, the present study complements previous studies conducted by Termini and Moramarco (2018) restricting the attention to a high-amplitude meander bend, for which the role of curvature and of its downstream variation on the velocity pattern are especially important. The aforementioned general goal is achieved by three specific objectives: (1) to investigate, for large and small width-to-depth ratios, if the velocity-dip and its variation along the bend could be related to the effect of the cross-sectional flow evolution along the bend; (2) to verify how such an effect could be identified by applying the entropy concept by the spatial distribution of entropic parameter M , characterizing the velocity profile shape; (3) to analyze if the entropy theory is able to provide useful insights in terms of velocity-dip distribution along straight and bended channels, just starting from the monitoring of surface velocity across the site.

The paper is organized as follows: Section 2. describes the experimental data used and the experimental results are discussed in Section 3. The application of the entropic parameter are presented in Section 4. Discussion is reported in Section 5. and finally conclusion are summarized in Section 6.

2. Materials and methods

2.1. Experimental data

The data used in the present work were collected in the ambit of a broader research program aimed to analyze the flow pattern in a high-amplitude meandering channel. In particular, data taken from Termini (2009) have been used. The data were collected in a meandering channel 25.12 m long following the sine-generated curve with a deflection angle at the inflection section $\theta_o = 110^\circ$. The channel cross-section is rectangular with width $W = 0.50$ m; the fixed vertical sidewalls are of plexiglass and the bed is of quartz sand (d_{50} =medium sediment diameter=0.65 mm).

In order to isolate the effect of channel's curvature, in the present work only the experimental runs conducted over the rigid and flat bed (runs called as RB-1 and RB-2 in Termini, 2009) have been considered. In particular, RB-1 run (hereon indicated as Run1) was conducted with a flow discharge of $Q = 0.019$ m³/s (channel-averaged flow depth $h = 5.5$ cm; $W/h = 9.09 < 10$); RB-2 run (hereon indicated as Run2) was conducted with a flow discharge $Q = 0.007$ m³/s (channel-averaged flow depth $h = 3.0$ cm, $W/h = 16.67 > 10$). Herein, in accordance with previous researches (among others Nezu and Rodi 1985; Yalin, 1992; da Silva et al., 2006; Termini and Piraino, 2011) $W/h < 10$ indicates “small” width-to-depth ratio and $W/h > 10$ indicates “large” width-to-depth ratio.

During each run, the water surface was measured using a Profiler Indicator PV09 (Delft Hydraulics) with a precision of 0.1 mm. The instantaneous local longitudinal, $u(t)$, and transversal, $v(t)$, velocity components were measured in sections spaced about 50 cm or so apart in points spaced of 1–1.5 mm along the verticals of nine transverse abscissas. The flow velocity measurements were carried out by using a 2D side-looking Acoustic Doppler Velocimeter (ADV - by SonTek Inc.), identified symmetrically to the channel axis (see details in Termini, 2009). Bearing in mind the specific aim of the present work, our attention is focused only on the longitudinal velocity component, $u(t)$, measured in the central bend between the two consecutive inflection sections A and I indicated in Fig. 2. By using the time series of the measured longitudinal flow velocity component, $u(t)$, the corresponding time-averaged longitudinal velocity, u , was estimated along each measurement vertical of

the considered sections. The analyses reported in the present work have been performed by considering the local reference system reported in Fig. 2 (s =coordinate in the longitudinal direction; ξ =coordinate in the transversal direction).

3. Experimental results

3.1. Variation of the u -profile shape along the bend and relation with the velocity-dip

As indicated in the “Introduction”, the results obtained in the previous works (Termini and Moramarco, 2018) have suggested that the formation of the velocity-dip and its variation along the bend could be related to the evolution of the cross-sectional motion whose entity also varies as a function of the width-to-depth ratio (among others Whiting and Dietrich, 1993; Termini, 2015).

This also suggests that the knowledge of the variation of the u -profile shape along the bend could be important in order to identify the evolution of secondary motion along the bend itself.

For this reason, in the present work the measured u -values have been first used in order to analyze how the shape of the u -profiles varies along the bend for both the considered width-to-depth ratios. The u -profile shape has been examined, for each measurement vertical v , by estimating not only the velocity-dip (i.e. the distance δ of the local maximum velocity $u_{max,v}$ from the free surface) but also the difference du between the local maximum value of the velocity $u_{max,v}$ and the local free-surface velocity $u_{s,v}$ (see the scheme of Fig. 1).

Figs. 3 and 4 report, respectively, the distributions along the bend of the difference $du = u_{max,v} - u_{s,v}$ and of the velocity-dip δ , normalized by the channel-averaged flow depth h , obtained for both the width-to-depth ratios. It can be observed that, generally, du assumes for $W/h < 10$ values lower than those for $W/h > 10$; the normalized velocity-dip, δ^* , assumes for $W/h < 10$ values greater than those obtained for $W/h > 10$. In particular, Fig. 3 indicates that, for both the width-to-depth ratios, du increases along the inner (right) bank (i.e. passing from section A to section I); along the outer (left) bank a different behavior can be observed between the case of $W/h < 10$ and the case of $W/h > 10$. In the case of $W/h < 10$, the lower values of du , which are found close to the inner bank of the inflection section A, gradually cross the channel reaching the outer side of section D (bend entrance) at a certain distance from the outer bank itself. It should be noted that section D is slightly downstream of the section where the tangent to the inflection section intersects the outer bank. Then, the lower values of du are maintained close to the outer (left) bank until to reach the inflection section downstream (section I). In the case of $W/h > 10$ the lower values of du are also found close to the inner (right) bank of section A, but they cross the channel and reach the outer side at section C, which is slightly upstream of the section where the tangent to the inflection section intersects the outer bank; passing from section C to section D they are maintained close to the outer (left) bank. Then, the difference du increases in value and the lower values are obtained not too close to the outer bank until to reach section F; from this section to the inflection section downstream (section I) the lower values of du are found close to the outer bank. The comparison between Figs. 3 and 4 highlights that, generally, the lower values of δ^* are found where the lower values of du are obtained. A different behavior between the case of $W/h < 10$ and the case of $W/h > 10$ can be observed also for δ^* along the outer (left) bank. In particular for $W/h < 10$ the normalized velocity-dip δ^* assumes the higher values close to the outer bank of the section D (bend entrance) and between sections G and H (i.e. close to the bend exit) while low values of the velocity-dip are found close to the apex section. Thus, in this case, δ^* increases close to the outer (left) bank passing from section A (where almost constant values of the velocity-dip occur) to section D, then it slightly decreases until to reach the apex section; passing from the apex section to the bend exit (between sections G and H) it slightly increases and then it decreases until to reach the inflection section downstream I where it assumes almost constant

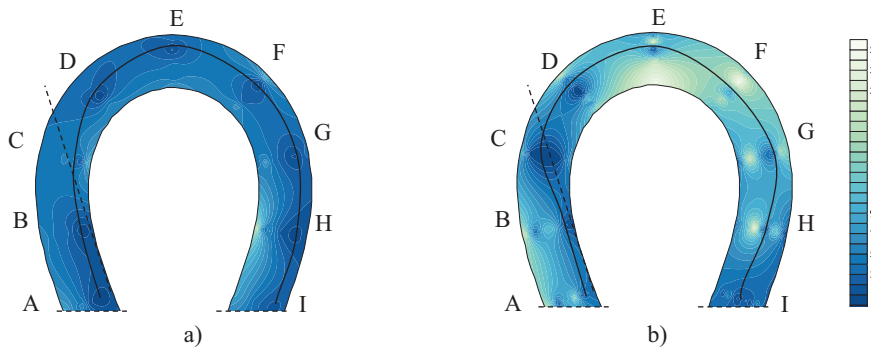


Fig. 3. Map contour of the difference $du = u_{max,v} - u_{s,v}$ (cm/s): a) $W/h < 10$; b) $W/h > 10$ (the black line indicates the location of lower values of du).

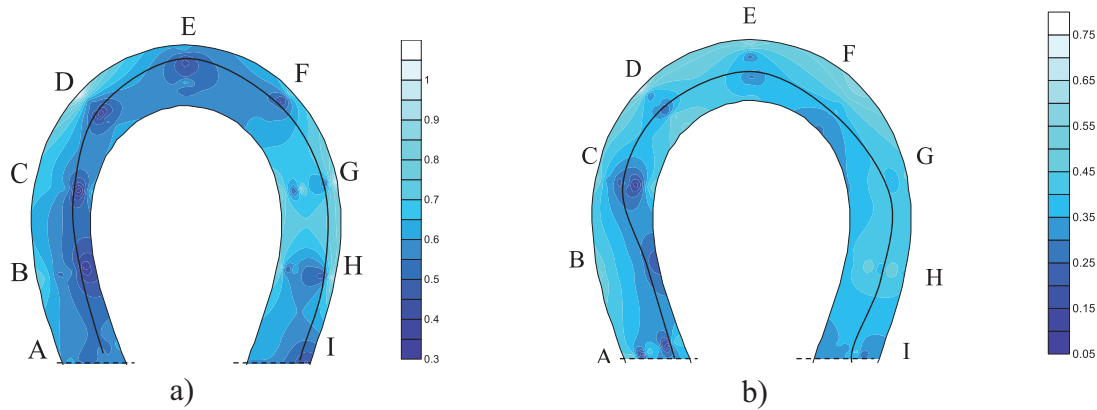


Fig. 4. Map contour of the normalized velocity-dip $\delta^* = \delta/h$: a) $W/h < 10$; b) $W/h > 10$ (the black line indicates the location of lower values of δ^*).

low values. For $W/h > 10$, upstream of the apex section the normalized velocity-dip δ^* assumes a behavior similar, but less accentuated, than that for $W/h < 10$. At downstream of the apex section, δ^* assumes high values, with a peak close to the outer bank of section F (bend exit), until to reach the inflection section downstream I. The different behavior observed between $W/h < 10$ and $W/h > 10$ could be related to the fact that the two components of the transversal velocity (i.e. the cross-circulation and the convective acceleration) assume a different relative importance in the two considered cases, as verified by Termini (2009) by using the same data of the present work and by Termini and Piraino (2011). In particular, their results indicated that, for $W/h > 10$ the convective acceleration component is more significant than $W/h < 10$; in this last case, the cross-circulation component attenuates the contribution of the convective acceleration component. Furthermore, comparing Figs. 3 and 4, it's evident that the maximum du occurs at the outer bank of sections D and E where the maximum velocity-dip occurs, while at the inner bank the phenomenon is the opposite. In other words, the cross-circulation component increases its effect moving from the inner to the outer bank as the channel's curvature increases, driving the location of the maximum velocity towards the bed near the outer side of section D.

In summary, Figs. 3 and 4 show that the difference du assumes low values where also the normalized velocity-dip assumes low values and thus where the maximum velocity is closer to the free surface. The difference in the shape of the u -profiles between $W/h < 10$ and $W/h > 10$ occurs especially in the region close to the outer bank downstream of section D, where for $W/h > 10$ du assumes values higher than those obtained for $W/h < 10$. This means that for $W/h > 10$ the u -profiles are strongly concavely curved near the free surface. The results also suggest that the u -profile assumes a more concave shape near the free surface for lower water depths.

As consequence of the aforementioned behavior, the location of the maximum velocity $u_{max,v}$ varies along the bend, for both W/h , moving from the inner (right) bank to the outer (left) one as one passes

from section A to section E, as Fig. 5 shows. At the bend entrance (section D), while for $W/h > 10$ the higher value of the maximum of velocity is found close to the outer bank, for $W/h < 10$ it is still maintained far from the outer bank itself. Such a different behavior can be explained by the fact that a counter-rotating circulation cell (see Fig. 1) initiates in the outer-bank region of the bend entrance (i.e. close to section D) as demonstrated by Termini and Piraino (2011). Thus, especially along the channel reach between sections D and F the action of the central-region circulation cell is limited by the presence of the counter-rotating circulation cell.

Figs. 3 and 4 give also an indication on the redistribution of the maximum velocity not only in the stream-wise direction (as Fig. 5 shows) but also in the vertical one. It is clear that the higher values of $u_{max,v}$ cross the bend passing from the inner bank of the inflection section A to outer bank of the inflection section I as well as the lower values of the normalized velocity-dip δ^* ; this suggests that the higher values of the maximum velocity, $u_{max,v}$, are close to the free surface. In the case of $W/h > 10$, downstream of the apex section, where high velocity-dip values occur, such a behavior is less evident, thus indicating that the maximum velocity can be also found far from the free surface along the bend-exit reach. This could be related to the fact that, in the case of $W/h > 10$, the effect due to the cross-circulation on the longitudinal velocity is not significant and decreases especially along the bend-exit reach (Yalin, 1992; Da Silva et al., 2006; Termini, 2009). Furthermore, focusing the attention to the region close to the outer bank, it can be observed that, for both the width-to-depth ratios, passing from the inflection section upstream (section A) to the bend entrance (section D), the normalized velocity-dip assumes high increasing values with a peak around the outer bank of section D, meaning that there the maximum velocity is localized near the bed. The observed behavior also suggests that the pattern of the velocity-dip reflects the effect of circulation cells due to the cross-sectional motion. Indeed, where the velocity-dip is minimum (sections B and C of Fig. 4) only the effect of a central-region cir-

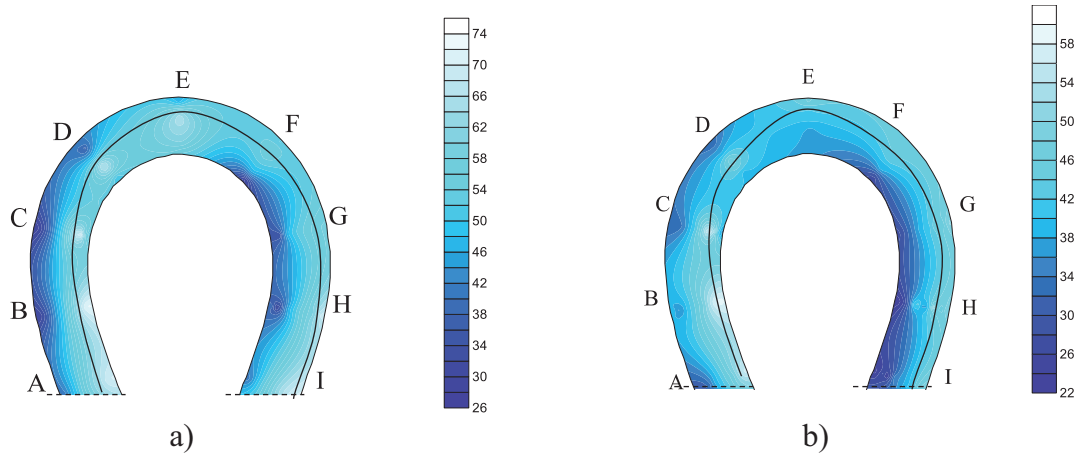


Fig. 5. Map contour of the maximum velocity $u_{max,v}$ (cm/s): a) $W/h < 10$; b) $W/h > 10$ (the black line indicates the location of higher values of $u_{max,v}$).

lation cell moving from the inner to outer bank occurs; as the channel's curvature increases (i.e. at the bend entrance - section D), especially for $W/h < 10$, an outer-bank circulation cell initiates close to the free surface, as shown by Termini and Piraino (2011) analyzing the evolution of cross-sectional flow along the same meandering bend, and the central-region circulation cell seems to split into two weak counter-rotating circulation cells. This effect could drive the location of the maximum velocity close to the bed near the outer side of section D determining high values of the velocity-dip.

This result is particularly interesting for analysing the erosion action of flow to the bed. In particular, the fact that close to the outer bank of the bend entrance the velocity-dip is significant and the maximum velocity is localized just close to the bed could explain why at such a location the maximum bed erosion was observed in previous works (Whiting and Dietrich, 1993; Termini, 2009). In particular, Termini (2009) measured the equilibrium bed deformation obtained at the end of mobile bed runs with hydraulic conditions corresponding to Run1 and Run2. She verified that, in both cases, the maximum bed scour occurs approximately where the tangent to the inner bank at the upstream inflection section A intersects the outer bank (see Fig. 3), and thus close to the outer bank of section D. This behavior is also consistent with previous results obtained by Blanckaert and Graf (2004) indicating that the momentum transport by cross-circulation contributes to redistribute the maximum velocity along the bend affecting the consequent erosion processes. To better understand this phenomenon, the role of the water depth on the shape of the velocity profile is also analyzed by the value of the ratio $u_{m,v}/u_{max,v}$, where $u_{m,v}$ and $u_{max,v}$ are respectively the depth-averaged velocity and the maximum velocity referring to the vertical profile v . Specifically $u_{m,v}$ is estimated on the basis of velocity profile measured by ADV. Fig. 6 reports the value of the ratio $u_{m,v}/u_{max,v}$ against the ratio W/h_v , where h_v indicates the local water depth. From this figure it appears clear that, with exception of few points, for Run1 ($W/h < 10$) the values of the ratio $u_{m,v}/u_{max,v}$ vary in the range 0.7–0.9 with a variation of the ratio W/h_v in the range 8–12; for Run2 ($W/h > 10$) the values of the ratio $u_{m,v}/u_{max,v}$ are generally lower than those observed for Run1 and vary in the range 0.4–0.75 while the values of the ratio W/h_v vary in the range 12–20. The different values of the ratio $u_{m,v}/u_{max,v}$ between $W/h < 10$ and $W/h > 10$ further indicates how the different magnitude of the momentum transport by the cross-circulation, which is more significant for $W/h < 10$, could affect the distribution of the longitudinal velocity.

3.2. Experimental distribution of the bed shear stress

It is important to estimate the bed shear stress along the bend which reshapes the bed topography. In the present work, according to other au-

thors (see as an example Blanckaert et al., 2008; Termini, 2016), in a first approximation the “experimental” bed shear stress has been estimated by using the depth-averaged velocity for vertical profile v as $\tau_{bs} = \rho c_f u_{m,v}^2$ (with $c_f = \frac{\chi}{\sqrt{g}} = \text{Ch\`ezy type dimensionless friction coefficient}$).

Fig. 7, which reports the contour-maps of the term $\tau_{bs}/c_f = \rho u_{m,v}^2$ estimated for both runs, highlights that, for both runs, the higher bed shear stress lies near the inner (right) bank at the inflection section A. Along the bend, it crosses gradually the channel and, at the inflection section downstream, it is found near the outer (left) bank. This behavior is especially evident in the case of $W/h < 10$ where, along the bend, the higher values of τ_{bs}/c_f gradually shift from the inner bank to the outer bank. In the case of $W/h > 10$ the higher values of τ_{bs}/c_f shift towards the outer bank passing from section A to section E; then values of τ_{bs}/c_f decrease until to reach section F, increasing again until to reach the bend exit between sections F and G; passing from section G to the inflection section downstream I the values of τ_{bs}/c_f slightly decrease. It is important to note that this pattern of bed shear stress is consistent with that obtained by Termini (2009) by using the law of the wall. According with Termini (2009), the different behavior observed for $W/h < 10$ and $W/h > 10$ might be caused by the fact that the cross-circulation, which is more significant for $W/h < 10$ than for $W/h > 10$, attenuates the effect of the convective acceleration at the outer bank. This entails the shift of the high values of τ_{bs}/c_f from the inner bank to the outer bank more gradual along the bend in the case of $W/h < 10$.

4. Entropic model application

4.1. Entropic parameter distribution along the bend

In this section a short description of entropic model and its application has given along with the relation between the entropic parameter M (or the entropic function $\phi(M)$) and the kinematic characteristics of the flow.

The entropic model (Chiu, 1988, 1991; Chiu and Murray, 1992; Ciu and Hsu, 2006) correlates the cross-sectional mean velocity, u_m , to the cross-sectional maximum velocity, u_{max} , using the linear relation-ship:

$$u_m = \left(\frac{e^M}{e^M - 1} - \frac{1}{M} \right) u_{max} = \phi(M) u_{max} \quad (1)$$

with M the entropic parameter. u_{max} is considered as the maximum value of all velocity points sampled in the cross-sectional flow area during a measurement (Moramarco et al., 2004). u_m is obtained by applying the velocity-area method to the measured velocity profiles. Therefore, for a gage site in which a velocity dataset is available $\Phi(M)$ and, hence, M can be estimated on the basis of pairs (u_m, u_{max}) .

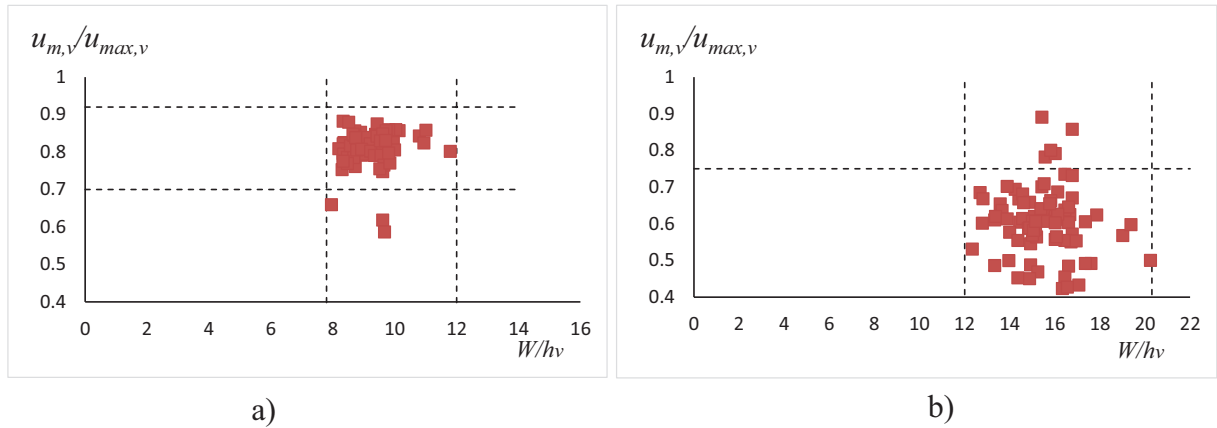


Fig. 6. Ratio $u_{m,v}/u_{max,v}$ against the ratio W/h_v : a) Run1 ($W/h < 10$); b) Run2 ($W/h > 10$).

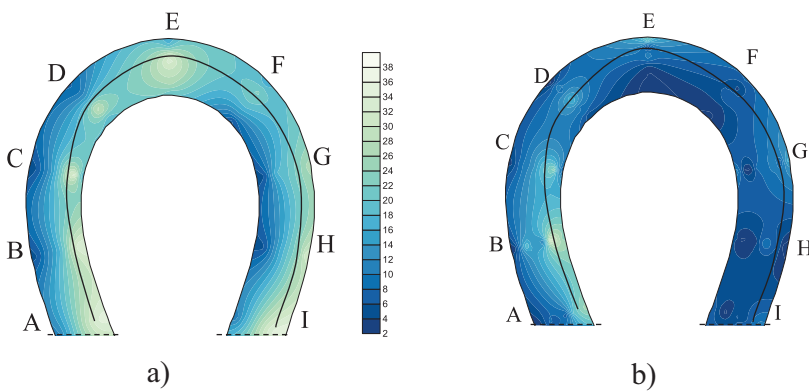


Fig. 7. contour maps of the term $\tau_{bs}/c_f = \rho u_{m,v}^2$: a) $W/h < 10$; b) $W/h > 10$ (the black line indicates the location of higher values of τ_{bs}/c_f).

As regards the theoretical ground of Eq. (1), this comes from the expression of mean flow velocity:

$$u_m = \int_0^{u_{max}} u(x, y) p(u) du \quad (2)$$

where $u(x, y)$ is the velocity in the cross-sectional flow area; x, y represent the distance from the left side of bank and from the bed, respectively; $p(u)$ is the probability density function obtained by assuming an *a-priori* cumulative probability distribution function $F(u) = \frac{y}{D}$, (Chiu, 1988):

$$p(u) = \frac{dF(u)}{du} = \frac{dF(u)}{dy} \frac{dy}{du} = \frac{1}{D} \frac{dy}{du} \quad (3)$$

This hypothesis (i.e., the relation between the cumulative probability $F(u)$ and the ratio y/D) was tested for a number of natural rivers and was found to be approximately true (Singh, 2011). The *a-posteriori* $p(u)$ is obtained by maximizing the entropy (Chiu, 1988; Jaynes, 1957), yielding:

$$p(u) = e^{\lambda_1 - 1} e^{\lambda_2 u} \quad (4)$$

with λ_1, λ_2 are the Lagrange multipliers. For more details we refer the reader to Chiu (1988).

By replacing Eq. (4) in (3) and integrating, after some algebraic manipulation one has:

$$u_m = \frac{u_{max} e^{\lambda_2 u_{max}}}{e^{\lambda_2 u_{max}} - 1} - \frac{1}{\lambda_2} e^{\lambda_2 u} \quad (5)$$

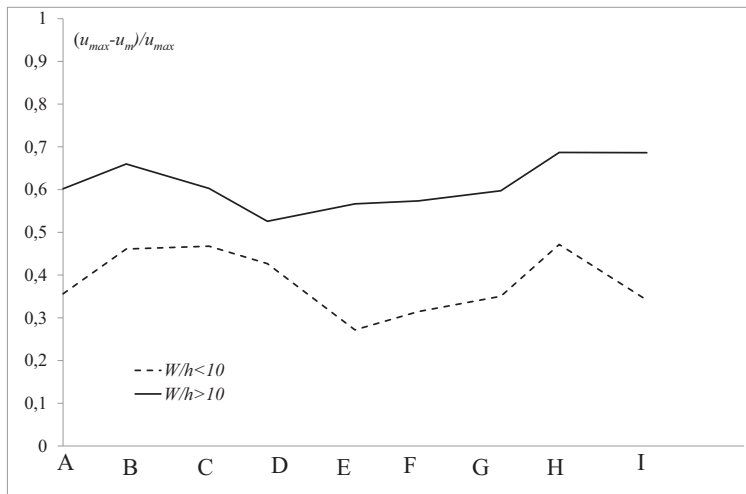
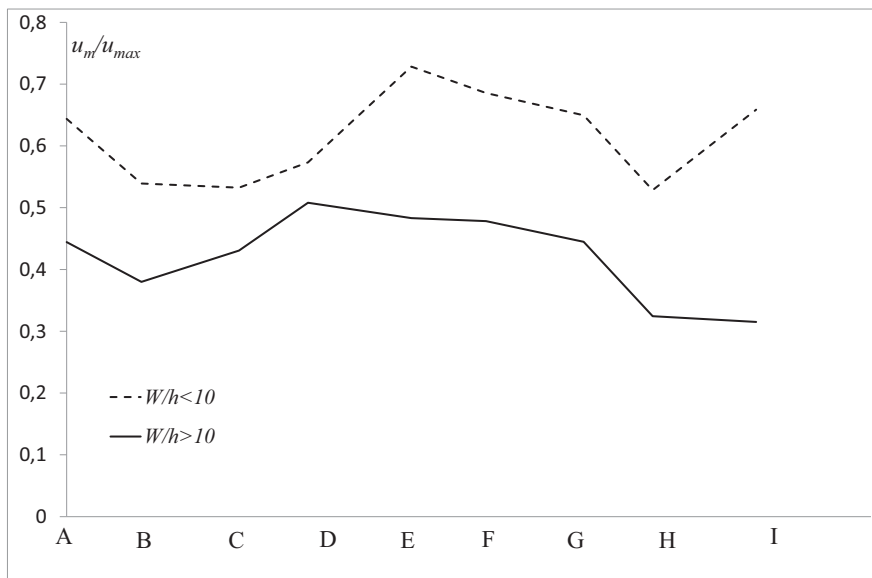
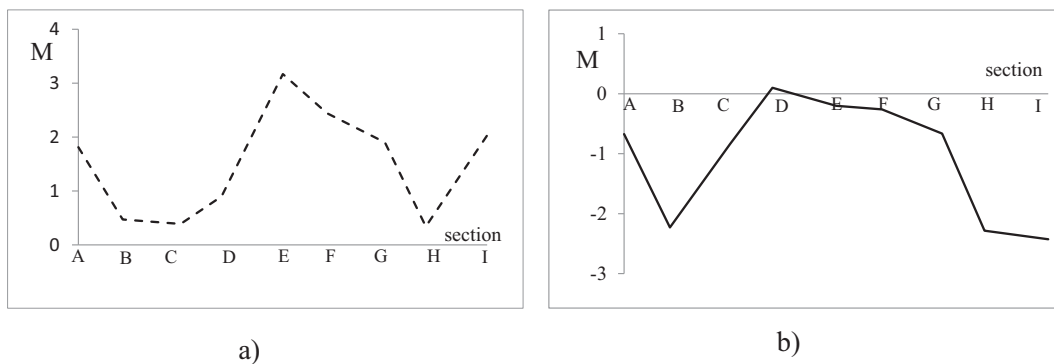
assuming $M = \lambda_2 u_{max}$ and replacing λ_2 in (4), Eq. (1) is thus inferred (Chiu, 1988).

Termini and Moramarco (2017), by using only data from Run1 ($W/h < 10$), verified that the function $\Phi(M)$ of Eq. (1) varies along the

meandering bend. However, Fig. 6 suggests that the relation between u_m and u_{max} could vary also as a function of the width-to-depth ratio. In order to verify this behavior, the cross-sectional maximum value of velocity u_{max} and the cross-sectional mean velocity u_m have been also estimated for each examined section and for both runs. In particular, the mean velocity, u_m , has been obtained by integrating the measured u -profiles on the cross-section. Fig. 8 reports the values of the normalized difference $(u_{max} - u_m)/u_{max} = 1 - \Phi(M)$ estimated along the bend for both runs.

From this figure it can be observed that, for both the width-to-depth ratios, the normalized difference $(u_{max} - u_m)/u_{max}$ increases, i.e. $\Phi(M)$ decreases, as the radius of curvature decreases. In particular, $(u_{max} - u_m)/u_{max}$ increases passing from the inflection section upstream A to section C for $W/h < 10$ and to section B for $W/h > 10$; then it decreases until to reach the apex section E for $W/h < 10$ and the section D for $W/h > 10$. Then, in both cases, $(u_{max} - u_m)/u_{max}$ increases again until to reach the bend exit (section H) after which it decreases towards the inflection section downstream. In the case of $W/h > 10$ the normalized difference $(u_{max} - u_m)/u_{max}$ assumes generally values higher than those obtained for $W/h < 10$. This suggests that for $W/h > 10$ the entropic function $\Phi(M) = u_m/u_{max}$ assumes values lower than those obtained in the case of $W/h < 10$. This can be also observed from Fig. 9 where the values of the ratio $u_m/u_{max} = \Phi(M)$ estimated for each section have been reported for both runs.

Literature studies (see as an example Moramarco and Singh, 2010; Greco and Mirauda, 2015), based on data measured in steady flow conditions either in gauged sections along natural channels or in laboratory flumes, verified that M is constant in the same section for different flow rate ranges. For the examined case, the value of the entropic parameter M in each section can be estimated from Eq. (1) by knowing the corresponding value of the ratio u_m/u_{max} , as reported in Fig. 10. By com-

Fig. 8. Normalized difference $(u_{max} - u_m)/u_{max}$ along the bend.Fig. 9. Ratio $u_m/u_{max} = \Phi(M)$ along the bend.Fig. 10. Estimated values of the entropic parameter M: a) $W/h < 10$; b) $W/h > 10$.

paring Fig. 10 with Fig. 9 it is clear that the value of the parameter M follows that of the entropic function $\Phi(M) = u_m/u_{max}$, i.e. M decreases as the ratio u_m/u_{max} decreases, in accordance with previous works (see as an example in Kumbhakar et al., 2016). The lower value of M is reached at section C for $W/h < 10$ and at section B for $W/h > 10$. Then the values of M increase until to reach section E for $W/h < 10$ and section D for $W/h > 10$; downstream of section D, while for $W/h < 10$ M reaches a sec-

ond minimum value at section H, for $W/h > 10$ M decreases until to reach section I.

The observed behavior suggests that as the flow enters the bend the entropic parameter M reduces, adjusting the magnitude of u_m and both the magnitude and the location of u_{max} . This is coherent with the structure of the secondary currents that influence the magnitude of u_m and u_{max} and for the latter the location in the cross-sectional flow area. To

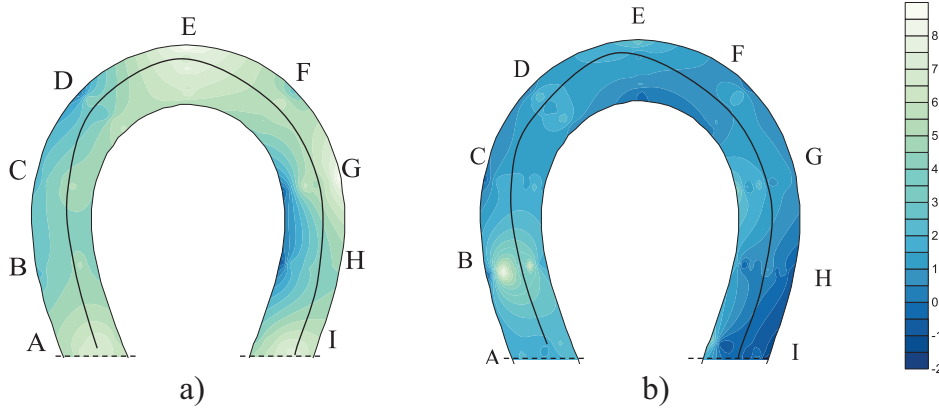


Fig. 11. Contour map of the local entropic parameter M_v : a) $W/h < 10$; b) $W/h > 10$ (the black line indicates the location of higher values of M_v).

better investigate this aspect, following Moramarco et al. (2004), the formulation of Eq. (1) is also written for the generic vertical v as:

$$u_{m,v} = \left(\frac{e^{M_v}}{e^{M_v} - 1} - \frac{1}{M_v} \right) u_{\max,v} = \varphi_v(M_v) u_{\max,v} \quad (6)$$

where M_v indicates the so-called “local” entropy parameter, which is constant along the vertical v .

It is worth noting that Eq. (6) is inferred by assuming Eq. (1) holds also locally such as proposed by Moramarco et al. (2004), who showed the parameter M_v can be considered a signal of the boundary effects on the velocity distribution.

Thus, the values of the local parameter M_v have been estimated for both runs by using Eq. (6). From Fig. 11, which reports the contour-maps of the estimated values of M_v , it can be observed that in the case of $W/h < 10$ the values of M_v are generally higher than those obtained for $W/h > 10$. For $W/h < 10$ the higher values of M_v are found close to the inner (right) bank at the inflection section A; then they gradually cross the channel until that the higher values of M_v are found close to the outer (left) bank at the apex section E; then the higher values of M_v are maintained close to outer bank until to reach the inflection section downstream I.

For $W/h > 10$, M_v assumes high values close to the inner bank of section A; passing from this section to section B it crosses the channel increasing in value; it slightly decreases passing from section C to section E; then the higher values of M_v maintain at the outer side of the cross-section reaching the outer bank at the apex section E; passing from the apex section E to section I the values of M_v decrease close to the outer (left) bank and increase along the inner (right) bank.

Thus, Fig. 11 suggests that, especially for $W/h < 10$, the pattern of the entropic parameter M_v is similar to that of the term τ_{bs}/c_f , which is, in turn, affected by the momentum transport by cross-circulation (Termini, 2015). Inspecting Fig. 11, for both values of W/h , one can also infer that the minimum values of M_v are located close the outer bank of section D where the maximum value of the velocity-dip occurs and the maximum velocity is near the bed. Moreover, comparing Fig. 11 with Figs. 4 and 5, it can be observed that along the bend, the local entropic parameter M_v assumes the higher values where the lower values of the velocity-dip and the higher values of the maximum velocity $u_{\max,v}$ occur. This could be due to the fact that the system responds to the changing curvature by varying the value of the entropic parameter M_v and adjusting the magnitude and the location of $u_{\max,v}$. Fig. 11 also shows that in the case of $W/h < 10$ at the bend entrance, the higher values of M_v are found at a certain distance from the outer bank, as well as it was observed for the values $u_{\max,v}$. This could be related to the presence of the aforementioned counter-rotating cell in the outer-bank region which limits the action of the central-region circulation cell, as verified by Termini and Piraino (2011) in the same meandering channel as that used in the present work.

4.2. Entropic model application to estimate velocity-dip and bed-shear stress distribution

The results presented in the previous subsection suggest that the entropy model could be used to simulate the u -profiles and the effect of secondary circulation on the bed shear stress distribution along the bend.

The 2D velocity distribution can be estimated by applying the entropy profile such as proposed by Moramarco et al. (2004) (see also in Termini and Moramarco, 2018):

$$u(\xi, z) = \frac{u_{\max,v}}{M} \ln \left[1 + (e^M - 1) \frac{z}{h_v - \delta} \exp \left(1 - \frac{z}{h_v - \delta} \right) \right] \quad (7)$$

with

$$u_{\max,v} = \frac{u_{s,v}}{\frac{1}{M} \ln [1 + (e^M - 1) \delta e^{1-\delta}]} \quad (8)$$

with ξ the horizontal distance of vertical v from the outer left bank and z the distance of velocity along v from the bottom; h_v and $u_{s,v}$ represent the flow depth of vertical v and the surface flow velocity, respectively.

Eq. (7) is a simplification of entropy-based 2D model developed by Chiu (1988) who, by exploiting Principle Of Maximum Entropy (POME), established a bridge between the probability domain, wherein a probability distribution of the velocity is surmised, and the physical space. In particular, Chiu obtained the velocity distribution as:

$$u = \frac{u_{\max}}{M} \ln [1 + (e^M - 1) F(u)] \quad (9)$$

where $F(u) = \frac{\gamma - \gamma_0}{\gamma_{\max} - \gamma_0}$ represents the cumulative probability distribution function and γ is a function of the spatial coordinates in the physical space; $\gamma_{\max}$ at the point where $u_{\max}$ occurs; γ_0 at the channel bed where $u = 0$.

As Eq. (9) involves many parameters, Moramarco et al. (2004) simplified the entropy-based 2D velocity distribution assuming that the entropy velocity distribution written for the flow depth where $u_{\max}$ occurs, named y -axis, can be applied everywhere in flow area, thus obtaining Eq. (7).

Therefore, for a gage site for which M is given by historical velocity dataset once $u_{\max,v}$ is measured across the river and the cross-section geometry is known, the 2D velocity distribution can be easily estimated by using Eq. (7). In this way, all information are available, included the velocity dip, δ , to get the 2D velocity distribution. In the case of high floods, when only surface velocities are sampled, $u_{s,v}$, the value of $u_{\max,v}$ to be used in Eq. (7), can be estimated by Eq. (8).

In this context, the velocity-dip, δ , is determined for each u -profile through an iterative procedure by minimizing an objective function in estimating the error between observed values of M and computed ones, M_{com} , such as proposed by Moramarco et al. (2017). Shortly, for a gauged site for which M is known, the velocity dip is estimated by an iterative procedure assigning an initial value of δ at z -axis. Then an

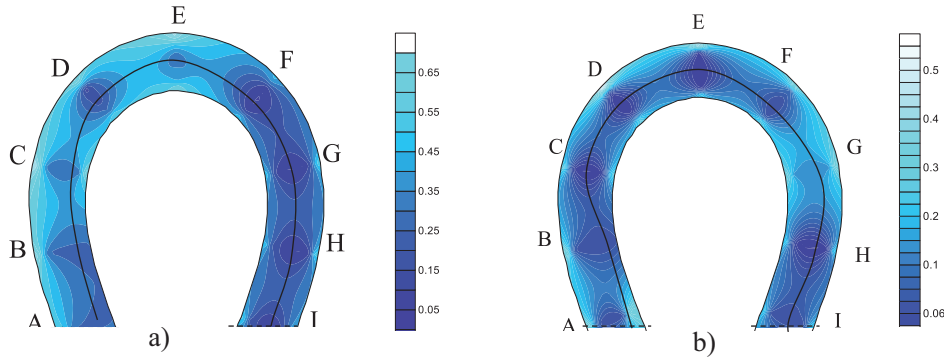


Fig. 12. Map contour of the normalized estimated velocity-dip $\delta^*_{com} = \delta_{com}/h$ estimated by Eqs (3)-(4): a) $W/h < 10$; b) $W/h > 10$ (the black line indicates the location of lower values of δ^*_{com}).

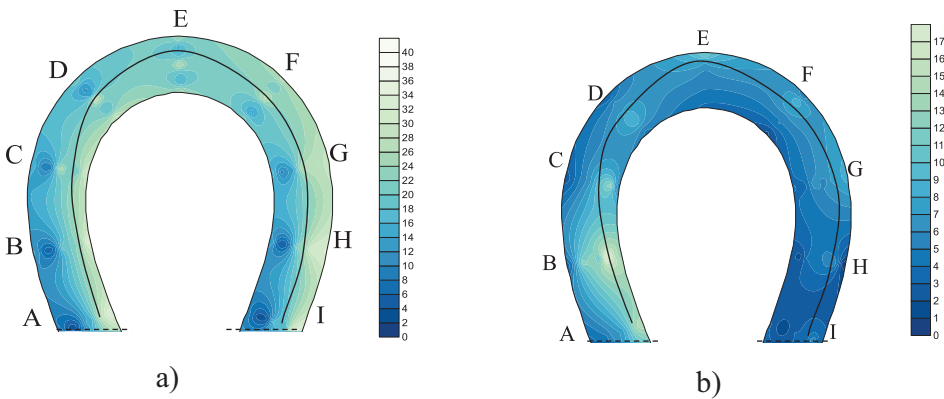


Fig. 13. Map contour of computed term $\tau_{bs,com}/c_f = \rho u_{m,v,com}^2$: a) $W/h < 10$; b) $W/h > 10$ (the black line indicates the location of higher values of $\tau_{bs,com}/c_f$).

experimental distribution law δ developed by Yang et al. (2004) is applied across the river starting from the assigned initial value at z -axis. Once δ is estimated in the cross-sectional flow-area and $u_{s,v}$ is measured, through Eq. (8) $u_{max,v}$ is assessed and the 2D velocity can be evaluated by Eq. (7), from which the computed mean flow velocity, $u_{m,com}$, is assessed. Considering that $u_{max,com} = \max(u_{max,v})$ is known, M_{com} can be inferred using Eq. (1) as the ratio $u_{m,com}/u_{max,com}$. The velocity dip, δ , is updated until the error between M and M_{com} is lower than a threshold. For more details, we refer the reader to Moramarco et al. (2017).

Fig. 12 reports the contour maps of the computed entropy-based velocity-dip, δ_{com} , normalized by the channel-averaged water depth h along the bend for both runs.

The comparison between Figs. 12 and 4 indicates that the entropic model well estimates the pattern of the velocity-dip along the bend for both W/h . Moreover, even in this case it's evident that just by sampling the surface velocity the entropy-based 2D velocity model is able to simulate the effect both of the central-region circulation cell and the counter-rotating cell.

After having integrated along the u -profiles computed by the entropic model and applied Eq. (7), the corresponding values of the depth-averaged velocity $u_{m,v,com}$ have been also determined. Thus, in order to verify the applicability of the entropic model for determining the bed shear stress distribution, the values of computed depth-averaged velocity, $u_{m,v,com}$ have been used to evaluate the “computed” bed shear stress $\tau_{bs,com}$. Fig. 13 reports the contour-maps of the term $\tau_{bs,com}/c_f = \rho u_{m,v,com}^2$ for both runs.

Fig. 13 highlights that, for both runs, the distribution of the bed shear stress estimated by applying the entropic model compares well with that evaluated by using the experimental data shown in Fig. 7. Even in this case one can figure out the coherence of computed $\tau_{bs,com}$ and the presence of central-region circulation cells and counter-rotating cells. It's worth noting that this results came out from the sampling of $u_{s,v}$ only, and applying Eqs. (7) and (8).

In summary, the present application have allowed us to verify the existence of a relation between the entropic parameter M (or of the entropic function $\phi(M)$) and the kinematic characteristics of the flow (i.e. the feedback effect of secondary flow on the primary flow and bed shear stress distributions and, thus, the localization of the maximum velocity) along the bend. This is particularly interesting in a high-curved channel where the effects of secondary flow is especially important.

5. Validation and comparison

The applicability of the entropic model has been assessed by using data from FCF-UK, collected in a compound straight large scale flume of the University of Birmingham (<http://www.birmingham.ac.uk>). The flume was 56 m long with the main section trapezoidal with depth $h = 0.15$ m, width equal to 1.5 m and 45° bank slope; the width of flood plain was equal to $B = 10$ m. For the present work, runs 1, 2, 3 from series A-01, reported in Mohanty and Khatua (2018), see also in Devi and Khatua (2016) and, are used. The Manning's coefficient was equal to ≈ 0.01 . Table 1 reports, for these runs, the values of flow discharge Q (first column), of the flow depth H in the main channel (second column) and of the ratio β (third column) between the depth of flow on flood plain and the depth of flow in the main channel H .

By using the experimental values of the ratios $u_{d,e}/u_{max}$ ($u_{d,e}$ =experimental depth-averaged longitudinal velocity) reported for the considered runs in Mohanty and Khatua (2018), in order to

Table 1
Details of experimental conditions FCFA-1.

Run	$Q(m^3/s)$	$H(cm)$	β	$\phi(M)$
1	0.2082	15.899	0.056544	0.311
2	0.2337	16.519	0.091955	0.309
3	0.2852	17.563	0.145932	0.345

determine the values of the entropic function verifying Eq. (1) for each run, an iterative procedure has been applied. The mean velocity has been determined as the ratio between the flow discharge Q and the cross-sectional area A ($u_m = Q/A$). Thus, assuming a first value of the entropic function $\Phi(M) = 0.6$ (according to previous literature findings – see as an example in Moramarco et al., 2017), the maximum cross-sectional velocity has been evaluated as $u_{max} = \Phi(M)u_m$. Then, for each run, this value of the maximum cross-sectional velocity u_{max} has been used to calculate, by using the experimental values of the ratios $u_{d,e}/u_{max}$, the depth-averaged velocity $u_{d,c}$ for each measurement vertical. Once that the horizontal distribution of $u_{d,c}$ has been integrated in order to determine the mean velocity $u_{m,c}$ using the velocity-area method, for each run the final value of the entropic function $\Phi(M)_c$ has been calculated by minimizing the difference $\Delta = (u_m - u_{m,c})$. The obtained values of the entropic function $\Phi(M)_c$ are reported in the fourth column of Table 1. As it can be observed, an almost constant value of $\Phi(M)_c = 0.3$ has been obtained; to such a value of $\Phi(M)_c$, a value of the parameter $M = -2.49$ corresponds. It should be noted that this value of $\Phi(M)_c$ is different from that obtained in previous works (Moramarco and Dingmann, 2017; Moramarco et al., Singh, 2010) especially in straight flat-bed channels but it is very close to those obtained in laboratory channels characterized by significant bed roughness (i.e. in curved deformed-bed and in straight vegetated-bed channels – see Moramarco and Termini, 2015; Termini and Moramarco, 2017). This suggests that the value of $\Phi(M)_c = 0.3$ obtained for the present application could be related to the roughness conditions of the channel considered for the application. Finally, by using the obtained $\Phi(M)_c$, the corrected values of the maximum cross-sectional velocity have been also determined as $u_{max,c} = u_{m,c}/\Phi(M)_c$. Fig. 14 reports, for each run, the comparison between the horizontal distribution of the calculated ratios $u_{d,c}/u_{max,c}$ (normalized calculated depth-averaged velocity) and that of the experimental ones $u_{d,e}/u_{max}$ (normalized experimental depth-averaged velocity). Then, by using the calculated $u_{d,c}$, the values of the boundary shear stress have been also estimated as described in the sub-Section 3.2. Fig. 15 reports for each considered run the comparison between the calculated ratio $\tau_c/\tau_{max,c}$ (normalized calculated boundary shear stress) and the experimental one $\tau_e/\tau_{max,e}$ (normalized experimental boundary shear stress). Figs. 14 and 15 show the good agreement between the experimental and the estimated values of the normalized depth-averaged velocity and of normalized boundary shear stress, confirming the applicability of Eq. (1) to identify not only the velocity distribution but also the bed shear stress distribution.

6. Discussion

The results presented in the previous sections have substantially demonstrated that the shape of the u -profile varies along the bend and such a variation is strongly related to the variation of the cross-sectional motion, whose entity depends on the width-to-depth ratio, W/h . This is consistent with the fact that, as literature demonstrates (among others Yalin, 1992; Whiting and Dietrich, 1993; Da Silva et al., 2006; Termini, 2015), the width-to-depth ratio is one the most important parameters affecting the entity of the cross-sectional flow motion.

In particular, for both the examined width-to-depth ratios ($W/h < 10$ and $W/h > 10$), the maximum value of the longitudinal velocity $u_{max,v}$ has been found below the free surface indicating that the u -profiles present the velocity-dip. Generally, the velocity-dip assumes for $W/h < 10$ values greater than those obtained for $W/h > 10$ and the local difference between the local maximum value of the longitudinal velocity $u_{max,v}$ and the local free-surface velocity $u_{s,v}$, assumes for $W/h < 10$ values lower than those obtained for $W/h > 10$. This suggests that passing from the free surface to the location of the maximum velocity, the u -profiles present a concave shape, that is more pronounced near the free surface for lower values of the water depth ($W/h > 10$).

For both the width-to depth ratios, the entity of the velocity-dip varies along the bend, thus suggesting that the location of the maxi-

mum velocity $u_{max,v}$, also varies along the bend. The variation of the shape of the velocity profiles along the bend has been analyzed through the variation of the velocity-dip (i.e. the vertical distance of the maximum longitudinal velocity from the free surface) and of the difference du between the maximum velocity and that at the free surface. This analysis has allowed us to have an indication on the redistribution of the maximum velocity not only on the plane but also along the vertical. In particular, it has been verified that the higher values of the maximum velocity cross the bend passing from the inner bank of the inflection section upstream A to outer bank of the inflection section downstream I; the lower values of the velocity-dip have a pattern similar to that of the higher values of the maximum velocity. This suggests that the higher values of the maximum velocity, $u_{max,v}$, are localized close to the free surface. Such a behavior is less evident for $W/h > 10$, especially downstream of the apex section where high values of the velocity-dip occur.

The different behavior between $W/h < 10$ and $W/h > 10$ seems to be related to the fact that in a meandering bend the two components of the cross-sectional motion (i.e. the cross-circulation and the convective component, according to Yalin (1992)) assume a different relative importance in the two cases, as demonstrated by Termini (2009) using the same data of this study and by Termini and Piraino (2011). In particular, their results indicated that the convective acceleration component is more significant for $W/h > 10$ than for $W/h < 10$; on the contrary, in this last case, the cross-circulation component is more important attenuating the contribution of the convective acceleration component. As consequence, while in the case of $W/h > 10$ the shifting of the maximum velocity towards the outer bank occurs along the channel reach between the inflection section upstream to the apex section, in the case of $W/h < 10$ the aforementioned shifting occurs gradually passing from the inflection section upstream to the inflection section downstream maintaining the maximum velocity not too far from the free surface.

Results also show that, at the bend entrance (section D), while for $W/h > 10$ the higher maximum of velocity is found close to the outer bank, for $W/h < 10$ it is still maintained far from the outer bank itself. This could be due to the fact that, as highlighted by Termini and Piraino (2011) in the same meandering bend as that considered in the present work, at the bend entrance a counter-rotating circulation cell initiates in the outer-bank region so that, especially in the case of $W/h < 10$, the central-region circulation cell seems to split into two weak counter-rotating circulation cells. The counter-rotating cell avoids the reaching of the higher velocity to the outer-bank thus limiting the action of the central-region circulation motion.

The analysis of the velocity-dip and the maximum velocity distributions has also confirmed that the pattern of the velocity-dip reflects the aforementioned effects of the cross-sectional motion and of the corresponding circulation cells. Indeed, the velocity-dip is minimum where only the effect of a central-region circulation cell moving from the inner to outer bank occurs, driving the maximum velocity not too far from the free surface; at the bend entrance, especially for $W/h < 10$, because of the presence of outer-bank circulation cell, the maximum velocity is driven close to the bed near the outer bank determining high values of the velocity-dip. This result is particularly interesting for analysing the erosion action of flow. In fact, such a location coincides with the location where Termini (2009) measured, in the same meandering flume, the maximum value of the bed erosion.

This behavior further confirms that the momentum transport by cross-circulation, contributing to redistribute the maximum velocity along the bend, also affects the consequent erosion processes.

Literature indicates (among others Chang, 1988; Whiting and Dietrich, 1993) that the convective acceleration of flow is associated with an imbalance between the transverse pressure gradient and the centrifugal acceleration and, in a meander wave, such an imbalance is continuously modified by the downstream changing channel curvature. For $W/h < 10$ (small width-to-depth ratios) such an unbalance is attenuated by the momentum transport by the cross-circulation developing in each cross section of the bend, determining that the shifting of the maximum ve-

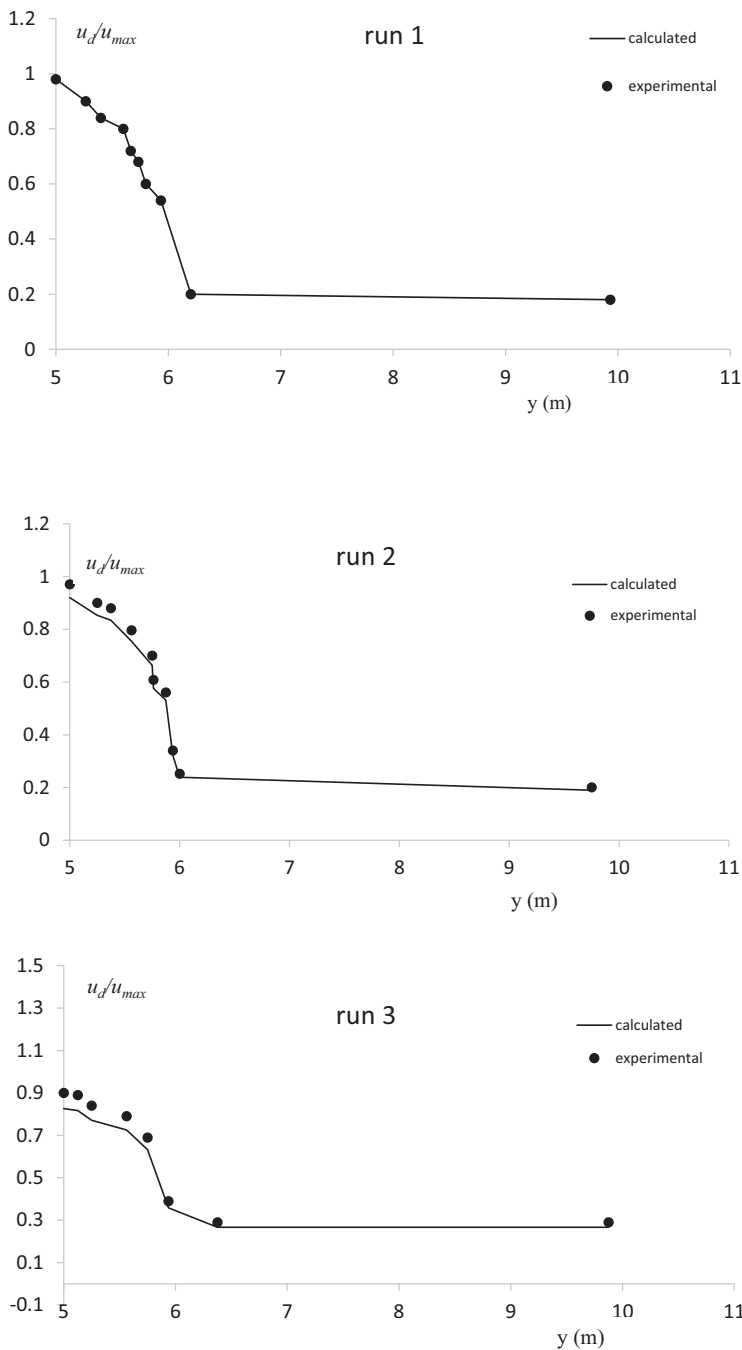


Fig. 14. Comparison between calculated ratio $u_{d,c}/u_{max,c}$ and the experimental one, $u_{d,e}/u_{max}$.

locity along the bend is more gradual in the case of $W/h < 10$ than in the case of $W/h > 10$.

As consequence, also the distribution of the bed shear stress τ_{bs}/c_f shows the shifting of the higher values of τ_{bs}/c_f from the inner bank to the outer bank along the bend: for $W/h < 10$, the higher values of the bed shear stress τ_{bs}/c_f gradually shift towards the outer bank from the inflection section upstream (section A) to the inflection section downstream (section I); for $W/h > 10$ the higher values of the bed shear stress τ_{bs}/c_f cross the channel passing from the inner bank of the inflection section upstream (section A) to the outer bank of the apex section (section E); then the bed shear stress τ_{bs}/c_f slightly decreases in value but maintaining the higher values close to the outer bank until to reach the inflection section downstream.

The aforementioned considerations indicate that the difference in the entity of the cross-circulation between the case of $W/h < 10$ and the

case of $W/h > 10$ affects the shape of the u -profiles and, consequently, the local ratio between the local vertical mean velocity and the local maximum velocity, $u_{m,v}/u_{max,v}$. In particular, it has been found that, while the case of $W/h < 10$ the ratio $u_{m,v}/u_{max,v}$ vary in the range 0.7–0.9, in the case of $W/h > 10$ the values of the ratio $u_{m,v}/u_{max,v}$ assume lower values and vary in the range 0.4–0.75.

By knowing the values of the local ratio $u_{m,v}/u_{max,v}$, the application to the generic vertical v of the Chiu's (1991) entropic model relating the flow velocity to the entropy function, has allowed us to estimate the value of a “local” entropic parameter M_v . The analysis of the contour maps of the estimated M_v values has shown that, especially for $W/h > 10$, M_v increases where flow accelerates and decreases where flow decelerates. Furthermore, the entropic parameter M_v generally follows a trend similar to that of τ_{bs}/c_f . This suggests that the local entropic parameter could reflect the unbalance between the transverse pressure gradient

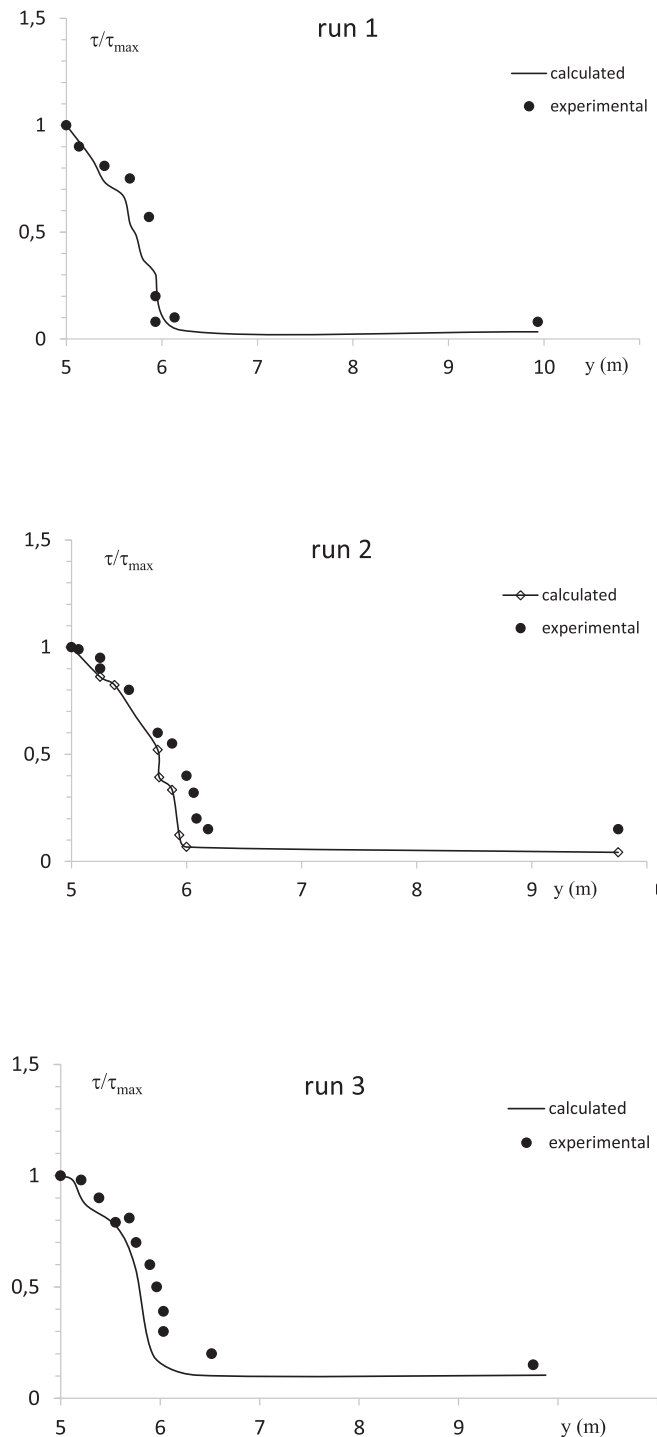


Fig. 15. Comparison between calculated ratio $\tau_c/\tau_{max,c}$ and the experimental one, $\tau_e/\tau_{max,e}$.

and the centrifugal acceleration and its variation along the bend depends on the variation of the cross-sectional motion.

According with the Chiu's (1991) entropic model, the ratio between the cross-sectional mean velocity and the maximum cross-sectional velocity, u_m/u_{max} , gives the entropic function $\Phi(M)=u_m/u_{max}$. The analysis of the variation of $\Phi(M)$ and of the entropic parameter M along the bend has highlighted that, for both the width-to-depth ratios, they follows the same pattern, in accordance with previous works (see as an example in Kumbhakar et al. (2016)). For $W/h<10$, the lower values of M are found

close to the bend entrance and close to the bend exit and the higher values close to the apex section E; for $W/h>10$ the lower and higher values are found slightly upstream of those for $W/h<10$. The observed behavior of the entropic parameter M suggests that as the flow enters a bend it reduces, adjusting the magnitude of u_m and both the magnitude and the location of u_{max} .

Thus, the application of the entropy-based 2D velocity distribution proposed by Moramarco et al. (2004) has allowed us also to estimate the values of the bed shear stress and the velocity-dip just by sampling the surface velocity. The comparison between the estimated values of both the bed shear stress and the velocity-dip with the experimental ones has demonstrated that the entropic model well approximates the experimental distributions. This means that the shape of the u -profile, which is affected by the entity of the cross-sectional flow, and the bed shear stress distribution, could be directly estimated from the entropic model starting from acknowledgement of the surface velocity only.

7. Conclusion

Based on the obtained results, with respect to the three specific objectives of the present work the following findings can be drawn.

With respect to the objective 1), the results indicate that:

- the shape of the u -profile varies along the bend as a function of the width-to-depth ratio. In particular, it has been verified that for low water depths and large width-to-depth ratios the u -profile is more concave near the free surface and the dip δ assumes low values.
- for "small" values of the width-to-depth ratio ($W/h<10$) the values of the local ratio between the mean velocity and the maximum velocity ($u_{m,v}/u_{max,v}$) vary in the range 0.7–0.9; for "large" values of the width-to-depth ratio ($W/h>10$) this ratio ($u_{m,v}/u_{max,v}$) assumes lower values, varying in the range 0.4–0.75. This suggests that the higher values of the maximum velocity, $u_{max,v}$, are localized close to the free surface. Such a behavior is less evident downstream of the apex section for $W/h>10$ where high dip values occur, indicating that in such a channel the maximum velocity can be also found far from the free surface along the exit-reach of the bend.

Thus, it can be concluded that the pattern of the velocity-dip reflects the effect of cross-sectional motion and of the related circulation cells.

With respect to the objective 2), the results indicate that:

- the ratio between the cross-sectional mean and maximum velocity u_m/u_{max} , which is related to the entropic parameter M by the relation $u_m/u_{max}=\Phi(M)$, varies along the bend by having for $W/h>10$ values lower than those obtained in the case of $W/h<10$.
- by assuming that the relation between the mean velocity and the maximum one through the entropic parameter can be considered valid for each vertical v , as $u_{m,v}/u_{max,v}=\Phi_v(M_v)$, the map of the variation of the "local" entropic parameter M_v along the bend has been analyzed. Results have shown that the local entropic parameter M_v is slightly different between the case of $W/h<10$ and the case of $W/h>10$ and generally follows the pattern of the bed shear stress τ_{bs}/c_f . The bed shear stress, τ_{bs}/c_f , in turn, assumes a different behavior for $W/h<10$ and $W/h>10$ due to the fact that the cross-circulation, which is more significant for $W/h<10$ than for $W/h>10$, attenuates the effect of the convective acceleration at the outer bank. This determines the shift of the higher values of τ_{bs}/c_f from the inner bank to the outer bank along the bend, to be more gradual in the case of $W/h<10$ than in the case of $W/h>10$.
- especially for $W/h>10$, M_v increases where flow accelerates and decreases where flow decelerates. This suggests that the local entropic parameter could reflect the unbalance between the transverse pressure gradient and the centrifugal acceleration.

Thus, it can be concluded that the feedback effect of cross-sectional flow on the longitudinal velocity distribution could be qualitatively

identified by applying the entropy concept and by analyzing the spatial distribution of entropic parameter M .

With respect to the objective 3), the results indicate that:

- the plane distributions of the velocity-dip and the bed shear stress estimated by applying the entropic model (Moramarco et al., 2004) to the investigated cases are in good agreement with the experimental ones.

Thus, it can be concluded that the entropy theory is able to provide useful insights in terms of velocity-dip, just starting from the monitoring of surface velocity across the site.

In summary, the results have shown that the pattern of the entropic parameter is similar to that of the term τ_{bs}/C_f which is, in turn, affected by the advective transport by cross-circulation, in accordance to Termini (2015). The shape of u -profile and the effect of the variation of the transversal motion along the bend could be directly estimated by the entropic model starting from the acknowledgement of the surface velocity only. This is especially important for its practical relevance allowing us to estimate not only the shape of the velocity profile but also the evolution of the primary velocity and of the bed shear stress distribution along the bend starting from the knowledge of the surface velocity only.

It should be noted that this paper investigated on the possibility to analyze the variation of velocity-dip and the variation of the transversal motion effect through the entropy profile, restricting the attention to a high-amplitude meander bend, for which the role of curvature and of its downstream variation on the velocity pattern are especially important. On the basis of the results obtained in previous works (see as an example in Moramarco et al., 2004; Moramarco and Termini, 2015; Termini and Moramarco, 2017) we can assume that the methodology can be applied for different sinuosity values and, as extreme case, in straight channels. However, future research will be addressed to verify the effect of the two components of the transversal motion (i.e. the cross-circulation and the convective acceleration) and the velocity-dip also varying the planimetric configuration of the bend.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Donatella Termini: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Supervision, Validation, Writing - original draft, Writing - review & editing. **Tommaso Moramarco:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Software, Supervision, Validation, Writing - review & editing.

Acknowledgments

This work has been supported by Italian National Research Programme PRIN 2017, with the project “*IntEractions between hydrodyNamics flows and bioTic communities in fluvial Ecosystems: advancement in dischaRge monitoring and understanding of Processes Relevant for ecosystem sustaInability by the development of novel technologieS with fIeld observaTioNs and laboratory testinG (ENTERPRISING)*”.

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