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Real Time Localization Systems for autonomous navigation

Modelling, Analysis and Control

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Preface

Real-Time Localization of mobile objects and people in indoor and outdoor scenario has gained increasing interest and popularity in the academic and industrial control community because of the growing demand of location based services. A system that is able to accurately localize (determine speed and position) items or persons in real time is called Real Time Localization System better known by the acronym RTLS. Smart industries warehouses, Healthcare industry, Transportation and Logistics, Defense, mobile robot guidance and autonomous drive and flight are only some fields of application where RTLS can be applied to improve skills and safety.

For example, in Smart Factory RTLS can be used for inventory, people and asset tracking and management. In the Healthcare industry, specially with the ongoing pandemic, due to automated tracking capability, they could be used for contact tracing, tracking of medical equipments, medication administration and patients monitoring. In the Transportation and Logistics fields RTLS can be employed to track shippings, container management, delivery confirmation, supply chain efficiency maximization and to monitor sensitive shipments which enable faster intervention if there are any requirements. In Defense department RTLS can be used to recover aeronautical, automotive and naval equipments or to develop autonomous vehicles (drone or rover) for high risk operations. Last but not least in the Autonomous drive and flight field of application RTLS is the key component around which the entire navigation system is built.

Since localization systems can be used to acquire information on the speed and position of a large class of mobile objects the first categorization of this kind of systems can be made on the desired precision and update frequency of the requested information. While in some low level applications, like warehouse asset tracking, localization systems are used to acquire information of stationary objects (or object whose position change at very low rate in time) for the purpose of other high level applications like autonomous driving of car, aircraft and generally mobile robots, since data from localization systems are used to close the feedback loop of high frequency control schemes they need to be faster, more

accurate and reliable.

Generally an RTLS is made up by some hardware and software components. Hardware components are composed by microprocessors and sensors able to read environmental physical quantities like distances from some specific reference positions named way points. They are divided, in turn, in two categories, the ones that are stationary in space at known position, commonly named beacons, and the other ones, applied on the mobile object to be tracked, commonly named transponders. Transponders can also integrate sensors able to measure body acceleration, angular velocity and Earth magnetic field. Software component are made of algorithms that collect data from sensors (commonly with wireless techniques), compute them and offer a visual interface to present the results to the operator or send the computed measures to other electronic equipment for further data elaborations.

It is important to highlight that, talking about localization (of an object in an environment) leads to the definition of an a priori reference system in the same scenario where the mobile object can move. For example, if the localization goal is to determine the position of an aircraft moving on the Earth, one can refer to the World Geodetic Standard reference system (WGS). Instead, if the intention is to track a mobile robot moving in a custom indoor environment one can refer to a tailor-made reference system built, for instance, on the floor plan of the building.

In any case, to calculate the position of an object in an area (both for indoors and outdoors scenario) it is necessary that the transponder (carried by the mobile object) is able to measure some physical quantities that reflect its disposition relatively to the fixed devices (beacons). To cope with this, based on the nature of the sensors, two kind of methods are available to calculate the position starting from these measures: *trilateration* and *triangulation*. The first one is based on the range measurements between beacons and transponder, instead, the second one, compute the position relying on the measured angles between beacons and transponder. Anyway, in this thesis, a novel method inspired from the robotic context and based on a closed loop scheme will be proposed as an alternative to trilateration.

Noted that the key component of a localization system is the ranging system and that, for practical purposes, the measurement needs to be wireless, let us explore the available technological solutions to accomplish this task. Radio frequency, Bluetooth, Wi-Fi, Ultra-sound and Infrared signals are just some of the available technologies to achieve wireless data transmission. Ultra Wide Band (UWB) instead is a relatively new radio technology that can use a very low energy level for short-range and high-bandwidth communications over a large portion of the radio spectrum. However, the wireless communication system

alone is not sufficient to obtain a distance or angle measurement between the beacons and the transponder. To compute the distance between two radio stations, one acting as transmitter and the other one acting as receiver, it is necessary to use some techniques like for example the ToF (Time of Flight), ToA (Time of Arrival), AOA (Angle of Arrival) and RSSI (Received signal strength indication) which will be extensively described in the course of the next chapters.

Each communication method and technique, for measuring angles or distances, has pros and cons in terms of accuracy, response speed, cost. By the way, depending on the target application, position update needs to satisfy a minimal frequency of update (bandwidth) and a maximum error tolerance. If the object to be tracked is a person moving at slow speed probably a system based only on range measures can be good for the position estimation, instead, if the localization system is the feedback loop of an autonomous driving mobile robot it may not be enough.

Until now, we have talked about the object to be located in the most possible general way. Radio equipment and ranging technique can be used indifferently on any type of target, be it a person rather than an aircraft, a vehicle or a robot. Moreover nothing has been said respect to the possibility to estimate the position of the mobile object relying only on local measures like speed or acceleration. Indeed, by using a cinematic and/or a dynamical model of the target, starting from local measures of speed and acceleration, it is possible to estimate the position at more high frequency. This task can be achieved by installing additional instrumentation on board of the object to be tracked. Common IMU equipment (Inertial Measurement Unit) are small enough to be placed on any target and are able to measure acceleration, angular velocity and Earth's magnetic field while for speed measurement more advanced sensors based on vision system are needed.

Assuming that the dynamical property of the mobile object are not know, the first way to locally estimate the position of an object knowing only its acceleration is to apply a cinematic model and therefore performing a double integration. First integration allows to obtain the speed of the object while the second one allows to obtain the position. While this method is really simple and relies only on acceleration measurement, because of the noise on the measures and double integration, it suffers of drift over the time. This problem will be addressed in the Strap-Down model relative section.

So, while the position estimation computed starting from range measurements and trilateration has a bounded absolute error but is available at low refresh rate, the position estimation coming from the double integration of the acceleration are generally faster but suffers of divergent drift error. From this consideration follows the needs to mix these information all together in order to obtain a position estimation that is at the higher

possible frequency and with the lower possible error. The branch of automatic control science that deals with the study of observers/estimators based on measurements from various sensors, at different refresh rates, is called sensor fusion and it is the main topic of this thesis.

Prior to proceed on the explanation of the contents of this thesis a further consideration must be made respectively to the use of the localization systems for indoor or outdoor applications. Generally outdoor localization systems rely on the GPS (Global Position System) equipment. It is made by a network of satellites in geostationary orbits (whose position is known) acting as beacons that allow, with a minimal computational power device, to know the position of an object on the Earth using the Time of Flight technique and trilateration. When using a localization system in an indoor environment GPS signals are not accessible because the presence of wall or more generally object that block the radio waves coming from the satellites. This leads to the need to build a custom system of beacons based on the previously described techniques. For these reasons, generally, while an indoor localization system can be used also for outdoor applications, the opposite is not possible if the outdoor localization system use the GPS signals.

This thesis is organized into five main parts. The first part deals with an introduction to the state of art for ranging systems and wireless communication technologies highlighting the sporadic nature of the measures, modeling and observing techniques used for the estimation problem with a focus on the hybrid modeling framework. In the second part implementation of the experimental setup for indoor and outdoor localization applications are presented. The third part is devoted to the synthesis of two sensor fusion location estimators based on the hybrid framework and a novel technique as alternative to trilateration is presented. In the fourth part an hybrid observer suitable both for indoor and outdoor applications is derived with particular focus to the analysis of Internal State Stability (ISS) even in the presence of measurement noise. Finally, in the fifth part, a trajectory robust controller based on model decoupling and disturbance estimation will be presented as an experimental workbench for the testing of the proposed observers with an autonomous flight quadcopter application. A topological scheme of the contents in this work and a list of the scientific production submitted during the PhD period close this preface on the following pages.

TOPOLOGICAL SCHEME OF THE CONTENTS IN THIS THESIS

	INDOOR	BOTH	OUTDOOR
INTRODUCTION		RANGE MEASURE MODELLING HYBRID SYSTEMS	
EXPERIMENTAL SETUP	UWB/IMU/ROBOT INDOOR SETUP	IMU SETUP CALIBRATION ATTITUDE EST. QUADRATOR	GPS/IMU/VEHICLE OUTDOOR SETUP
POSITION ESTIMATION	INDOOR HYBRID OBSERVER	KINEMATIC TRILATERATION	OUTDOOR HYBRID OBSERVER
SENSOR FUSION		NOISE ROBUST HYBRID OBSERVER	
APPLICATIONS		QUADRATOR ROBUST CONTROL	

The research activity during the PhD period has resulted in the following publications:

INTERNATIONAL JOURNAL

- [1] F. Alonge, F. D'Ippolito, G. Garraffa, A. Sferlazza. "A hybrid observer for localization of mobile vehicles with asynchronous measurements". In: Asian Journal of Control. Volume 21. pp 1506-1521. 2019.
- [2] G. Garraffa, A. Sferlazza, F. Alonge, F. D'Ippolito. "Localization Based on Parallel Robots Kinematics as an Alternative to Trilateration". In: IEEE Transactions on Industrial Electronics.
- [3] F. Alonge, F. D'Ippolito, A. Fagiolini, G. Garraffa, A. Sferlazza. "Trajectory Robust Control of Autonomous Quadcopters Based on Model Decoupling and Disturbances Estimation". In: International Journal of Advanced Robotic Systems.

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- [4] F. Alonge, F. D'Ippolito, G. Garraffa, A. Sferlazza. "Hybrid Observer for Indoor Localization with Random Time-of-Arrival Measurements". In: 2018 IEEE 4th International Forum on Research and Technology for Society and Industry (RTSI).
- [5] F. Alonge, F. D'Ippolito, A. Fagiolini, G. Garraffa, F.M. Raimondi, A. Sferlazza. "Tuning of Extended Kalman Filters for Sensorless Motion Control with Induction Motor". In: 2019 AEIT International Conference of Electrical and Electronic Technologies for Automotive (AEIT Automotive).
- [6] A. Sferlazza, L. Zaccarian, G. Garraffa, F. D'Ippolito. "Localization from Inertial Data and Sporadic Position Measurements". In: 21st IFAC World Congress 2020.

IN PREPARATION OR UNDER REVIEW

- [7] A. Sferlazza, L. Zaccarian, G. Garraffa, F. D'Ippolito. "Localization from Noisy Inertial Data and Sporadic Position Measurements: Theory and Experiments". In: IEEE Transactions on Control Systems Technology.

List of symbols

Table 1: Mathematical symbols and notations

$\mathbb{R}^n$	n-dimensional Euclidean space;
$\mathbb{R}_{\geq 0}$	Set of nonnegative real numbers;
$\mathbb{Z}$	The set of all integers;
$\mathbb{Z}_{\geq 0}$	The set of nonnegative integers;
$\mathbb{B}$	The closed unit ball, in the Euclidean norm;
$ x $	Euclidean norm of a vector $x \in \mathbb{R}^n$;
B, N	BODY and NED (North-East-Down) reference frames, respectively;
O_b, O_n	Origin of the B and N reference frames, respectively;
$\mathbb{S}^3$	The 3-sphere $\mathbb{S}^3 := \{\xi \in \mathbb{R}^4 : \xi = 1\}$.
$SO(3) \subset \mathbb{R}^{3 \times 3}$	The special orthogonal group of rotation matrices.
$\mathbf{p}_{b/n}^n \in \mathbb{R}^3$	Position vector in N -frame;
$\mathbf{v}_{b/n}^n \in \mathbb{R}^3$	Velocity vector in N -frame;
$\mathbf{v}_{b/n}^b$	Velocity of O_b relative to O_n in B frame;
$\boldsymbol{\omega}_{b/n}^b \in \mathbb{R}^3$	Angular velocity of B with respect to N , expressed in B ;
$\boldsymbol{\omega}_{b/n,m}^b \in \mathbb{R}^3$	Angular velocity of B with respect to N , measured by gyroscope in B ;
$\boldsymbol{\Omega}_{b/n,C}^b \in \mathbb{R}^3$	Cconstant angular velocity bias of the gyroscope;
$\boldsymbol{\gamma}^b \in \mathbb{R}^3$	acceleration in the B frame;
$\boldsymbol{\gamma}_m^b \in \mathbb{R}^3$	acceleration in the B frame measured by accelerometer;
$\boldsymbol{\phi}^b \in \mathbb{R}^3$	magnetic field in the B frame;
$\boldsymbol{\phi}_m^b \in \mathbb{R}^3$	magnetic field in the B frame measured by magnetometer;
$\boldsymbol{\Theta}_b^n \in \mathbb{R}^3$	roll, pitch and yaw angles between B and N ;
$\boldsymbol{\xi}_b^n \in \mathbb{R}^4$	unit quaternion between B and N ;
$\mathbf{A} \otimes \mathbf{B}$	Kronecker product between matrices $\mathbf{A}$ and $\mathbf{B}$;
$\mathbf{I}_n$	Identity matrix with dimension $n \times n$;
$\mathbf{0}_n$	Matrix with all zero elements with dimension $n \times n$;
$\lambda_m(S)$	Minimum eigenvalues of a positive definite symmetric matrix S ;
$\lambda_M(S)$	Maximum eigenvalues of a positive definite symmetric matrix S ;
x^+	The state of a hybrid system after a jump;

Acronym:

RTLS Real Time Localization system

ECEF Earth Centered Earth Fixed

NED North East down

ENU East North up

AOA Angle of arrival

TOA Time of Arrival

TDOA Time Difference of Arrival

RSS Received Signal Strength

TOF Time of Flight

RSPM Received Signal Phase Method

TWR Two Way Ranging

LOS Line of Sight

UWB Ultra Wide Band

GPS Global Position System

EPM Elliptical Propagation Model

DOF Degree of Freedom

RPY Roll Pitch Yaw

IMU Inertial Measurement Unit

ECI Earth Centered Inertial

INS Inertial Navigation System

MARG Magnetic, Angular Rate and Gravity

AHRS Attitude and Heading Reference System

Chapter 1

Introducing and modeling RTLS

OUTLINE. In this chapter an overview about localization systems will be given. We will start with a brief introduction of RTLS system with a focus on the measurement principles, wireless communication systems, position calculation algorithms and attitude estimation. Then, after a recap about the ECEF and NED/ENU geographical coordinate system, the model of Inertial Navigation System is presented and, in the final section, an introduction of the hybrid framework formalism, widely used in this thesis, will be given.

1.1 Overview of Real Time Localization System

The term RTLS generally refer to a set of hardware and software components all of which work together to estimate the position of a mobile object. To formalize the terminology that will be used in this manuscript we will refer to the following in Figure 1.1 in which the devices, involved in the implementation of the system, are schematized. The element in the center of the image, drawn as a mobile robot, represents the object whose position p_{xyz} , relatively to the Cartesian Coordinate reference system O_{XYZ} , is to be estimated. In order to obtain information on its disposition in space, the mobile object, through its radio equipment called transponder, can exchange information with the beacons $A_1, \dots, A_4$ whose position, relatively to the same reference system, is known a priori. Measured distances between the mobile robot and each beacon A_i are marked with d_i . A data processing unit, not included in the figure, collects the data from the system and, applying some algorithms, calculates the position of the robot in real time.

There exists two main method to cope with position calculation, triangulation and trilateration. The first one use a geometric approach to estimate the position of an object by measuring its angle from multiple reference points as in Figure 1.2(a) where position

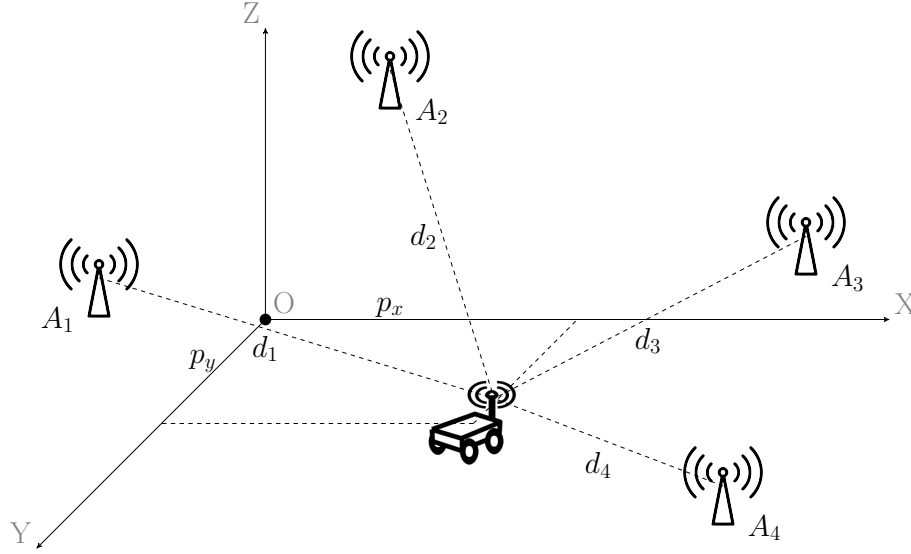


Figure 1.1: Schematization of a RTLS system.

p respect to a $2D$ scenario is calculated starting from the measure of angles θ_1 and θ_2 . The second one, instead, measuring its distance d_1 , d_2 and d_3 as the radius of circles with center on the reference Anchor points, Figure 1.2(b).

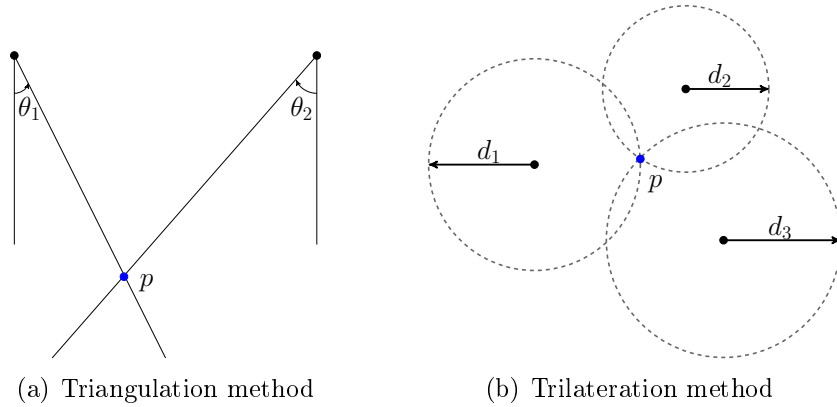


Figure 1.2: Position estimation technique.

A clarification must be made regarding the minimum number of fixed devices necessary for the calculation of the position. This, first of all, depends on the dimension of the working scenario coordinate reference system. If localization only on a floor or a plane is requested, $2D$ scenario can be used, instead, if spatial localization is required it is necessary to use a $3D$ reference system. In case of a $2D$ scenario, with the triangulation, two angle measurement a time, between the mobile object and three beacon are sufficient to calculate the position of the mobile object on the reference plane [8], so it need for

at least three device, and a minimal number of three angle measures are needed for 3D scenario, corresponding to at least four beacons. Using the trilateration technique, a minimal number of three range measures from three different beacons is needed in case of 2D scenario and a minimal number of four devices for 3D scenario. In the case the trilateration technique is used, it is possible to reduce the number of minimal needed range measure by restricting the working area of space. In case of 2D trilateration only two beacons are needed, instead of three, if the localization problem is reduced to one of the half plane identified by the straight line passing for the two beacons. Respectively, in case of 3D trilateration only three beacon are sufficient instead of four, if the localization problem is reduced to one side of the plane that pass for the three beacons.

1.1.1 Measuring principles

Angles and distances evaluation, used in triangulation and trilateration, rely on different measurement principles. For the triangulation AOA (Angle of Arrival) technique are used, while for trilateration several techniques can be used like the TOA (Time of Arrival), TDOA (Time Difference of Arrival), RSS (Received Signal Strength), TOF (Time of Flight) and RSPM (Received Signal Phase Method). It follows a brief description of these principles.

Angle measure technique

Angle-based localization systems are a little bit complex in the hardware respect to the range-based one because, since the task is to find a direction, this has to be accomplished with directional antennas or with an array of antenna for each beacon. Main advantage of this technique is that not require time synchronization between stations. Disadvantage are the relatively large and complex system required and the degradation of accuracy when transponder is far from the measurement station.

Ranging technique

Time-based range measurement techniques rely on the direct proportionality by the travel distance and the propagation time of a signal. Multiplying the speed of light c ($c = 299792458m/s$ in the vacuum) by the time it needs to the signal to travel, it is possible to obtain the distance between the two radio stations. About constant c , apart from the advice to use the better approximation value depending on local environmental variables, nothing additionally is possible to say. About the time interval capture,

instead, there are some different technique that differ for the presence of a synchronization clock in the system, one-way or two-way communication and possibility to know the strength of received signal. In TOA (Time of Arrival) the one-way propagation time is measured and then the distance between transmitting and receiver station is calculated. This technique suffers from two main problems, all radio stations in the system have to be synchronized and, an additional payload, containing the timestamp string, has to be transmitted together with the range signal thus increasing the load on the bandwidth. On the contrary, due to the one-way communication this technique can reach the best performance in terms of update frequency. Synchronization for TOA can be achieved by sharing a clock cable (this causes the system to lose the property of being wireless) or by implementing some additional synchronization protocols. TDOA (Time Difference of Arrival) technique try to reduce the need for receiver synchronization relying on the measure of the difference between the time arrival of two signals from different station. In this case, only the transmitters has to be synchronized thus avoiding the synchronization task on the mobile object. TOF (Time of Flight) technique measure the time of flight of the signal in a Two-Way Ranging (TWR) communication between the beacon and the transponder. In the TWR approach, only local calculated duration of time enter in the ranging calculation and, since local period of time is the same for every device, regardless of their own clock references, there is no need for the synchronization as in TOA and TDOA techniques. However, because of tolerance of crystal oscillators in the real physic world, a clock drift happen even if they are perfectly tuned at startup. This clock drift cause inaccuracy in the ranging measure specially when the application require high precision, this because $1nS$ error in time measure, multiplied for the speed of light, can give an approximate error of $30cm$ in the distance calculation. Explained ranging technique rely all on the calculation of the distance starting from the measure of the time of flight. Direct proportionality between the distance from two stations and the propagation time is true only if the station are in the Line Of Sight (LOS) between them. In case of obstacle in the LOS there can be multi path effect that corrupt the measure. An alternative approach, used in trilateration, is to estimate the distance of the mobile object using the attenuation of emitted signal strength. RSS-Based (Received Signal Strength) method can be of two type, the first one use path-loss model to estimate the distance, the second one use RSS-based fingerprint to estimate the location by comparison with a pre-filled database.

1.1.2 Wireless communication system for RTLS

In order to ensure the freedom of movement of the mobile object in the designated area, wireless communication is the only applicable way for the exchange of information between the beacons, the transponder on the object to be tracked and the data processing system. Starting from the classic radio frequency transceiver to the most modern Ultra Wide Band systems, each communication device has its own specific characteristics regarding the range, the energy required for operation, the size of the equipment, the cost, the sensitivity, noise robustness and the available bandwidth on the channel.

Wi-Fi device based on IEEE 802.11 standard and Bluetooth technologies are both communication system working on 2.4 Ghz bandwidth that are suitable for the exchange of information in a limited coverage area like a field or a building. Due to the popularity of these wireless standards and to the small size of the transceivers, common mobile device like tablet, smartphone and watch has them built-in and ready to use. Both of these two technologies also integrate a way to obtain the RSSI (Received Signal Strength Indicator) that, under some assumptions, can be used to estimate the distance between two communicating stations. Due to the lower energy needed to work in short range communications Bluetooth is more suitable when transmission of packet is relative to a small area, while with Wi-Fi technologies more communication distance can be achieved.

Ultra Wide Band is a newer short range radio technology that, with a very low level energy consumption, is suitable for battery powered devices. Differently from common radio communication systems where a central sinusoidal carrier is modulated by the transmitting signal, in UWB transceivers the informations are transmitted with a pulse technique for specific time intervals and occupying a large bandwidth.

GPS Global Position System, for its diffusion in the industrial and consumer market, is the most know localization system in the world. In order to work, a radio equipment on the mobile object constantly receive time signals from a constellation of satellites. Since satellites time references are synchronized with each other there is no need to implement a two-way transmission between the receiver and the satellites, well the communication is one-way from the satellite to the receiver on the Earth using carrier at 1.17, 1.22 and 1.57 Ghz. Use of this range of frequency for the carrier leads to the need of the Line Of Sight between the satellites and the receiver in order to the localization algorithm to works properly, for this reason, GPS can only be used in outdoor scenario. Even if the use of GPS system is free of charge for civil, commercial and scientific uses it is property of the U.S. Government. The Department of Defense of the United State of America periodically release a document called "Global Positioning System Standard Positioning

Service Performance Standard" where the performance in terms of precision of the system are reported. At the time of writing last update of this document from April 2020 reports a medium accuracy of 10m that however depends strictly on the number of visible satellites by the receiver. Clearly, even if the receiver section on the Earth is not that expensive, the same cannot be said for the entire satellite infrastructure which setup and management can only be carried out by a large organization. For the purpose of this work we will refer to the GPS system like a black box device able to return, with a low precision and refresh rate, information about the position of an object on the Earth that carry the GPS receiver.

1.1.3 Position calculation algorithms

Once the measures have been taken, there is the need to apply some algorithms to calculate the position of the target starting from angles or ranges. In this section, some algorithms, different for the nature of the input measure, will be presented. These can be divided into two main categories. Triangulation algorithms are used to calculate the position starting from angle measures while Trilateration algorithms are used when range measurements are available.

Triangulation technique

Triangulation is based on the measure of the angle "seen" by the mobile object between a couple of beacons. Each measured angle determine an arc where the mobile object can be found. An additional arc is then obtained starting from a new angle measurement between a new couple of beacons (one can be the same of the last iteration) and finally the mobile object is in the intersection of this two arches. This scheme has been called *three object triangulation* by *Cohen and Koss* [9]. To deal with this kind of geometric approach there are some common restrictions to consider. First it must be present a Line Of Sight radio path between the mobile object and at least three beacons in order to perform localization on a plane. The mobile object and the beacons, moreover, must not be on the same circumference because, in this case, localization on the plane is not possible since the intersection of the two arc is an other arc and not a point [8].

Several triangulation algorithms have been proposed, they can be classified as *Geometric Triangulation*, *Geometric Circle Intersection*, *Iterative Method* and *Multiple Beacons Triangulation* [9]. In the *Geometric Triangulation* an intensive use of trigonometric functions is made to calculate the position [10, 11]. In *Geometric Circle Intersection* instead first the radius and center of the arc passing through the beacons and the robot are determined and then the intersection between them is computed [12, 13]. In *Iterative Method*

the trigonometric relations are linearized to let the mobile object position converge after some iteration from a starting point [14, 15, 16]. *Multiple Beacons Triangulation* cope with the estimation of the position starting from more than three angle measurements (usually corrupted by error) which is an over determined problem [17],[18],[19].

Each of the many available triangulation algorithms have its own limitation in addition to the common one in *three object triangulation* problem. Some of them can require a particular beacon ordering or can work only on a restricted area, for example, only inside the triangle defined by the three beacons. More improved triangulation methods are available, however, they present an increased complexity in the computation and may require a particular spatial arrangement of the beacons.

Trilateration technique

Trilateration is based on the measurement of the distance by the mobile object and each beacon. Referring to the study case in Figure 1.1, the common approach in trilateration is to treat the coordinate of the mobile object p_{xyz} as the point of intersection of several spheres centered in the location of the n beacon $A_i(x, y, z)$ with $i = 1, 2, \dots, n$ whose radius is the distance between the object to be tracked and the i_{th} beacon d_i . For each of these sphere it has to be:

$$(p_x - A_{ix})^2 + (p_y - A_{iy})^2 + (p_z - A_{iz})^2 = d_i^2 \quad (1.1)$$

In the case of a 3D scenario, as in Figure 1.1 where a minimal number of four anchors are needed, (1.1) represent a system of four non linear quadratic equations to solve in order to obtain the point p_{xyz} where all spheres make the intersection.

One way to cope with this computation is to geometrically linearize the system by converting it into the new problem to find the point of intersection of several planes [20, 21]. In this case one of the i_{th} constrain is used as a linearizing tool for the problem that becomes a system of three equation with three unknowns. Solution p_{xyz} exists and is unique but only if measures from the beacons are not corrupted by noise. While in the real world applications measured distances are corrupted by noise this way to solve the problem is not longer acceptable. Some other iterative technique are available in this case based for example on linear and non linear least square [22, 23] or on the use of Cayley-Menger determinant [24, 25].

Depending on the chosen ranging technique as in Subsection 1.1.1 particular algorithms can be used to exploit some particular property of the measure system. For ToA (Time of Arrival) ranging, measure can be taken using different signaling techniques as in [26]

with direct sequence spread-spectrum or in [27] with Ultra Wide Band radio signal. Other algorithms for ToA based indoor localization systems such as closest-neighbor (CN) and residual weighting (RWGH) can be found in [28].

For TDoA (Time Difference of Arrival) ranging in [29] a solution to the position estimation is to linearize the equations through the use of a Taylor-series expansion with an iterative algorithm, while, in [30] a TDoA measuring method for overlaid geolocation system, which depends on delay measurement using an high precision timer instead of initial synchronization, is proposed.

About RSS based ranging systems, some solutions can be found in [31] where an elliptical directional propagation model (EPM) has been designed to avoid multi path effect and in [32] where a fuzzy logic algorithm is used to improve the location accuracy.

Regard the ToF (Time of Flight) based ranging system in [33] a concept of modulated reflection is proposed to correct error in the propagation time measure and in [34] an algorithm to measure ToF with wireless LAN packets is proposed.

1.1.4 6DOF Position/Attitude localization

Localization made using only ranging (or angle) measurement techniques and relative trilateration or triangulation algorithms as in Section 1.1.3 leads to know only the position of the mobile object (or its coordinate in the chosen reference frame) but nothing is possible to say about the orientation of the target. Since an object moving on a 3D space has six degree of freedom (three for the position and three for attitude), to completely know the pose of the target also the attitude must be taken into account.

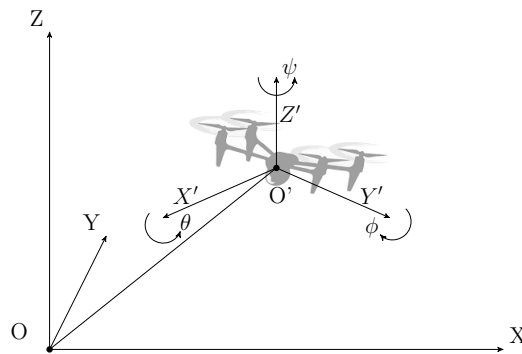


Figure 1.3: Reference scheme for 6DOF localization system.

Figure 1.3 shows a reference scheme for a 6DOF localization system. Main reference coordinate system $O - xyz$ is the one used to localize the target which O' represents the

position of the center of mass. Auxiliary reference system $O' - x'y'z'$, fixed on the body of the target, is the one used to keep track of its orientation. The target, schematized as a quadcopter in the figure, can rotate by an angle Θ around the x' axis performing a roll motion, can rotate by an angle Ψ around the y' axis performing a pitch motion and can rotate by an angle Ψ around the z' axis performing a yaw movement. In order to completely know the pose of the target at least three position coordinate in the reference frame $O - xyz$ and three orientation angle in the reference frame $O' - x'y'z'$ must be known. In the robotic context this kind of problem is commonly known as *Pose of a rigid body* and it can be described making use of *Homogeneous Transformations* that take in account a *translation* and a *rotation* handled by use of *Rotation Matrices*. The attitude of the rigid body (rotation) can be expressed in several way, by *Euler Angles* that includes ZYZ and RPY (Roll, Pitch, Yaw) representation, by *Angle and Axis* method or by *Quaternion* that are also the commonly used [35].

1.1.5 Attitude Estimation

The accurate measurement of the orientation, that plays a critical role in pose estimation, has various application sectors like aerospace, robotics, navigation and so on. Inertial devices called IMU (Inertial Measurement Unit) consist of accelerometers, gyroscope and magnetometer in a self contained device (Figure 1.4) and are the main candidates to cope with Attitude estimation. In order to measure in three dimensions they are made up by three tri-axis sensors (one for acceleration, one for rotational speed and the other one for the Earth magnetic field) each one consisting of three mutually orthogonal sensitive axes, for this reason they are also called 9-DOF IMU. A single tri-axes accelerometer can be sufficient to measure an attitude relative to the direction of gravity when the target is still or is moving at constant speed without any rotation [36]. When the target is subjected to external acceleration the result of the measure in the accelerometer is the sum of the static and the dynamical ones. A single tri-axes gyroscope that measure body angular velocity vector referred to the Earth Centered Inertial (ECI) frame, can be sufficient to know the attitude of the target by simple integration of the angular speed. However, since gyro measure are corrupted by a bias, integration leads to a drift error after a short time. In order to obtain an improved attitude estimation starting from acceleration, angular speed and magnetometer measure a process called *Sensor Fusion* is applied to data to obtain orientation of the target [37, 38]. Sensor fusion can be obtained either using complementary filter [39], Kalman filters [40, 41, 42, 43]. In [44] an adaptive Kalman filter is designed to compensate external acceleration from the residual in the accelerometer.

In [45] accelerometer and magnetometer data are mixed using Gauss-Newton iteration algorithm to obtain the orientation and, in order to avoid the singularity problem of Euler angles, quaternions are used. Some other filtering technique make use of particle filtering [46], fuzzy logic [47], explicit complementary filter (ECF) [48] and gradient based orientation filter [49].

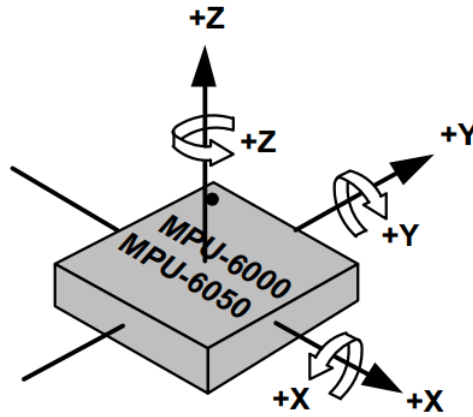


Figure 1.4: IMU axes disposition on MPU6050 by InvenSense.

1.2 Inertial Navigation System

The capability to calculate the position, orientation and velocity of a mobile object by dead reckoning of acceleration and rotation sensor represents an important field of research and, for this reason, many efforts have been done in the study of the Inertial Navigation Systems (INSs) [48, 50, 51]. A crucial point in the INSs is the knowledge of the inertial variables, i.e. position, velocity, acceleration and attitude. In particular, in case of autonomous vehicles applications, they need to know their position and orientation, in order to track a trajectory or in order to cope with general tasks involving navigation and positioning [52, 53, 54]. Most of INSs are based on 9-DOF (Degree Of Freedom) Inertial Measurement Unit (IMU).

A common data fusion process is the strap-down method consisting of: a) integration of the gyro signals for obtaining orientation in the chosen reference frame, using also the magnetometer data; b) determination of the accelerations by means of projection of the accelerometer signals, measured in the body frame, on the desired frame; c) two consecutive integrations of these linear accelerations to obtain speed and then position of the vehicle under consideration [55, 51, 56]. The strap-down method, however, suffers from measurement inaccuracy due to the characteristics of the above sensors. In particular, the bias of these sensors produce drift into the generated output variables, i.e. orientation, angular velocity, accelerations, linear speed and position. Consequently, the IMU-based INS works correctly when the motion has a short time duration, but its behavior degrades rapidly at the increasing of the motion duration [56].

1.2.1 Geographical Coordinate System

Since in the majority of localization applications it is important to estimate the position (and attitude) of some mobile objects on the Earth it is convenient to define some standardized coordinate reference systems referred on the local vertical direction and the Earth's axis of rotation. ECEF (Earth-Centered Earth-Fixed) in Figure 1.5 from [57] for example is a standard 3D reference frame whose origin coincides with the center of the Earth and its axes X, Y are aligned with the international reference pole (IRP) and international reference meridian (IRM) that are fixed with respect to Earth's surface. X axis intersect the Earth at 0^0 latitude and 0^0 longitude and Z axis is directed through the North pole. ECEF rotates with the Earth and allows to know position and altitude of an object on it.

To track and/or control object moving on the Earth's surface, in the point intercepted

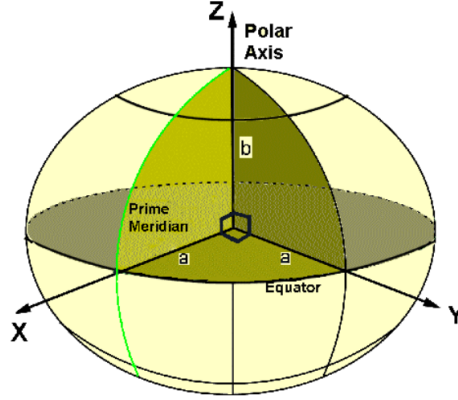


Figure 1.5: ECEF coordinate reference system.

by ECEF coordinate system it is possible to place an additional local reference system such as the NED or ENU ones. NED (North, East, down) and ENU (East, North, up) are two kinds of convention used to represent the motion of an object referred to Earth's surface, they differ essentially for axes direction placement. While in NED convention x_n axis point to North, y_n axis point to East and z_n axis point down to the center of the Earth, in ENU convention, x_e axis point to East, y_e axis point to North and z_e axis point radially out from the Earth's center as in fig 1.6 from [58].

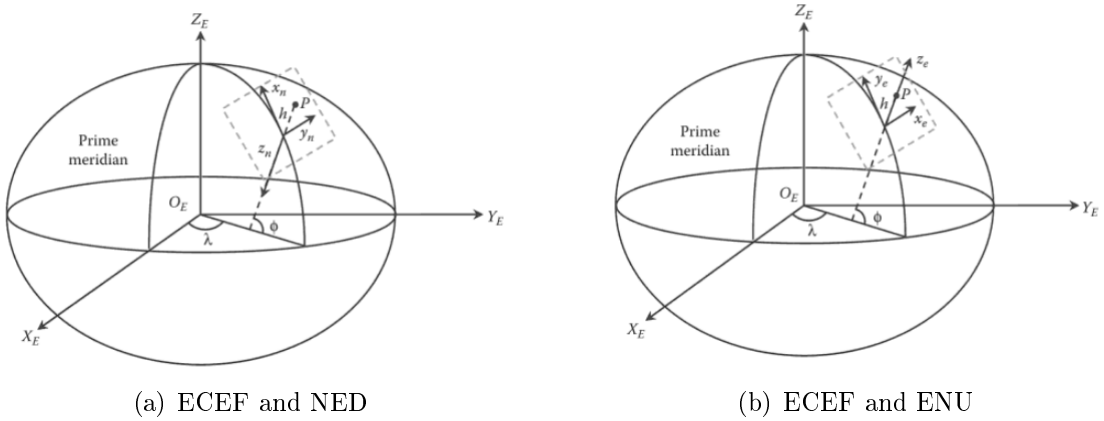


Figure 1.6: NED and ENU reference system.

1.2.2 Mathematical model of the INS

The 6 DOF kinetic model referred to the N -frame (NED frame), in terms of quaternions, is given by [52, 50, 59]:

$$\dot{\boldsymbol{\xi}}_b^n = \frac{1}{2} \mathbf{H}(\boldsymbol{\omega}_{b/n}^b) \boldsymbol{\xi}_b^n, \quad (1.2)$$

$$\dot{\mathbf{v}}_{b/n}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) (\dot{\mathbf{v}}_{b/n}^b + \mathbf{S}(\boldsymbol{\omega}_{b/n}^b) \mathbf{v}_{b/n}^b), \quad (1.3)$$

$$\dot{\mathbf{p}}_{b/n}^n = \mathbf{v}_{b/n}^n, \quad (1.4)$$

where $\boldsymbol{\xi}_b^n = [\xi_0 \ \xi_1 \ \xi_2 \ \xi_3]^\top$ is the quaternion vector, describing the orientation of the B -frame (body frame) relative to the N -frame, $\mathbf{p}_{b/n}^n = [p_x \ p_y \ p_z]^\top$ and $\mathbf{v}_{b/n}^n = [v_x \ v_y \ v_z]^\top$ are respectively the position and velocity vectors expressed in N coordinates (x, y, z) , $\boldsymbol{\omega}_{b/n}^b = [p \ q \ r]^\top$ is the angular velocity expressed in the B -frame, matrix $\mathbf{H}(\boldsymbol{\omega}_{b/n}^b)$ is the propagation matrix of the quaternion, and it is defined as:

$$\mathbf{H}(\boldsymbol{\omega}_{b/n}^b) = \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix},$$

matrix $\mathbf{V}(\boldsymbol{\xi}_b^n)$ is the rotation matrix from the B -frame to N -frame, expressed in terms of the quaternion, and it is defined as:

$$\mathbf{V}(\boldsymbol{\xi}_b^n) = \begin{bmatrix} \xi_0^2 + \xi_1^2 - \xi_2^2 - \xi_3^2 & 2(\xi_1 \xi_2 - \xi_0 \xi_3) & 2(\xi_1 \xi_3 + \xi_0 \xi_2) \\ 2(\xi_1 \xi_2 + \xi_0 \xi_3) & \xi_0^2 - \xi_1^2 + \xi_2^2 - \xi_3^2 & 2(\xi_2 \xi_3 - \xi_0 \xi_1) \\ 2(\xi_1 \xi_3 - \xi_0 \xi_2) & 2(\xi_2 \xi_3 + \xi_0 \xi_1) & \xi_0^2 - \xi_1^2 - \xi_2^2 + \xi_3^2 \end{bmatrix},$$

and matrix $\mathbf{S}(\boldsymbol{\omega}_{b/n}^b)$ is defined as:

$$\mathbf{S}(\boldsymbol{\omega}_{b/n}^b) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}.$$

Note that the term $\mathbf{V}(\boldsymbol{\xi}_b^n) \mathbf{S}(\boldsymbol{\omega}_{b/n}^b) \mathbf{v}_{b/n}^b$ in (1.3) represents the Coriolis term.

From the quaternion it is possible to obtain the Euler angles by means of the following equations:

$$\boldsymbol{\Theta}_b^n = \begin{bmatrix} \arctan(\frac{V(3,2)}{V(3,3)}) \\ -\arcsin(V(3,1)) \\ \arctan(\frac{V(2,1)}{V(1,1)}) \end{bmatrix}, \quad (1.5)$$

where $V(i, j)$ is the element of $\mathbf{V}(\boldsymbol{\xi}_b^n)$ at row i and column j .

The INS is based on an IMU, connected to the rigid body under study, and it consists of three gyros, three accelerometers and three magnetometers. The gyros give the angular

velocity vector referred to the Earth Centered Inertial (ECI) frame, $\boldsymbol{\omega}_{b/i}^b$, and consequently the vector $\boldsymbol{\omega}_{b/n}^b$ can be obtained as $\boldsymbol{\omega}_{b/n}^b = \boldsymbol{\omega}_{b/i}^b - \boldsymbol{\omega}_{ei}^b$, where $\boldsymbol{\omega}_{ei}^b$ is the velocity of the Earth referred to the B -frame. Considering that the angular velocity of the Earth around the z -axis of the ECI frame is very small, $\boldsymbol{\omega}_{ei}^b$ is usually neglected, so $\boldsymbol{\omega}_{b/n}^b \approx \boldsymbol{\omega}_{b/i}^b$ [56, Remark 2]. The accelerometers measure the global acceleration along the B -axes, i.e. the acceleration due to the motion and the gravity acceleration, and consequently, it is necessary to compensate for gravity. Then, denoting the acceleration measurement provided by the IMU as:

$$\boldsymbol{\gamma}^b = \begin{bmatrix} \gamma_x & \gamma_y & \gamma_z \end{bmatrix}^\top = \dot{\mathbf{v}}_{b/n}^b + \mathbf{S}(\boldsymbol{\omega}_{b/n}^b) \mathbf{v}_{b/n}^b - \mathbf{V}(\boldsymbol{\xi}_n^b) \mathbf{g}, \quad (1.6)$$

where $\mathbf{g} = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^\top$ is the gravity acceleration referred to the N -frame and $\mathbf{V}(\boldsymbol{\xi}_n^b) = \mathbf{V}^{-1}(\boldsymbol{\xi}_b^n)$, then the speed equation for describing the strap-down model can be written as:

$$\dot{\mathbf{v}}_{b/n}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g}. \quad (1.7)$$

Then, we conclude that the strap-down model of INS, mainly used in this manuscript, is given by [59, Eq.s (1)-(3)]:

$$\dot{\boldsymbol{\xi}}_b^n = \frac{1}{2} \mathbf{H}(\boldsymbol{\omega}_{b/n}^b) \boldsymbol{\xi}_b^n, \quad (1.8)$$

$$\dot{\mathbf{v}}_{b/n}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g}, \quad (1.9)$$

$$\dot{\mathbf{p}}_{b/n}^n = \mathbf{v}_{b/n}^n. \quad (1.10)$$

1.3 The Hybrid system framework

Since localization problems exhibit both continuous-time dynamical characteristic (the flow of the mobile object) and discrete-time dynamical one (i.e. range or angle measure arrives at discrete time) there is the need to use a unified mathematical framework able to manage both of them at same time. The *Hybrid Dynamical System framework* [60] is the main tool used in this thesis to cope both with continuous and discrete time dynamics. Here is reported a brief introduction on the modeling and analysis techniques that will be used in this manuscript as an extract from [61] and [60].

The general form of a hybrid system model can be represented in the following form:

$$\begin{cases} x \in C, & \dot{x} \in F(x) \\ x \in D, & x^+ \in G(x) \end{cases} \quad (1.11)$$

In this representation, the state of the system x can evolve continuously by the differential inclusion $F(x)$ while in the set C and discretely by difference inclusion $G(x)$ while in the set D . Notation $\dot{x}$ so represents the continuous-time derivative of the state of the system while x^+ represents the value of the state after a discrete time update. By substituting set-valued mappings and differential and difference inclusions with differential and difference equations 1.12 can be expressed in the following simpler and less general form:

$$\begin{cases} x \in C, & \dot{x} = f(x) \\ x \in D, & x^+ = g(x) \end{cases} \quad (1.12)$$

According to 1.11, in 1.12 the state x can change continuously in time by differential equation $\dot{x} = f(x)$ and discretely by difference equation $x^+ = g(x)$. Regardless of the representation used, the behavior of a dynamical system that can be described by a differential equation or inclusion is referred to as *flow* and C and F (or f) are respectively the *flow set* and the *flow map*. The behavior of a dynamical system that can be described by a difference equation or inclusion is referred to as a *jump* and D and G (or g) are respectively the *jump set* and the *jump map*. The collection of the element C , F , D and G is called the *data* of the hybrid system.

Scientific literature is rich of application making use of *The Hybrid system framework* to model physical systems. In [62] the bouncing ball tracking and state estimation with impact is treated. In [63] it is used to model the dynamics of a 3D bipedal walking robot, in [64, 65] modeling and control of a snake robot using the Hybrid framework is proposed. Other application in the robotic field like an event-triggered tracking control

of unicycle mobile robots [66], modeling and control of a flying robot interacting with the environment [67] and an impedance control of an aerial manipulator [68] are present in literature. Also biologic and medical system has mixed continuous and discrete dynamics that can be both mixed using the hybrid framework [69].

Recalling the concept of *set-valued mapping* and *domain* of a set-valued mapping ([61],2.1), data of a hybrid system in $\mathbb{R}^n$ consist of the following four elements:

- a set $C \subset \mathbb{R}^n$, called the *flow set*
- a set-valued mapping $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ whit $C \subset \text{dom}F$ called the *flow map*
- a set $D \subset \mathbb{R}^n$, called the *jump set*
- a set-valued mapping $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ whit $D \subset \text{dom}G$ called the *jump map*

Hybrid system with the data as above will be represented by the notation $\mathcal{H} = (C, F, D, G)$ or, briefly, by $\mathcal{H}$.

While in continuous time systems solution are parametrized by $t \in \mathbb{R}_{\geq 0}$ or the time evolution, and in discrete time systems solutions are parametrized by $j \in \mathbb{N}$ or the number of discrete steps, in hybrid systems, since they cope with both continuous time and discrete time dynamics, solutions are parametrized by some subset of $\mathbb{R}_{\geq 0} \times \mathbb{N}$ properly named *Hybrid time domains*.

Since it is not the scope of this Thesis, for a detailed treatment regarding the Hybrid System Framework, the solutions of a hybrid system and their properties, convergence and stability analysis, the reader is postponed to [60].

Chapter 2

Experimental Setup

OUTLINE. Since results presented in this thesis work have been verified by means of experimental tests, in this chapter, a description of the workbench setup are presented. We will start by introducing the processing units used to perform the various tasks like IMU sensors reading, attitude estimation and IMU calibration used both for indoor and outdoor applications. Then, an indoor setup based on UWB ranging devices and an anthropomorphic manipulator used as reference system and an outdoor setup based on IMU/GPS measurements logger will be described.

2.1 Calculator unit

Calculator (or processing) units are the components of an RTLS system responsible for sensor readings and data elaborations. Processing unit choice is mainly based on the characteristics of the process to be performed in terms of the required computing power, footprint, weight, supply power, wired or wireless communication and ability to interconnect with other devices. For the purpose of this thesis work microcontroller units from *Atmega*, *Texas Instrument* and *Broadcom* have been chosen. They are built-in in prototype boards like *Arduino Nano*, *Arduino Mega*, *Raspberry Pi 3* and *F28379D* from Texas Instrument. In detail, *Arduino Nano*, in Figure 2.1 (a), has been chosen to build the UWB beacons in the indoor experimental setup for its smaller dimension and low power consumption that allows to power them from a battery for a long time. *Arduino Mega*, in Figure 2.1 (b), has been used to build a GPS/IMU logger mainly because of the direct compatibility with an IMU sensor by Bosch. *Raspberry Pi*, in Figure 2.2 (a), has been used to build the UWB transponder hardware in the indoor experimental setup mainly because with its WiFi capabilities it allows to maintain a stable wireless connec-

tion between the mobile object and the PC running Matlab/Simulink. Finally *F28379D*, in Figure 2.2 (b), from Texas Instrument has been chosen to perform Hardware in The Loop tests due to its capability to interact with Matlab/Simulink environment and to the presence of multiple core that allow to simulate at the same time the behavior of two systems with different dynamic like for example a controller or observer and a model of the plant.

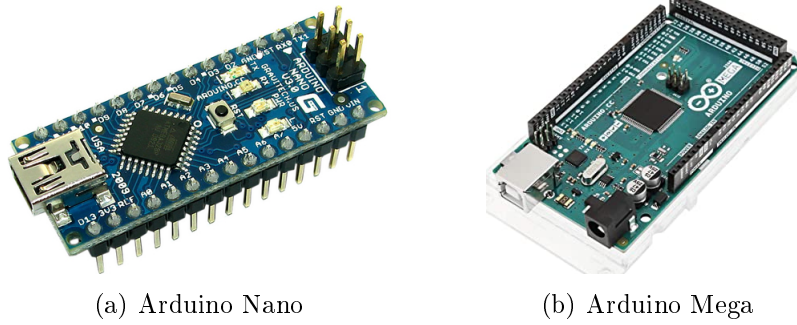


Figure 2.1: Arduino prototype boards.

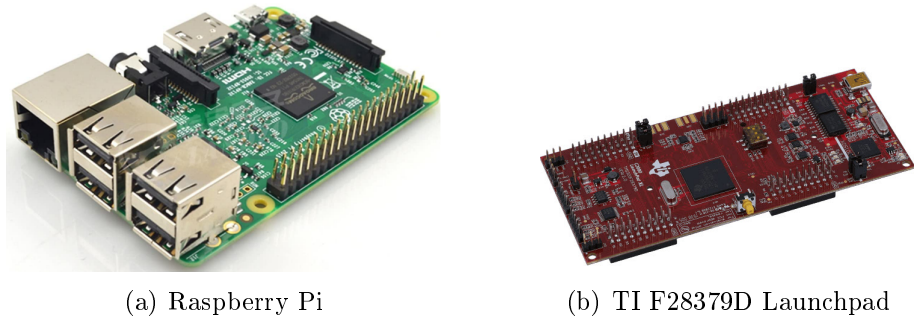


Figure 2.2: Raspberry and Texas Launchpad.

2.2 IMU devices

In order to measure accelerations, body angular speed and magnetometer data on the mobile object, to perform attitude estimation as in Section 1.1.4, an IMU device is needed both for indoor and outdoor applications. Moreover, in addition to the sensor device, a calculator unit is needed in order to read the data from the device, elaborate them, and return information to other connected systems in the chain. About the IMU device itself two of the ones available on the electronic market have been chosen in this work, they are the *BNO055* from Bosch Sensortec 2.3 (a) and the *LSM9DS1* from ST Microelectronics 2.3

(b). Both are system-in-package chips that include 3D digital linear acceleration sensor, 3D digital angular rate sensor and 3D digital magnetic sensor in a very small footprint, indeed, the package dimensions of the Bosch one are 3.8x5.2x1.13mm and 3.5x3.0x1.0mm for the ST ones.

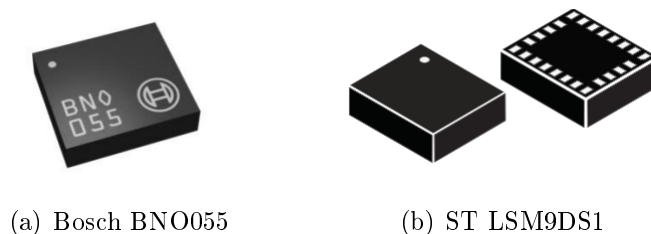


Figure 2.3: IMU chips.

Due to the small dimensions IMU chips are not really easy to handle, moreover they need some external electronic components for power stabilization and bus level translation, for this reason, some breakout boards called *shield*, that has built-in additional electronic and give to the final user the possibility to put them on other boards or fix them by screws, are available on the market. About the selected devices, regarding the BNO055, a breakout board from Arduino in Figure 2.4(a) has been chosen mainly because its layout and pin arrangement are fully compatible with the controller board *Mega* by Arduino. Regarding the LSM9DS1 a breakout board by Adafruit, as in Figure 2.4(b), has been chosen also for its smaller footprint.

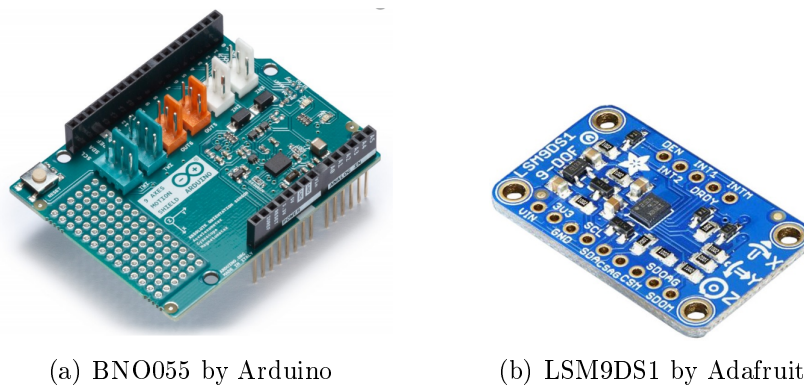


Figure 2.4: IMU breakout boards.

The BNO055 IMU, respect to the LSM9DS1, has an additional software feature. Instead of just reading the inertial sensor it integrates a calibration and sensor fusion algorithm. This make possible, for any upper software layer, to know directly the attitude of the sensor and therefore of the object where it is placed on, however, due to the com-

pany's commercial policies these algorithms are not public and therefore are armored for any change. Since calibration and data fusion algorithms are key components for a good attitude estimation they will be discussed in more detail in this chapter and, an implementation for the LSM9DS1 (that not has built-in calibration and sensor fusion) will be addressed.

By the way, LSM9DS1 respect to the BNO055 has the advantage to be fully compatible with Matlab/Simulink environment when used with Raspberry Hardware. This feature has been used to build the indoor test setup as will be showed on the next sections.

2.2.1 IMU calibration

IMU are generally affected by systematic error given by imprecise scaling factors, axes misalignments, non uniform cross-axis sensitivities and non zero biases that decrease accuracy in the position and attitudes estimation. IMU calibration procedures refers to the process of identifying these quantities in order to obtain a more accurate measure. Since each different sensor in the IMU (gyroscope, accelerometer and magnetometer) presents a different behavior while in static condition or dynamical motion, each of them will be treated separately. A reference input is a fundamental requirement of any sensor calibration, for instance, the local gravity can be used to compare accelerometers data [70] or the local geomagnetic field can do the same for magnetometers [71, 72, 73]. However is quite difficult to find a reference sensor input for gyroscopes, as the Earth rotation rate is too small in magnitude to be used for consumer sensors.

Gyroscope calibration

Since for the gyroscope sensor no reference system are available, in the literature many efforts have been made to find some other calibration ways. In [74] a gyroscope calibration method in the homogeneous magnetic field by the help of magnetometer has been proposed. It is shown that, with sufficient rotation excitation, the homogeneous magnetic field vector can be exploited to serve as a good reference for calibrating low-cost gyroscopes. In [75] with a continuous excitation of two axes of the gyroscope and using continuous-time mode reversal technique a reduction of bias drift of a MEMS pendular gyroscope is obtained. In [76] a prediction algorithm based on Kalman filtering and the computation of moving averages is proposed to predict Gyroscope bias. In [77] a tri-axial MEMS Gyroscope Calibration Method that take account of temperature change is presented. However, for the purpose of this thesis work, only static bias estimation has been taken in account to calibrate the gyroscope measure. To obtain the gyroscope bias

calibration constant vector $\boldsymbol{\Omega}_C^b = \begin{bmatrix} p_x & q_y & r_z \end{bmatrix}^\top$, each of the three gyroscope axis measure $\boldsymbol{\omega}_{b/n,m}^b = \begin{bmatrix} \omega_{m,x} & \omega_{m,y} & \omega_{m,z} \end{bmatrix}^\top$ has been integrated for a minute and then the mean value has been calculated. So, in the following, with the term calibrated gyroscope measurements we will refer to gyroscope measurements corrected by subtracting the constant bias, according to equation (2.1).

$$\boldsymbol{\omega}_{b/n}^b = \boldsymbol{\omega}_{b/n,m}^b - \boldsymbol{\Omega}_C^b \quad (2.1)$$

Accelerometer calibration

Since accelerometer measures, with even small error and biases, could lead in a fast position drift in INS systems, they have to be calibrated as accurate as possible. Moreover, the calibration parameters can change under different environmental conditions [78] and temperature change [79] so accelerometers have to be calibrated before each use or when environmental conditions and temperature are significantly changed. The conventional method to calibrate a tri-axes accelerometer is to rotate it on some fixed known tilt angles and rely on the principles that, when in stationary state, the sum of the outputs is equal to the gravity vector [80, 81, 82]. Other methods can involve an external devices as a reference such an actuator [83] or position sensor [84]. There exists also other methods that do not require precise reference measurement or external devices, for instance, in [85] the measurement data from eighteen different attitudes and a least square estimation algorithm are used to obtain a *on-the-field* method. In [86] a multi-position calibration method that does not require special aligned mounting specifications was designed for MEMS IMU. In [87] a semi-automatic calibration technique, using six randomly chosen tilt angles, has been used to find three gain factors and three biases for a triaxial accelerometer. Moreover in [88, 89] two procedures for online calibration are exposed.

In this thesis work, an *on-the-field* autocalibration method based on Gauss-Newton nonlinear optimization from [90] has been used to compute bias and scale factor for each axis and the cross-axis symmetrical factors. A mathematical model that describe the accelerometer calibrated output $\boldsymbol{\gamma}^b$ can be described in matrix form as:

$$\boldsymbol{\gamma}^b = S_a(\boldsymbol{\gamma}_m^b - O_a) \quad (2.2)$$

where

$$S_a = \begin{bmatrix} S_{a,xx} & S_{a,xy} & S_{a,xz} \\ S_{a,yx} & S_{a,yy} & S_{a,yz} \\ S_{a,zx} & S_{a,zy} & S_{a,zz} \end{bmatrix}, O_a = \begin{bmatrix} O_{a,x} \\ O_{a,y} \\ O_{a,z} \end{bmatrix} \quad (2.3)$$

are the scale factor matrix and the bias vector respectively and γ_m^b are the uncalibrated sensor measures. S_a and O_a in (2.3) allow to take account of both axis misalignment and the crosstalk effect between different channels caused by the sensor electronics. In particular, the diagonal elements of S represent the scale factors along three axes, whereas the other element of S_a are the cross-axis factors. Imposing the symmetry constraint on S_a , that is $S_{a,xy} = S_{a,yx}$, $S_{a,xz} = S_{a,zx}$ and $S_{a,yz} = S_{a,zy}$ the resulting model has nine independent parameters. To perform autocalibration the sensor has to be placed in N different random orientations and the sensor output are evaluated while maintaining it in a strictly static condition.

Magnetometer calibration

When no magnetic disturbances are present the magnetometer measure a constant local magnetic field vector. Since this vector point to the local magnetic North it can be used for absolute heading estimation in opposition to the gyroscope integration that can measure only relative change in orientation because of the drift. The problem to find bias, scale gains and cross-axes factors has been addressed in literature in many works. In [91, 92, 93, 94, 95] various algorithms have been exploited to compute calibration parameters in stand-alone mode where only magnetometers data are used, moreover, other algorithms combine magnetometers and accelerometers data to improve calibration parameters estimate like in [96]. In this work, an ellipsoid fitting method from [97] has been used to perform the sensor calibration. State that the uncalibrated magnetic field measurements in the body (sensor) frame can be grouped in a vector $\phi_m^b = [\phi_x \ \phi_y \ \phi_z]^\top$ a mathematical model that describe the calibrated magnetometer measures can be described in matrix form as in (2.4):

$$\phi^b = S_m(\phi_m^b - O_m) \quad (2.4)$$

where

$$S_m = \begin{bmatrix} S_{m,xx} & S_{m,xy} & S_{m,xz} \\ 0 & S_{m,yy} & S_{m,yz} \\ 0 & 0 & S_{m,zz} \end{bmatrix}, O_m = \begin{bmatrix} O_{m,x} \\ O_{m,y} \\ O_{m,z} \end{bmatrix} \quad (2.5)$$

are respectively the upper triangular matrix taking account of scaling factor, three-axis orthogonality and iron interference and the bias vector. Measurements ϕ_m^b are taken on a number of positions N and combined to calculate the parameter matrix by a least square method. Once obtained S_m and O_m with the calibration procedure, calibrated magnetometer measures ϕ^b can be obtained applying (2.4).

2.2.2 IMU attitude estimation

As already shown in Section 1.1.5 attitude estimation is a key component of a RTLS system. For the purpose of building up an equipment to perform experimental test in this thesis work, attitude estimation algorithm from [38] has been considered. In [38] two implementations are given in order to estimate attitude of the sensor respect to the reference frame, first one is referred as IMU filter and make use only of gyroscope and accelerometer measurements (imposing a predefined reference direction for the magnetic field in the Earth frame) in order to obtain the attitude estimation. Second one, referred as MARG filter (Magnetic, Angular Rate and Gravity), and also known as AHRS (Attitude and Heading Reference System), make also use of magnetometer measures to estimate a complete heading and attitude of the sensor frame respect to the reference one. Both filter derivations rely on a quaternion representation of the attitude in order to eliminate singularity associated with Euler angles representations and make use of a gradient descent algorithm and a gain based fusion process to mix gyroscope, accelerometer and magnetometer measurement and obtain the attitude estimation. Filter derivations start by analyzing the orientation from angular rate sensing (gyroscope), then introducing the gravity vector observations (accelerometer) and therefore presenting the IMU fusion algorithm. Following, the magnetic distortion compensation is taken in account and the final MARG attitude estimator, that, respect to the IMU one, doesn't need for a predefined reference system of the Earth's magnetic field is presented.

Starting from calibrated angular rate measurements $\omega_{b/n}^b$ in (2.1), they can be arranged into a vector S_ω as defined in 2.6. The rate of change in orientation of the reference frame N relative to the sensor frame B can therefore be calculated by (2.7).

$$S_\omega = \begin{bmatrix} 0 & \omega_{m,x} & \omega_{m,y} & \omega_{m,z} \end{bmatrix}^\top \quad (2.6)$$

$$\dot{\xi}_b^n = \frac{1}{2} \hat{\xi}_b^n \otimes S_\omega \quad (2.7)$$

and finally the orientation of the Earth's frame relative to the sensor frame $\hat{\xi}_b^n$ can be

computed by integration of (2.7) provided that initial conditions are known.

Regarding the observation of orientation starting by measure of the gravity vector, a consideration to be made is that if the direction of the gravity is known in the N frame, a measurement in the sensor frame allow to obtain the orientation of the sensor frame relative to the N reference frame. However, for any given measurements there will be infinite solutions represented by all orientations achieved by the rotation around an axis parallel with the field. Since a quaternion representation require a complete solution to be found, this can be achieved by the formulation of an optimization problem where an orientation of the sensor $\hat{\xi}_b^n$ is that which aligns a predefined reference direction of the field in the Earth frame. Using a gradient descent alghoritm to calculate the quaternion update leads, in [38], to the following equation for attitude estimation by gravity vector observation:

$$\xi_{b\nabla,t}^n = \hat{\xi}_{b\nabla,t-1}^n - \mu_t \frac{\nabla f}{\|\nabla f\|} \quad (2.8)$$

where μ_t is a scalar parameter responsible for the convergence speed and ∇f is the gradient objective function defined by sensor measurements (cfr. 34 in [38]).

The orientation $\xi_{b,\omega}^n$ calculated starting by integration of (2.7) for gyroscope measures and $\xi_{b\nabla,t}^n$ calculated from for gravity measurements by (2.8) are mixed in the fusion equation (2.9):

$$\xi_{b,t}^n = \beta_t \xi_{b\nabla,t}^n + (1 - \beta_t) \xi_{b,\omega}^n, 0 \leq \beta_t \leq 1 \quad (2.9)$$

where β_t is the weight applied to each orientation calculation. Equation (2.9) represents the first IMU filter solution that rely only on gyroscope and accelerometer measurements and, in order to be implemented, needs a predefined reference direction for the Earth's magnetic field to be chosen. To take account also of magnetometer measurement and obtain the MARG filter an additional consideration have to be made. The measured direction of the Earth's magnetic field in the Earth frame at time t , $\hat{\phi}_t^n$, can be computed as the normalised magnetometer measurement, ϕ_t^b , rotated by the estimated orientation of the sensor provided by the filter, $\xi_{b,t-1}^n$, as described by equation (2.10).

$$\hat{\phi}_t^n = \begin{bmatrix} 0 & \hat{\phi}_x & \hat{\phi}_y & \hat{\phi}_z \end{bmatrix} = \xi_{b,t-1}^n \otimes \phi_t^b \otimes \xi_{b,t-1}^{n*} \quad (2.10)$$

This approach, which leads to determination of the complete scheme block in Figure 2.5 from [38] where both gyroscope accelerometer and magnetometer measures are used. This complete approach also eliminates the need for the reference direction of the earth's

magnetic field to be predefined.

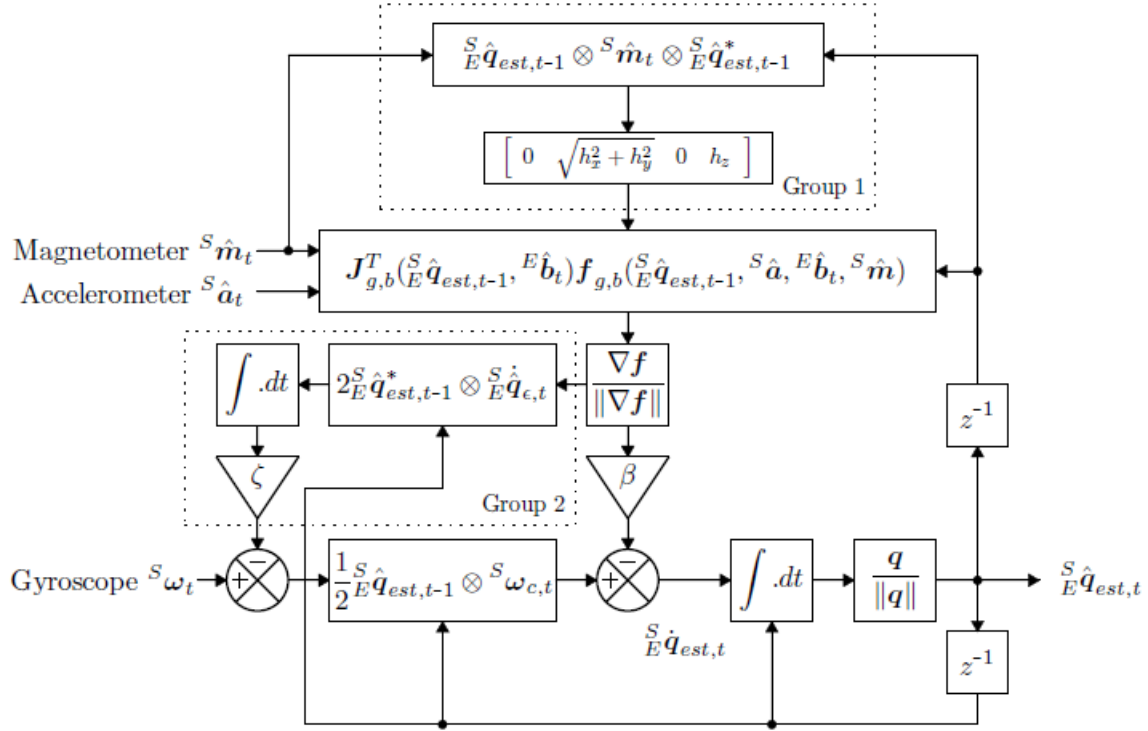


Figure 2.5: MARG attitude estimation by Madgwick.

2.3 Experimental Test Setup

In order to experimentally verify the estimators proposed during this thesis work two test setup have been built. One for indoor tests (where GPS signals are not available and an UWB beacon system has been used) and an other one for the outdoor tests. In the following subsections a description of how they were designed is shown.

2.3.1 Indoor test setup

About the indoor test setup it consists of:

1. *Ranging system*: it is composed by four fixed beacons placed on the testing scene and a mobile beacon mounted on the object to be tracked.
2. *Attitude estimator*: it consists of a 9-axis IMU that includes: gyroscope, accelerometer and magnetometer sensor.
3. *Processing unit*: it consists of a Raspberry Single Board Computer with Wifi connection.
4. *Reference system*: it is a 6-axes industrial robot system.

Figure 2.6 shows a photo of the experimental setup. In Figure 2.6(a) a fixed beacon unit is showed, in (b) the mobile plate to be tracked (carrying the transponder, IMU and processing unit), in (c) an overview of the indoor scenario.

Ranging system. This system is composed by four fixed devices placed on the scene and a mobile device placed on the object to be tracked. The beacon hardware is based on DW1000 IC from Decawave, a single chip IEEE802.15.4-2011 UWB compliant device with internal high precision counter for time-of-flight measurement (each LSB of the timestamp timer is 15.65 pS that leads to maximum theoretical accuracy of $\pm 5\text{mm}$). Since a two-way ranging technique has been used to perform ToF measures, more time captures are needed to compute a single distance measurement. This lead to a measurements accuracy, as indicated on the DW1000 manufacturer website, of $\pm 10\text{cm}$ instead of the maximum theoretical accuracy of $\pm 5\text{mm}$. Moreover each beacon device is equipped with an external Atmel microcontroller to perform Two Way Ranging algorithms. System precision is about 10cm with an actual equivalent refresh rate up to 12Hz.

Attitude estimator. The attitude estimator block consists of both hardware and software components. Hardware consists of a LSM9DS1, a ST Microelectronics chip that integrate 3 acceleration channels, 3 angular rate channels and 3 magnetic field channels

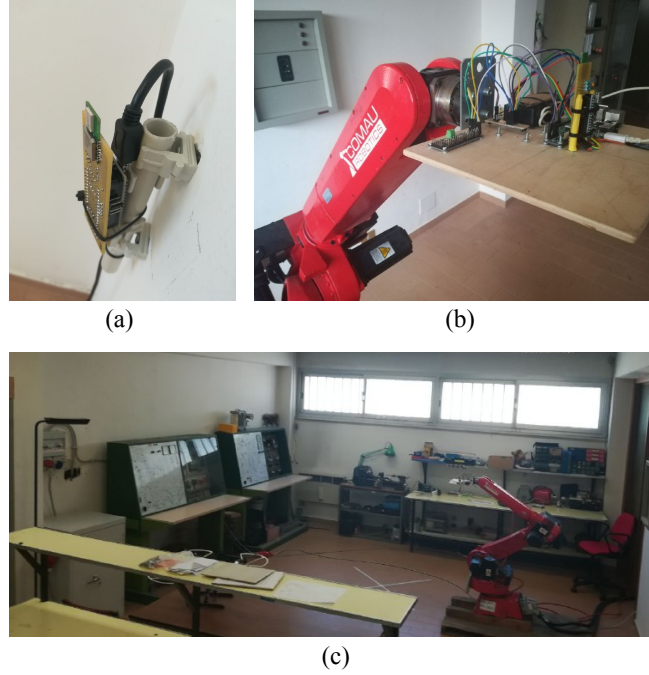


Figure 2.6: Photo of the indoor experimental setup.

with MEMS technology. The LSM9DS1 is also equipped with a I2C bus, so it can be connected to the bus of the processing unit. The ST-Microelectronics data-sheet gives $\pm 2g$ as linear acceleration full scale, $\pm 4G$ as full scale for the magnetic field and ± 245 degree-per-seconds as angular rate full scale for the gyroscope. About the sensitivity it is reported to be $0.061\text{mg} / \text{LSB}$ for the accelerometer, $0.14 \text{ mG} / \text{LSB}$ for the magnetometer and $8.75 \text{ mdps} / \text{LSB}$ for the gyroscope at 25°C operating temperature. Software elements, that are needed to estimate the attitude, are designed with Simulink and they run on the processing unit. Particular attention must have been paid to the calibration phase of the IMU in order to minimize gyro drift effects, acceleration norm errors and magnetometer misalignments.

Processing Unit. The processing unit consists of a Raspberry Pi3 board that runs compiled C code generated from Matlab/Simulink coder. Raspberry Pi3 is based on an ARM Cortex-A53, a 4-cpu 64bit superscalar processor running at 1.2Ghz able to communicate over I2C bus and a Wifi connection. Software that runs on the processing unit receives all the range data from UWB beacons and inertial measurements from IMU.

Reference System. All the explained components, except for fixed beacons, are placed on a plate that is fixed on the last link of a Smart Six-1.4 industrial robot Comau. It is an anthropomorphic robot with 6 degrees of freedom, 0.05mm repeatability and 6Kg maximum payload. Imposing a trajectory on the scene, it is used as reference for

algorithms quality evaluation. The robot is equipped with C4G controller that is also able to communicate with the Processing Unit, through wireless UDP protocol. The information shared with the processing unit are position and attitude of the plate where the measurement unit is fixed.

2.3.2 Outdoor test setup

About the outdoor test setup a platform able to store IMU and GPS measurements has been built to perform vehicle based experiments. In particular it consists of:

- a microcontroller board based on the ATmega2560;
- a 9-DOF IMU (Bosch BNO055);
- a GPS module (Sanav FV-U8).

As explained in Section 2.2 the BNO055 IMU from Bosch has a built-in Sensor Fusion algorithm for attitude estimation, however, to obtain more accurate data, the calibration procedure has been executed each time a new test has been made. Acquisition rate for IMU data and estimated attitude is 100Hz for the BNO055. Sanav FV-U8 is a commercial GPS with 1Hz update rate and a position accuracy of $\pm 2.5m$ by datasheet. Arduino Mega microcontroller board is here used with a custom firmware to acquire both IMU and GPS data from the sensor for further computations.

Chapter 3

Hybrid observer for Indoor Localization

OUTLINE. In this chapter, a novel observer solution for position estimation in indoor environment, is proposed. In particular the setting of the hybrid systems [60] is used, which provide an elegant framework for the proposed solution and allows to obtain a formal mathematical model of a system with sampled measurements and random time-of-arrival. In the proposed application the position is measured by means of a sensor network composed by fixed beacons placed on the indoor environment and a mobile beacon mounted on the object to be tracked. The mobile beacon communicates with all the fixed beacons by means of ultra wide-band signals, and the distance between them is computed by means of ToF techniques. Moreover also inertial measurements are used when the position measurements are not available. Two main problems have been considered in the proposed architecture: the fact that the beacons work with a lower update rate than the IMU, and that the mobile beacon can communicate with only one fixed beacon at a time.

3.1 Mathematical model and problem statement

Let us consider the classical strap-down problem in inertial navigation, already presented in Section 1.2.2. Model (1.8)-(1.10) is the classical model used in inertial navigation. It appears as a non-linear differentiable model, where the quaternions and the inertial velocity can be obtained by means of the body turn rate and inertial acceleration measurements. These measurements are carried out by means of the IMU. On the contrary the position is obtained by means of velocity integration. At this point it is evident that an observer based only on the model is not robust, since the integration of the speed, for the knowledge of the position, leads to drift problems, and for this reason a position measurement is required. In the proposed application the position is measured by means

of a sensor network composed by fixed beacons placed on the indoor environment and a mobile beacon mounted on the object to be tracked. The mobile beacon communicates with all the fixed beacons by means of ultra wide-band signals, and the distance between them is computed by means of ToF techniques. However two main problems are presented in this architecture:

- The beacons work with a lower update rate than the IMU, this means that for each position measurement, it is possible to collect more IMU-measurements.
- The mobile beacon can communicate with only one fixed beacon at a time.

For these reasons a suitable data fusion procedure is required in order to exploit all the available information correctly. In particular, an open loop estimation, based on the inertial model and IMU measurements, will be provided until a new position measurement is available. The fact that the mobile beacon can communicate with only one fixed beacon at a time represents a further problem because, in order to obtain one position measurement, it is necessary that the mobile beacon receives the distances from at least three fixed beacons, and then is possible to obtain the position with trilateration methods. However, if the object is moving, the first and the second measurements result delayed with respect to the time instant when the position is computed. This introduces errors that must be taken into consideration especially for very fast systems.

The main idea in this work is to perform a partial update of the state at the same time instant when the distance (and not absolute position) from one fixed beacon is available, and this means to consider different output maps depending of the fixed beacon involved in the measurement. The proposed mechanism is as follows. At the beginning an open loop estimation, based on the inertial model and IMU measurements, will be provided until a distance from one fixed beacon is available. Then the estimated position is only partially updated based on this distance measurement (in the following the update mechanism will be shown). Then the observer will work at open loop again, until a second distance from another fixed beacon is available. During this second phase the position estimation is again partially updated with the same mechanism, but using a different output map. And so on until all the distances from the fixed beacons have been exploited. Finally the procedure restarts from the first beacon. For these reasons, in the following, we speak about *sequential measurement*. In the following these concepts will be mathematically formalized.

Looking at model (1.8)-(1.10), it is evident that the quaternion dynamics is decoupled from the speed $\mathbf{v}$ and position $\mathbf{p}$ dynamics, so it can be treated independently from the

knowledge of speed and position since it depends only from the body turn rate ω . For this reason, since the main focus of this work is the localization, the following reduced state-space form is considered, naturally derived from (1.8)-(1.10):

$$\dot{\mathbf{s}} = \mathbf{A}\mathbf{s} + \boldsymbol{\varphi}(\mathbf{s}, \mathbf{u}, \mathbf{g}), \quad (3.1)$$

where the state vector $\mathbf{s}$ and the input $\mathbf{u}$ are:

$$\mathbf{s} = \begin{bmatrix} \mathbf{v} & \mathbf{p} \end{bmatrix}^T, \quad \mathbf{u} = \begin{bmatrix} \omega & \gamma \end{bmatrix}^T,$$

while matrix $\mathbf{A}$ and vector field $\boldsymbol{\varphi}$ are:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{I}_3 & \mathbf{0}_3 \end{bmatrix}, \quad \boldsymbol{\varphi}(\mathbf{s}, \mathbf{u}, \mathbf{g}) = \begin{bmatrix} \mathbf{V}(\boldsymbol{\xi})\boldsymbol{\gamma} + \mathbf{g} \\ \mathbf{0}_3 \end{bmatrix}.$$

It is assumed that an output of system (4.1) is accessible at discrete instants of time, resulting in the distance of the σ -th beacon, defined as:

$$d_\sigma(t_k) := \sqrt{(p_x(t_k) - x_\sigma)^2 + (p_y(t_k) - y_\sigma)^2 + (p_z(t_k) - z_\sigma)^2}, \quad (3.2)$$

where $\sigma \in \{1, 2, \dots, N\}$ denotes the σ -th beacon, $(x_\sigma, y_\sigma, z_\sigma)$ are the coordinates of the σ -th beacon, N is the number of beacons and t_k , $k \in \mathbb{Z}_{\geq 1}$, is a sequence of increasing non-negative real numbers that satisfies the following assumption:

Assumption 1 *There exist scalars T_m and T_M , with $0 < T_m \leq T_M$, such that:*

$$T_m \leq |t_{k+1} - t_k| \leq T_M, \quad \forall k \in \mathbb{Z}_{\geq 1}. \quad (3.3)$$

Assumption 1 considers the case of random time-of-arrival measurements with a sampling interval lower and upper bounded by two known positive constants T_m and T_M .

In order to formalize the above described mechanism of the sequential measurements,

the following hybrid model (the hybrid framework shown in [60] is used) is proposed:

$$\begin{cases} \dot{\mathbf{s}} = \mathbf{A}\mathbf{s} + \boldsymbol{\varphi}(\mathbf{s}, \mathbf{u}, \mathbf{g}), \\ \dot{\tau} = 1, \\ \dot{\sigma} = 0, \end{cases} \quad (\mathbf{s}, \tau, \sigma) \in \mathcal{C}_x, \quad (3.4a)$$

$$\begin{cases} \mathbf{s}^+ = \mathbf{s}, \\ \tau^+ = 0, \\ \sigma^+ = \text{mod}(\sigma/N) + 1, \end{cases} \quad (\mathbf{s}, \tau, \sigma) \in \mathcal{D}_x, \quad (3.4b)$$

where $\mathcal{C}_x$ and $\mathcal{D}_x$ are respectively the flow set and the jump set defined as:

$$\mathcal{C}_x := \mathbb{R}^6 \times [0, T_M] \times \{1, 2, \dots, N\}, \quad (3.5)$$

$$\mathcal{D}_x := \mathbb{R}^6 \times [T_m, T_M] \times \{1, 2, \dots, N\}. \quad (3.6)$$

The extra state variable τ in model (7.6a)-(7.6b) reproduces a timer keeping track of the elapsed time since the last sample, and the impulsive nature of the available measurement is represented by the extra property that output (3.2) is only available at jump times. Finally the extra state variable σ in model (7.6a)-(7.6b) identify the available beacon in that time instant. It is easy to verify that the proposed model (7.6a)-(7.6b) represents the measurement mechanism described above: only one distance at a time is available from one fixed beacon. Moreover the case of asynchronous measurements has been considered here in order to take into consideration the randomness in the ToA of the measurements.

3.2 Localization algorithm design

In this Section, an observer is proposed whose structure implicitly complies with both the restriction specified in Assumption 1 on the available output and that the mobile beacon can communicate with only one fixed beacon at a time. The proposed observer is capable of providing an asymptotic estimate of the plant state, regardless of the sequence of times t_k at which the sampled output is available. The hybrid structure of the proposed observer

is the following:

$$\begin{cases} \dot{\hat{\mathbf{s}}} = \mathbf{A}\hat{\mathbf{s}} + \boldsymbol{\varphi}(\hat{\mathbf{s}}, \mathbf{u}, \mathbf{g}), \\ \dot{\tau} = 1, \\ \dot{\sigma} = 0, \\ \dot{\boldsymbol{\pi}} = 0, \end{cases} \quad (\hat{\mathbf{s}}, \tau, \sigma, \boldsymbol{\pi}) \in \mathcal{C}_x \times \mathbb{R}^3, \quad (3.7a)$$

$$\begin{cases} \hat{\mathbf{s}}^+ = \hat{\mathbf{s}} + \mathbf{G}(\hat{\mathbf{s}}, \tau, \sigma, \boldsymbol{\pi}, d_\sigma), \\ \tau^+ = 0, \\ \sigma^+ = \text{mod}(\sigma/N) + 1, \\ \boldsymbol{\pi}^+ = \begin{bmatrix} \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix} (\hat{\mathbf{s}} + \mathbf{G}(\hat{\mathbf{s}}, \tau, \sigma, \boldsymbol{\pi}, d_\sigma)), \end{cases} \quad (\hat{\mathbf{s}}, \tau, \sigma, \boldsymbol{\pi}) \in \mathcal{D}_x \times \mathbb{R}^3, \quad (3.7b)$$

$$\mathbf{G}(\hat{\mathbf{s}}, \tau, \sigma, \boldsymbol{\pi}, d_\sigma) = \begin{bmatrix} (1-\alpha_v)\mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & (1-\alpha_p)\mathbf{I}_3 \end{bmatrix} (\boldsymbol{\Gamma}(\cdot) - \hat{\mathbf{s}}) \quad (3.7c)$$

where $\boldsymbol{\pi}$ keeps memory of the last position reset, the function $\boldsymbol{\Gamma}(\cdot)$ is responsible for the discrete output injection term, and $0 < \alpha_v \leq 1$ and $0 < \alpha_p \leq 1$ are two constants and they are chosen by design in order to achieve a good compromise between filtering and measurement update. Now the problem is how to compute $\boldsymbol{\Gamma}(\cdot)$ in order to ensure the convergence to zero of the estimation error.

When the distance from the σ -th fixed beacon is available, it is possible to claim that the position of the object belongs to the sphere of equation:

$$(x - x_\sigma)^2 + (y - y_\sigma)^2 + (z - z_\sigma)^2 = d_\sigma^2, \quad (3.8)$$

Now the key idea is to reset the position by means of the jump map, and to use as measured position of the system the point of the sphere (3.8) that minimize the distance with the point before the jump. This new point is obtained by intersection between the sphere (3.8) and the straight line passing through the centre of the sphere and the point corresponding to the estimated position before the jump. Considering the coordinates of the estimated position before the jump $(\hat{p}_x, \hat{p}_y, \hat{p}_z)$, the parametric equations of this line are:

$$\begin{cases} x = x_\sigma + (\hat{p}_x - x_\sigma)\rho, \\ y = y_\sigma + (\hat{p}_y - y_\sigma)\rho, \\ z = z_\sigma + (\hat{p}_z - z_\sigma)\rho, \end{cases} \quad (3.9)$$

Replacing (3.9) into (3.8) the values of ρ , that give the two points of intersection between

(3.9) and (3.8), are:

$$\rho_{1,2} = \pm \sqrt{\frac{d_\sigma}{(\hat{p}_x - x_\sigma)^2 + (\hat{p}_y - y_\sigma)^2 + (\hat{p}_z - z_\sigma)^2}}. \quad (3.10)$$

Since we are considering the point of the sphere with minimum distance from $\hat{\mathbf{p}}$, the sign of ρ is such that:

$$\rho = \min_{i=\{1,2\}} |1 - \rho_i|. \quad (3.11)$$

The (3.11) implies that ρ must be positive, so:

$$\rho = \sqrt{\frac{d_\sigma}{(\hat{p}_x - x_\sigma)^2 + (\hat{p}_y - y_\sigma)^2 + (\hat{p}_z - z_\sigma)^2}}. \quad (3.12)$$

With regards to the speed update, it is useful to reset also the speed in order to avoid drift problems. In particular a good choice is to use as value of the speed after the jump the mean value computed as ratio between the difference of $\hat{\mathbf{p}}^+(t_k)$ and $\hat{\mathbf{p}}^+(t_{k-1})$ and the elapsed time between them.

Based on these considerations, the term $\mathbf{\Gamma}(\cdot)$, of the jump map of (3.7), is as follows:

$$\mathbf{\Gamma}(\hat{\mathbf{s}}, \tau, \sigma, \boldsymbol{\pi}, d_\sigma) = \begin{bmatrix} \mathbf{\Gamma}_v^T & \mathbf{\Gamma}_p^T \end{bmatrix}^T. \quad (3.13)$$

where:

$$\mathbf{\Gamma}_v = \frac{\rho}{\tau} \begin{bmatrix} \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix} \hat{\mathbf{s}} + \begin{bmatrix} x_\sigma \\ y_\sigma \\ z_\sigma \end{bmatrix} \frac{1 - \rho}{\tau} - \frac{1}{\tau} \boldsymbol{\pi} \quad (3.14)$$

$$\mathbf{\Gamma}_p = \rho \begin{bmatrix} \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix} \hat{\mathbf{s}} + \begin{bmatrix} x_\sigma \\ y_\sigma \\ z_\sigma \end{bmatrix} (1 - \rho) \quad (3.15)$$

where ρ is chosen as in (3.12).

3.3 Experimental Results

The experimental test consists of a path with a rhombus shape in the indoor test setup environment described in Section 2.3.1. In particular the robot is actuated so that the plate mounted on its last link tracks a rhombus. The position of the robot is used as

reference. Figure 3.1 shows the estimated and measured positions. Starting from a different initial condition, it is evident that the observer converges to the measured value and it tracks correctly the path providing a good estimate also with sampled measurements. Actually the proposed observer merges properly the data from the measurements, and use the integration of the model when the measurements are not available, exploiting all the available information in order to provide an estimate as close as possible to the true value.

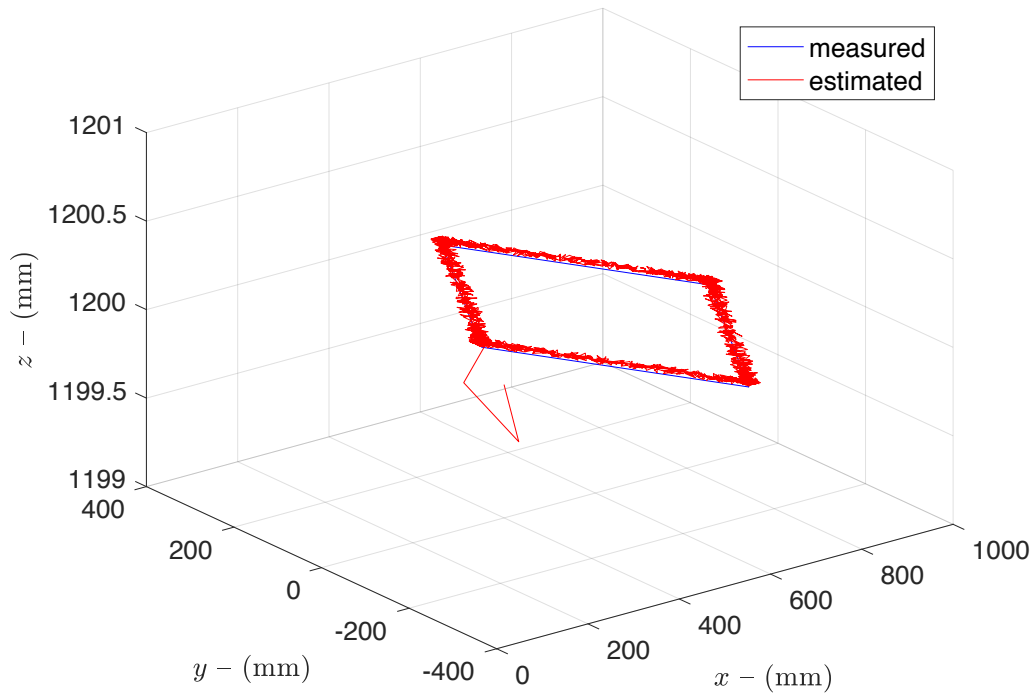


Figure 3.1: Estimated and measured positions.

Figure 3.2 shows the corresponding estimated speeds along x , y and z directions, Figure 3.3 shows the measured distances between the mobile and the fixed beacons, and Figure 3.4 shows quaternion components ξ_0 , ξ_1 , ξ_2 and ξ_3 .

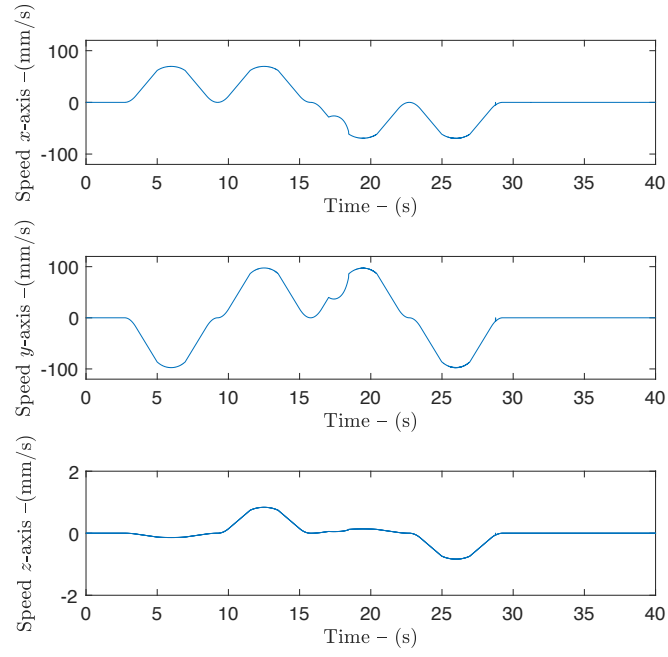
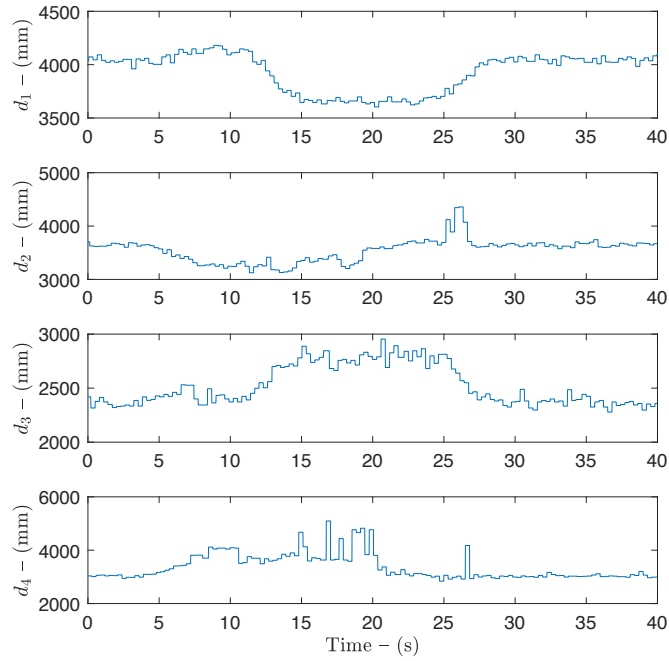
Figure 3.2: Estimated speeds along x , y and z directions.

Figure 3.3: Measured distances between the mobile and the fixed beacons.

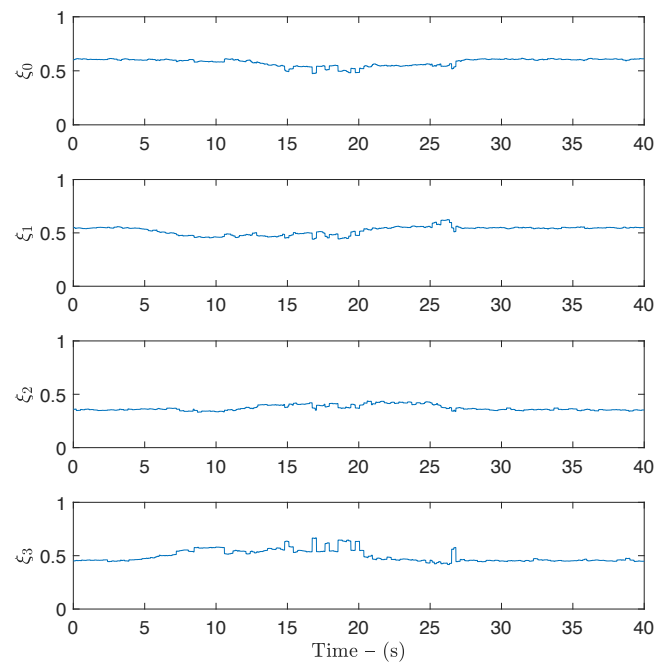


Figure 3.4: Quaternion components ξ_0 , ξ_1 , ξ_2 and ξ_3 .

Chapter 4

Hybrid observer for Outdoor Localization

OUTLINE. In this chapter the design of a hybrid nonlinear observer for mobile GPS based vehicles with asynchronous measurement is presented. The main problem is that position and velocity measurements are provided with a very low frequency, and the time between two consecutive measurements could be not constant, but it could vary randomly within a certain interval of time. For this reason the proposed observer has been contextualized in the hybrid systems framework. The convergence analysis of the estimation error has been carried out, and the sensitivity analysis has been performed in order to evaluate the bound of the estimation error when the measurements are biased and/or noisy. Simulation and experimental results, carried out on a mobile vehicle, show the feasibility of the proposed observer and the good behaviors in terms of estimation error that can be achieved. The use of the hybrid formalism [60] provides a natural setting for this type of observers, where both continuous-time and discrete-time dynamics coexist. Indeed a sampled-data observer can be modeled by a *flow map*, which describes the continuous-time dynamics when the measurements are not available, while the measurement can be considered as a discrete event and it can be modeled by a suitable *jump map*. Since this work deals with the study of hybrid observers for the data fusion between IMU and GPS, the continuous-time part will be a closed-loop observer based on the strap-down model of the IMU, which evolves until a new position measurement, generated from the GPS, arrives. At this instant of time, the discrete-time part updates the state of the continuous-time observer, according to the error between the state of the observer and the measurement. Moreover in this work it is assumed that the time between two measurements is not known and can vary randomly within a bounded interval. The convergence

of the observation error is proven in a straightforward way, without to use the Lyapunov direct method. To this regard, note that it can be difficult to choose a Lyapunov function which diminishes both during the flow and during the jump, and this difficulty principally lies in the fact that the time duration between two measurements is not constant.

A preliminary idea of the proposed observer has been developed in [98]. However, the case of asynchronous measurements is studied here, instead of that considered in [98] where the time interval between two measurements was known and constant. Moreover, new results about the sensitivity analysis of the observer are also presented, and well defined bounds on the estimation error have been given in presence of measurement errors. Experimental results are also shown with the aim of validating the theoretic approach.

4.1 Mathematical model of the inertial navigation system

In order to cope with the design of a suitable observer, model (1.8)-(1.9) presented in Section 1.2.2 is put in the following form:

$$\dot{\mathbf{x}} = \mathbf{A}(\boldsymbol{\omega}_{b/n}^b) \mathbf{x} + \boldsymbol{\varphi}(\boldsymbol{\xi}_b^n, \boldsymbol{\gamma}^b, \mathbf{g}), \quad (4.1)$$

where the state vector is $\mathbf{x} = [\boldsymbol{\xi}_b^{n\top} \quad \mathbf{v}^{n\top} \quad \mathbf{p}^{n\top}]^\top$, and matrix $\mathbf{A}(\boldsymbol{\omega}_{b/n}^b)$ and vector field $\boldsymbol{\varphi}(\boldsymbol{\xi}_b^n, \boldsymbol{\gamma}^b, \mathbf{g})$ are:

$$\mathbf{A}(\cdot) = \begin{bmatrix} \frac{1}{2} \mathbf{H}(\boldsymbol{\omega}_{b/n}^b) & \mathbf{0}_{4 \times 3} & \mathbf{0}_{4 \times 3} \\ \mathbf{0}_{3 \times 4} & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_{3 \times 4} & \mathbf{I}_3 & \mathbf{0}_3 \end{bmatrix}, \quad \boldsymbol{\varphi}(\cdot) = \begin{bmatrix} \mathbf{0}_{4 \times 1} \\ \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g} \\ \mathbf{0}_{3 \times 1} \end{bmatrix}.$$

4.2 Quaternion time-varying filter

The quaternion is measured by means of the IMU, and this solution guarantees a low computational complexity since no processing of the data is required. However, if the measurements provided by the IMU are too noisy, the adoption of a full-order strictly proper observer can be desirable since the high-frequency measurement noise can be filtered out. For this reason the following time-varying filter is proposed:

$$\dot{\hat{\boldsymbol{\xi}}}_b^n = \frac{1}{2} \mathbf{H}(\boldsymbol{\omega}_{b/n}^b) \hat{\boldsymbol{\xi}}_b^n + \frac{k_\xi}{2} (\boldsymbol{\xi}_b^n - \hat{\boldsymbol{\xi}}_b^n), \quad (4.2)$$

where $\hat{\boldsymbol{\xi}}_b^n$ is the estimate of $\boldsymbol{\xi}_b^n$, and k_ξ is a positive constant gain.

Proposition 1 *For each $\omega_{b/n}^b$ and $k_\xi > 0$, the function $V(\mathbf{e}_\xi) = \mathbf{e}_\xi^\top \mathbf{e}_\xi$ is a Lyapunov function for the model describing the dynamics of the observation error $\mathbf{e}_\xi = \boldsymbol{\xi}_b^n - \hat{\boldsymbol{\xi}}_b^n$. Moreover, the observation error is bounded during the transient and converges to zero exponentially, with convergence rate equal to k_ξ .*

Proof. The error dynamic can be written as:

$$\dot{\mathbf{e}}_\xi = \dot{\boldsymbol{\xi}}_b^n - \dot{\hat{\boldsymbol{\xi}}}_b^n = \frac{1}{2} (\mathbf{H}(\omega_{b/n}^b) - k_\xi \mathbf{I}_4) \mathbf{e}_\xi. \quad (4.3)$$

Computing the time derivative of V we have:

$$\dot{V}(\mathbf{e}_\xi) = 2\mathbf{e}_\xi^\top \dot{\mathbf{e}}_\xi = \mathbf{e}_\xi^\top (\mathbf{H}(\omega_{b/n}^b) - k_\xi \mathbf{I}_4) \mathbf{e}_\xi. \quad (4.4)$$

The term $\mathbf{e}_\xi^\top \mathbf{H}(\omega_{b/n}^b) \mathbf{e}_\xi$ is zero since $\mathbf{H}(\omega_{b/n}^b)$ is a skew-symmetric matrix. It follows that (4.4) becomes:

$$\dot{V}(\mathbf{e}_\xi) = -k_\xi V(\mathbf{e}_\xi). \quad (4.5)$$

The (4.5) implies that the evolution over the time of the Lyapunov function is:

$$V(t) = V(0)e^{-k_\xi t}. \quad (4.6)$$

This concludes the proof. □

The result in Proposition 1 is very appealing since the design of (4.2) can be performed by looking only at the gain k_ξ independently on the time-varying dynamics of $\omega_{b/n}^b$. Clearly, the error variables depend on $\omega_{b/n}^b$ and exhibit a peculiar time-varying transient, but the upper bound on their norm is a time-invariant function dependent only on k_ξ .

It is useful to note that $\hat{\boldsymbol{\xi}}_b^n$ belongs to a linear space and does not lie in the set of the unit quaternions during the transient. However, after the first transient, the variable $\hat{\boldsymbol{\xi}}_b^n$ converges to the true measured quaternion $\boldsymbol{\xi}_b^n$, which belongs to the set of the quaternions, as proved in Proposition 1.

Finally, another advantage due to the use of the above filter/observer, which will be shown in the following Subsection 5.2, consists in the fact that it allows to obtain an upper bound of the estimation error in presence of measurement error in the angular velocity if $k_\xi > 1$, and this bound can be reduced increasing k_ξ .

4.3 Hybrid nonlinear observer design

In this Section a Hybrid NonLinear Observer (HNLO) is proposed in order to cope with the problem listed before, i.e the localization of the autonomous vehicle by means of sampled and asynchronous measurements, i.e. the time between two consecutive measurements is not constant.

From equations (1.8)-(1.10) it is evident that the dynamics of the quaternion does not depend from those of linear speed and position. Indeed in (1.8) neither velocity nor position appears. However velocity and position depend from the quaternion, which can be considered as a time-varying quantity for the dynamics of velocity and position. In other words model (1.8)-(1.10) can be considered as the cascade of two subsystems, the first describes the dynamics of the quaternion and the second subsystem describes the dynamics of velocity and position. For this reason, it is convenient to define the following reduced order state-space model:

$$\dot{\bar{\mathbf{x}}} = \bar{\mathbf{A}}\bar{\mathbf{x}} + \bar{\boldsymbol{\varphi}}(\boldsymbol{\xi}_b^n, \boldsymbol{\gamma}^b, \mathbf{g}), \quad (4.7)$$

where the reduced state vector $\bar{\mathbf{x}}$ is defined as:

$$\bar{\mathbf{x}} = \begin{bmatrix} \mathbf{v}_{b/n}^{n\top} & \mathbf{p}_{b/n}^{n\top} \end{bmatrix}^\top, \quad (4.8)$$

and matrix $\bar{\mathbf{A}}$ and vector field $\bar{\boldsymbol{\varphi}}$ are:

$$\bar{\mathbf{A}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \otimes \mathbf{I}_3, \quad \bar{\boldsymbol{\varphi}}(\boldsymbol{\xi}_b^n, \boldsymbol{\gamma}^b, \mathbf{g}) = \begin{bmatrix} \mathbf{V}(\boldsymbol{\xi}_b^n)\boldsymbol{\gamma}^b + \mathbf{g} \\ \mathbf{0}_{3 \times 1} \end{bmatrix}. \quad (4.9)$$

Moreover, the state of system (4.7) is assumed to be accessible at discrete instants of time, resulting in a sequence of m dimensional vectors $\mathbf{y}_k$, $k \in \mathbb{Z}_{\geq 1}$, defined as:

$$\mathbf{y}_k := \bar{\mathbf{x}}(t_k), \quad (4.10)$$

where t_k , $k \in \mathbb{Z}_{\geq 1}$, is a sequence of increasing non-negative real numbers that satisfies the following assumption:

Assumption 2 *There exist scalars T_m and T_M , with $0 < T_m \leq T_M$, such that:*

$$T_m \leq |t_{k+1} - t_k| \leq T_M, \quad \forall k \in \mathbb{Z}_{\geq 1}. \quad (4.11)$$

Assumption 2 requires that we are considering the case of asynchronous discrete-time

measurements with a sampling interval lower and upper bounded by two known positive constants T_m and T_M . Note that T_m must be strictly greater than zero to avoid Zeno behaviors in the hybrid model developed below.

Using the the hybrid systems formalism of [60], the following hybrid dynamics is considered, where the symbol " $\hat{\cdot}$ " denotes the estimated variables:

$$\begin{cases} \dot{\hat{\mathbf{x}}} = \bar{\mathbf{A}}\hat{\mathbf{x}} + \bar{\boldsymbol{\varphi}}(\boldsymbol{\xi}_b^n, \boldsymbol{\gamma}^b, \mathbf{g}) + \mathbf{K}_x(\hat{\mathbf{s}} - \hat{\mathbf{x}}), \\ \dot{\hat{\mathbf{s}}} = \bar{\mathbf{A}}\hat{\mathbf{s}} + \bar{\boldsymbol{\varphi}}(\boldsymbol{\xi}_b^n, \boldsymbol{\gamma}^b, \mathbf{g}), \\ \dot{\tau}_r = -1, \end{cases} \quad \text{for } (\hat{\mathbf{x}}, \hat{\mathbf{s}}, \tau_r) \in \mathcal{C}, \quad (4.12)$$

$$\begin{cases} \hat{\mathbf{x}}^+ = \hat{\mathbf{x}}, \\ \hat{\mathbf{s}}^+ = \hat{\mathbf{s}} + \mathbf{K}_s(\mathbf{y}_k - \hat{\mathbf{s}}), \\ \tau_r^+ = T_m + (T_M - T_m)\nu, \end{cases} \quad \text{for } (\hat{\mathbf{x}}, \hat{\mathbf{s}}, \tau_r) \in \mathcal{D}, \quad (4.13)$$

where $\mathbf{K}_x$ and $\mathbf{K}_s$ are suitably chosen matrix gains, and $\mathcal{C}$ and $\mathcal{D}$ are respectively the flow set and the jump set defined as:

$$\mathcal{C} \in \mathbb{R}^6 \times \mathbb{R}^6 \times [T_M, 0], \quad (4.14)$$

$$\mathcal{D} \in \mathbb{R}^6 \times \mathbb{R}^6 \times \{0\}. \quad (4.15)$$

The variable $\hat{\mathbf{s}}$ is an extra state variables called "*discrete-time copy*" of $\hat{\mathbf{x}}$, while τ_r represents the "*timer*". Note that in order to model the random time of the arrival of the measurements a reverse timer has been used [99]. In particular the variable τ_r^+ represents the value of the time when the next measurement will arrive; starting from this value, τ_r evolves according to the third equation of (4.12) until it reaches the zero value, thus causing the next jump. The value assumed by τ_r^+ depends from the random variable ν , which belongs to the interval $[0, 1]$, so, by the third equation of (4.13), τ_r^+ belongs to the interval $[T_m, T_M]$. Obviously, due to the asynchrony of the time of arrival of the measurement data, in the practical application the instant in which the next data will arrive is not know a priori. Consequently the third equations of (4.12) and (4.13) have to be interpreted in terms of *modelling the functioning of the observer dynamics*. This is necessary in order to carry out convergence proofs and establish design methodologies. The τ_r dynamics does not compromise the practical implementation of the observer because only the first two equations of (4.12) and (4.13) are implemented and the jump will occur when the new measurement will be available.

Clearly, the dynamics (4.12)-(4.13) is composed by two different parts with different

characteristics. The first (second equation of (4.12)) is a copy of the model of the original system and it gives an open loop estimation based on the inertial model until a new measurement is available. When the new measurement is available ($\tau_r = 0$), then the state $\hat{\mathbf{s}}$ is initialized by means of a suitable law represented by the second equation of (4.13). This jump equation is driven by the error between the measured output at a generic discrete instant, and the state $\hat{\mathbf{s}}$ at the same instant. Obviously $\hat{\mathbf{s}}$ has impulsive behaviors due to the jump. The second part (first equation of (4.12)) is obtained constructing a model of the original system and forcing it with the residual $\hat{\mathbf{s}} - \hat{\mathbf{x}}$. This last part has a continuous-time solution, as confirmed by the first equation of (4.13), and it provides a kind of filtered estimate of $\hat{\mathbf{s}}$, removing the impulsive events that could produce problems when the estimate is used in a control system.

The intuition behind the proposed dynamics is to exploit all the available information of the problem (both the model and the measurements), and to provide a continuous-time estimate avoiding discontinuity in the estimated speed and position.

Before to prove the main results of this work it is useful to give the following technical lemmas:

Lemma 1 *If γ^b is bounded, then the vector field $|\bar{\varphi}|$ is bounded for each value of the attitude angles ξ_b^n , and in particular it results $|\bar{\varphi}| \leq |\gamma^b| + g$, $\forall \xi_b^n$.*

Proof. By using the expression of $\bar{\varphi}$ given in (4.9), we have:

$$|\bar{\varphi}| = \left| \begin{bmatrix} \mathbf{V}(\xi_b^n) \gamma^b + \mathbf{g} \\ \mathbf{0}_{3 \times 1} \end{bmatrix} \right| = |\mathbf{V}(\xi_b^n) \gamma^b + \mathbf{g}| \leq \|\mathbf{V}(\xi_b^n)\| |\gamma^b| + |\mathbf{g}|.$$

Now considering that $\mathbf{V}(\xi_b^n)$ is a rotation matrix, then $\|\mathbf{V}(\xi_b^n)\| = 1$, from which the statement follows. $\square$

Lemma 2 *Given a matrix $\mathbf{F}$, and consider its Jordan canonical form $\mathbf{J}$ such that $\mathbf{F} = \mathbf{T}^{-1} \mathbf{J} \mathbf{T}$, then:*

$$\|e^{\mathbf{F}t}\| \leq \|\mathbf{T}\| \|\mathbf{T}^{-1}\| \max_i e^{r_i t} \sum_{k=0}^{m_i-1} \frac{t^k}{k!}, \quad (4.16)$$

where m_i is the size of the i -th Jordan block of matrix $\mathbf{J}$ with eigenvalue λ_i , and $r_i = \Re\{\lambda_i\}$.

Proof. Since $\mathbf{F} = \mathbf{T}^{-1} \mathbf{J} \mathbf{T}$, then $e^{\mathbf{F}t} = \mathbf{T}^{-1} e^{\mathbf{J}t} \mathbf{T}$, so $\|e^{\mathbf{F}t}\| \leq \|\mathbf{T}\| \|\mathbf{T}^{-1}\| \|e^{\mathbf{J}t}\|$. Moreover it is well known that for the generic i -th Jordan block, with eigenvalue λ_i , $\|e^{\mathbf{J}_i t}\| \leq e^{\Re\{\lambda_i\}t} \sum_{k=0}^{m_i-1} \frac{t^k}{k!}$. From these considerations the proof comes trivially. $\square$

Lemma 3 Consider the matrix $\bar{\mathbf{A}}$ in (4.7), then:

$$\left\| e^{\bar{\mathbf{A}}t} \right\| \leq 1 + t, \quad \forall t \geq 0. \quad (4.17)$$

Proof. Considering the matrix $\bar{\mathbf{A}}$, it can be written as $\bar{\mathbf{A}} = \mathbf{T}_{\bar{\mathbf{A}}}^{-1} \mathbf{J}_{\bar{\mathbf{A}}} \mathbf{T}_{\bar{\mathbf{A}}}$, where $\mathbf{J}_{\bar{\mathbf{A}}}$ is the Jordan form of matrix $\bar{\mathbf{A}}$, $\mathbf{T}_{\bar{\mathbf{A}}}$ is the transformation matrix, and they are defined as:

$$\mathbf{J}_{\bar{\mathbf{A}}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{T}_{\bar{\mathbf{A}}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}. \quad (4.18)$$

From (4.18), the proof comes from Lemma 2, considering that $\|\mathbf{T}_{\bar{\mathbf{A}}}\| = \|\mathbf{T}_{\bar{\mathbf{A}}}\|^{-1} = 1$, $r_i = 0 \quad \forall i$, and $m_i = 2 \quad \forall i$. $\square$

Now the main results of this work can be given.

Theorem 1 Given the maximum update time of the localization device, T_M as in Assumption 2, then choosing $\mathbf{K}_s = \begin{bmatrix} k_s & 0 \\ 0 & k_s \end{bmatrix} \otimes \mathbf{I}_3$ such that $|1 - k_s|(1 + T_M) < 1$, and $\mathbf{K}_x$ such that $\bar{\mathbf{A}} - \mathbf{K}_x$ is Hurwitz, then the state $\hat{\mathbf{x}}$ converges asymptotically to $\bar{\mathbf{x}}$ along all solutions to (4.7), (4.12)-(4.13), independently of the measurement time satisfying Assumption 2.

Proof. The proof consists of two steps. In the first step, we proof that $\hat{\mathbf{s}}$ converges asymptotically to $\bar{\mathbf{x}}$ independently on $\hat{\mathbf{x}}$ and, in the second step, that $\hat{\mathbf{x}}$ converges asymptotically to $\hat{\mathbf{s}}$ and consequently to $\bar{\mathbf{x}}$.

In order to prove that $\hat{\mathbf{s}}$ converges to $\bar{\mathbf{x}}$, let us define the error:

$$\mathbf{e} = \bar{\mathbf{x}} - \hat{\mathbf{s}}, \quad (4.19)$$

Using (4.12)-(4.13) and (4.19), the error dynamics can be written as:

$$\begin{cases} \dot{\mathbf{e}} = \bar{\mathbf{A}}\mathbf{e}, & 0 < \tau_r < T, \\ \mathbf{e}^+ = (1 - k_s)\mathbf{e}, & \tau_r = 0, \end{cases} \quad (4.20)$$

where $T_m \leq T \leq T_M$. Now let us consider $|\mathbf{e}(t)|$, where $\mathbf{e}(t)$ is the solution of the differential jump equation (4.20), between two consecutive jumps: $kT \leq t \leq (k+1)T$, $k \in \mathbb{Z}_{\geq 1}$. In particular during flow, $kT \leq t < (k+1)T$:

$$|\mathbf{e}(t)| \leq \left\| e^{\bar{\mathbf{A}}(t-kT)} \right\| |\mathbf{e}(kT)|, \quad (4.21)$$

For Lemma 3, the (4.21) satisfies:

$$|\mathbf{e}(t)| \leq (1 + t - kT)|\mathbf{e}(kT)|. \quad (4.22)$$

Looking at (4.22), the upper bound of the norm of the error $|\mathbf{e}(t)|$ increases linearly with t during the flow. However when $t = (k + 1)T$ the jump occurs, and:

$$|\mathbf{e}((k + 1)T)|^+ = |1 - k_s||\mathbf{e}((k + 1)T)|. \quad (4.23)$$

So definitively we have:

$$\begin{cases} |\mathbf{e}(t)| \leq (1 + t - kT)|\mathbf{e}(kT)|, & kT < t < (k + 1)T, \\ |\mathbf{e}(t)|^+ = |1 - k_s||\mathbf{e}((k + 1)T)|, & t = (k + 1)T. \end{cases} \quad (4.24)$$

Note that, in (4.24), T is a random variable and it can assume all values between T_m and T_M . Looking at (4.24) the upper bound of the norm of the error $|\mathbf{e}(t)|$ reaches the maximum value when $t \rightarrow (k + 1)T$, so considering the worst case for the measurement time we have:

$$\lim_{t \rightarrow (k+1)T} |\mathbf{e}(t)| = (1 + T)|\mathbf{e}(kT)| \leq (1 + T_M)|\mathbf{e}(kT)|. \quad (4.25)$$

However during the jump it decreases of a quantity $|1 - k_s|$. So by extending recursively this procedure for $k = 0, 1, \dots$, and considering the worst case for the measurement time, we obtain:

$$|\mathbf{e}(t)| \leq \left(|1 - k_s|(1 + T_M)\right)^{\lfloor \frac{t}{T} \rfloor} (1 + T_M)|\mathbf{e}(0)|, \quad \forall t \geq 0 \quad (4.26)$$

where $\lfloor \frac{t}{T} \rfloor$ denotes the greatest integer smaller than $\frac{t}{T}$.

At this point, considering that the random variable T is strictly positive and bounded by assumption, if $|1 - k_s|(1 + T_M) < 1$, then:

$$\lim_{t \rightarrow \infty} |\mathbf{e}(t)| = 0. \quad (4.27)$$

So definitively $\hat{\mathbf{s}}$ converges asymptotically to $\bar{\mathbf{x}}$.

Now look at the $\hat{\hat{\mathbf{x}}}$ dynamics in (4.12)-(4.13). Since $\hat{\hat{\mathbf{x}}}$ does not contain jumps, it is straightforward to prove that $\hat{\hat{\mathbf{x}}}$ converges asymptotically to $\hat{\mathbf{s}}$ if $\bar{\mathbf{A}} - \mathbf{K}_x$ is Hurwitz. Actually $\dot{\hat{\mathbf{s}}} - \dot{\hat{\hat{\mathbf{x}}}} = (\bar{\mathbf{A}} - \mathbf{K}_x)(\hat{\mathbf{s}} - \hat{\hat{\mathbf{x}}})$ and it is asymptotically stable if $\bar{\mathbf{A}} - \mathbf{K}_x$ is Hurwitz.

Finally, since $\hat{\mathbf{s}}$ converges asymptotically to $\bar{\mathbf{x}}$, and $\hat{\hat{\mathbf{x}}}$ converges asymptotically to $\hat{\mathbf{s}}$, then also $\hat{\hat{\mathbf{x}}}$ will converge asymptotically to $\bar{\mathbf{x}}$, as to be proven. $\square$

Lemma 4 *If $\mathbf{K}_x$ in (4.12) is such that $\mathbf{K}_x = \begin{bmatrix} \frac{k_x}{2} & 0 \\ 1 & \frac{k_x}{2} \end{bmatrix} \otimes \mathbf{I}_3$ for any $k_x > 0$, then defining the error $\mathbf{e}_x = \hat{\mathbf{s}} - \hat{\hat{\mathbf{x}}}$ and function $V = \frac{1}{2} \mathbf{e}_x^\top \mathbf{e}_x$ we have:*

$$\dot{V}(\mathbf{e}_x) = -k_x V(\mathbf{e}_x), \quad (4.28)$$

along dynamics (4.12)-(4.13).

Proof. Since $\dot{\mathbf{e}}_x = (\bar{\mathbf{A}} - \mathbf{K}_x) \mathbf{e}_x$, we have that $\dot{V}(\mathbf{e}_x) = \mathbf{e}_x^\top \dot{\mathbf{e}}_x = \mathbf{e}_x^\top (\bar{\mathbf{A}} - \mathbf{K}_x) \mathbf{e}_x = \mathbf{e}_x^\top (-\frac{k_x}{2} \mathbf{I}_3) \mathbf{e}_x = -k_x V(\mathbf{e}_x)$. $\square$

Remark 1 *Lemma 4 gives an important tool in order to design the continuous-time part of the observer. Indeed the upper bound on the $\mathbf{e}_x$ norm is a time-invariant function dependent only on k_x , guaranteeing a well defined convergence time. This could suggest a practical rule for the k_x selection. For example, known the GPS maximum update time T_M , k_x can be chosen such that the error $\mathbf{e}_x$ approaches to zero before the jump. For example a good tradeoff between filtering and convergence speed could be if $\mathbf{K}_x$ is chosen as in Lemma 4 with $k_x \geq 4T_M$.*

Figure 4.1 shows the block diagram of the proposed observer.

4.4 Sensitivity analysis

4.4.1 Uncertainties on acceleration measurements

The main problem, for the speed and position estimation, is related to the acceleration measurement and an uncorrected compensation of the gravity component, since an error on γ^b leads to an error on the speed and consequently on the position. So the conditions given in Theorem 1 could not be sufficient in order to ensure the convergence of the proposed observer when an error on γ^b measurement occurs. Note that there are a lot of sources of errors in γ^b , i.e. noise, biases and an uncorrected compensation of the gravity acceleration. For this reason the measured acceleration and the gravity compensation are supposed affected by bounded errors:

$$\tilde{\gamma}^b = \gamma^b + \Delta\gamma, \quad (4.29a)$$

$$\tilde{\mathbf{g}} = \mathbf{g} + \Delta\mathbf{g}, \quad (4.29b)$$

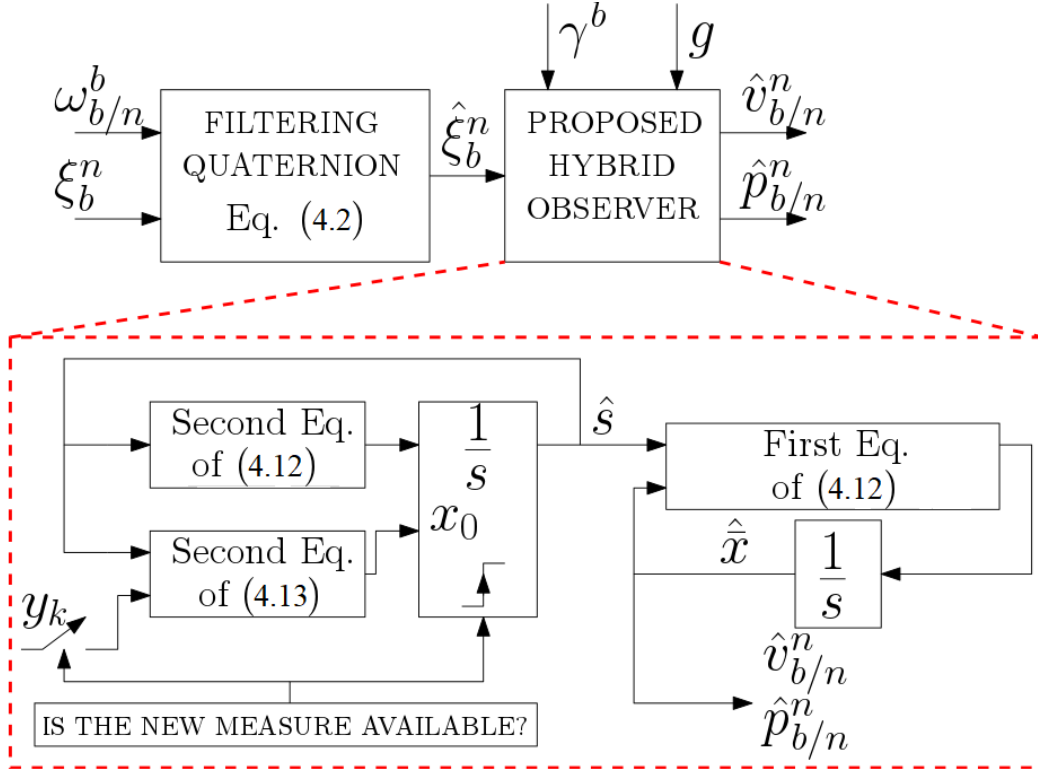


Figure 4.1: Block diagram of the proposed observer.

where $\Delta\gamma$ contains all the errors in the acceleration measurement, and $\Delta\mathbf{g}$ is the error in the gravity compensation. It is assumed that there exist two positive constants $\Delta_{\gamma,\max}$ and $\Delta_{\mathbf{g},\max}$ such that:

$$|\Delta\gamma| \leq \Delta_{\gamma,\max}, \quad (4.30a)$$

$$|\Delta\mathbf{g}| \leq \Delta_{\mathbf{g},\max}. \quad (4.30b)$$

Theorem 2 *If the conditions in Theorem 1 and inequalities (4.30) are satisfied, then the upper bound of the error (4.19) satisfies:*

$$|e(t)| \leq |1 - k_s|(\Delta_{\gamma,\max} + \Delta_{\mathbf{g},\max}) \left(T_M + \frac{T_M^2}{2} \right), \quad (4.31)$$

asymptotically when $t \rightarrow \infty$.

Proof. If the observer is driven by (4.29) instead of the nominal γ and $\mathbf{g}$, then the error dynamics can be written as:

$$\begin{cases} \dot{e} = \bar{A}e + \begin{bmatrix} \mathbf{v}(\xi_b^n) \\ \mathbf{0}_3 \end{bmatrix} \Delta\gamma + \Delta\mathbf{g} & 0 < \tau_r < T \\ e^+ = (1 - k_s)e & \tau_r = 0 \end{cases} \quad (4.32)$$

Now let us consider the same procedure used in Theorem 1, and let us evaluate $|\mathbf{e}(t)|$, where $\mathbf{e}(t)$ is the solution of the differential jump equation (4.32), between two consecutive jumps: $kT \leq t \leq (k+1)T$, $k \in \mathbb{Z}_{\geq 1}$. In particular during flow, $kT \leq t < (k+1)T$:

$$|\mathbf{e}(t)| = \left\| e^{\bar{\mathbf{A}}(t-kT)} \mathbf{e}(kT) + \int_{kT}^t e^{\bar{\mathbf{A}}((t-kT)-s)} \left(\begin{bmatrix} \mathbf{V}(\xi_b^n(s)) \\ \mathbf{0}_3 \end{bmatrix} \Delta \boldsymbol{\gamma}(s) + \Delta \mathbf{g}(s) \right) ds \right\|. \quad (4.33)$$

Using Lemma 1 and inequalities (4.30), the (4.33) can be upper bounded:

$$|\mathbf{e}(t)| \leq \left\| e^{\bar{\mathbf{A}}(t-kT)} \mathbf{e}(kT) \right\| + (\Delta_{\boldsymbol{\gamma}, \max} + \Delta_{\mathbf{g}, \max}) \int_{kT}^t \left\| e^{\bar{\mathbf{A}}((t-kT)-s)} \right\| ds. \quad (4.34)$$

Also in this case, this quantity increases with t , so using Lemma 3 and solving the integral, we can write:

$$\begin{aligned} |\mathbf{e}(t)| &\leq |\mathbf{e}((k+1)T)| = \\ &= (1+T)|\mathbf{e}(kT)| + (\Delta_{\boldsymbol{\gamma}, \max} + \Delta_{\mathbf{g}, \max}) \left(T + \frac{T^2}{2} \right), \\ &\quad \text{for } kT \leq t < (k+1)T. \end{aligned} \quad (4.35)$$

When the jump occurs, $t = (k+1)T$, we have:

$$\begin{aligned} |\mathbf{e}((k+1)T)|^+ &= |1-k_s|(1+T)|\mathbf{e}(kT)| \\ &\quad + |1-k_s|(\Delta_{\boldsymbol{\gamma}, \max} + \Delta_{\mathbf{g}, \max}) \left(T + \frac{T^2}{2} \right) \\ &\leq |1-k_s|(1+T_M)|\mathbf{e}(kT)| \\ &\quad + |1-k_s|(\Delta_{\boldsymbol{\gamma}, \max} + \Delta_{\mathbf{g}, \max}) \left(T_M + \frac{T_M^2}{2} \right). \end{aligned} \quad (4.36)$$

So using the same fashion as in Theorem 1, extending recursively this procedure for $k = 0, 1, \dots$, and considering the worst case for the measurement time, we definitively

obtain:

$$|e(t)| \leq \left(|1 - k_s|(1 + T_M) \right)^{\lfloor \frac{t}{T} \rfloor} (1 + T_M) |e(0)| \\ + |1 - k_s| (\Delta_{\gamma, \max} + \Delta_{g, \max}) \left(T_M + \frac{T_M^2}{2} \right), \quad \forall t \geq 0, \quad (4.37)$$

At this point, since $|1 - k_s|(1 + T_M) < 1$ because Theorem 1 holds, and T is strictly positive by assumption, then:

$$\lim_{t \rightarrow \infty} |e(t)| \leq |1 - k_s| (\Delta_{\gamma, \max} + \Delta_{g, \max}) \left(T_M + \frac{T_M^2}{2} \right). \quad (4.38)$$

As to be proven. □

Theorem 2 is particularly important since it permits to quantify the upper bound of the estimation error when maximum errors in the acceleration measurement and in the gravity acceleration are considered. As obviously expected this error increases with the GPS update time, moreover closer k_s is to one and smaller is the error due to the acceleration measurement. But in this case we have to pay attention when $k_s \rightarrow 1$, since if $k_s \rightarrow 1$ then the filtering effect on the GPS error measurement vanishes, so it is not a good choice. In conclusion, Theorems 1 and 2 represent two important tools in order to design an observer taking into consideration the convergence speed, the error due to the acceleration measurement, and the maximum time interval between two consecutive measurements.

4.4.2 Uncertainties on angular velocity measurements

If the $\omega_{b/n}^b$ is measured with good approximation, both equations (1.8) and (4.2) share the same matrix $H(\omega_{b/n}^b)$, so the results in Proposition 1 holds. However the angular velocity could be affected by errors, for example it could be biased or noisy, so the estimate of the quaternion can be affected by errors as well. In order to consider this aspect, let us assume that the measured angular velocity is affected by bounded errors:

$$\tilde{\omega}_{b/n}^b = \omega_{b/n}^b + \Delta\omega, \quad (4.39)$$

where $\Delta\omega$ contains all the errors in the angular velocity measurement, and it is upper bounded by a positive constant $\Delta_{\omega, \max}$ as follows:

$$|\Delta\omega| \leq \Delta_{\omega, \max}. \quad (4.40)$$

Under this assumption the Equation (4.2) becomes:

$$\dot{\hat{\xi}}_b^n = \frac{1}{2} \mathbf{H}(\tilde{\omega}_{b/n}^b) \hat{\xi}_b^n + \frac{k_\xi}{2} (\xi_b^n - \hat{\xi}_b^n), \quad (4.41)$$

where:

$$\mathbf{H}(\tilde{\omega}_{b/n}^b) = \mathbf{H}(\omega_{b/n}^b) + \mathbf{H}(\Delta\omega). \quad (4.42)$$

Note that also $\mathbf{H}(\Delta\omega)$ is a skew-symmetric matrix. In this condition the following result holds:

Proposition 2 *Assuming that the measurement of angular velocity is affected by error as in (4.39), then the estimation error $\mathbf{e}_\xi = \xi_b^n - \hat{\xi}_b^n$ satisfies asymptotically:*

$$|\mathbf{e}_\xi| \leq \frac{\|\mathbf{H}(\Delta\omega)\|}{k_\xi} \quad (4.43)$$

Proof. Let us consider the same Lyapunov function as in Proposition 1, $V(\mathbf{e}_\xi) = \mathbf{e}_\xi^\top \mathbf{e}_\xi$. Computing the time derivative of V , and considering dynamics (4.41) instead of (4.2), we have:

$$\dot{V}(\mathbf{e}_\xi) = -k_\xi \mathbf{e}_\xi^\top \mathbf{e}_\xi + \mathbf{e}_\xi^\top \mathbf{H}(\Delta\omega) \hat{\xi}_b^n, \quad (4.44)$$

where the equality (4.42) and fact the $\mathbf{H}(\omega_{b/n}^b)$ is a skew-symmetric matrix have been used. Since:

$$\mathbf{e}_\xi^\top \mathbf{H}(\Delta\omega) \hat{\xi}_b^n = \mathbf{e}_\xi^\top \mathbf{H}(\Delta\omega) (\xi_b^n - \mathbf{e}_\xi) = \mathbf{e}_\xi^\top \mathbf{H}(\Delta\omega) \xi_b^n,$$

because $\mathbf{H}(\Delta\omega)$ is skew-symmetric, we have:

$$\dot{V}(\mathbf{e}_\xi) \leq |\mathbf{e}_\xi| \|\mathbf{H}(\Delta\omega)\| |\xi_b^n| - k_\xi |\mathbf{e}_\xi|^2. \quad (4.45)$$

Since $|\xi_b^n| = 1$, the derivative $\dot{V}(\mathbf{e}_\xi)$ is less than zero if:

$$|\mathbf{e}_\xi| \geq \frac{\|\mathbf{H}(\Delta\omega)\|}{k_\xi}. \quad (4.46)$$

The (4.46) implies that $\mathbf{e}_\xi$ converges within a ball, centered in $\mathbf{e}_\xi = 0$ and radius equal to $\frac{\|\mathbf{H}(\Delta\omega)\|}{k_\xi}$, which in turns implies the (4.43), as to be proven. $\square$

Proposition 2 shows that the quaternion estimation error can be diminished by acting on the gain of the observer k_ξ . Obviously it is not possible to increase this gain behind a certain value, because this increasing produces increasing of the noise in the observed quaternion, so a good trade-off between filtering and estimation error have to be chosen

depending on the the maximum value $\Delta_{\omega, \max}$ assumed.

4.5 Simulation results

In this section simulation results are presented relatively to the path shown in Figure 4.2a. Two types of testes have been carried out. In *Test.1* the measurements have been corrupted by zero-mean Gaussian white noise, and no bias or other sources of errors have been considered in order to validate the theoretical results of Section III and Section IV. In *Test.2* the measurements have been purposely detuned in order to validate the results of Section V. In particular the following errors have been considered:

$$\tilde{\omega}_{b/n}^b = \eta_{\omega} \omega_{b/n}^b + \delta_{\omega}, \quad (4.47)$$

$$\tilde{\gamma}^b = \eta_{\gamma} \gamma^b + \delta_{\gamma}. \quad (4.48)$$

where $\tilde{\omega}_{b/n}^b$ and $\tilde{\gamma}^b$ are respectively the angular velocity and the acceleration given in input to the observer, $\omega_{b/n}^b$ and γ^b are the true angular velocity and the acceleration, η_{ω} and η_{γ} take into account scaling errors in the measurements and δ_{ω} and δ_{γ} take into account biases in the measurements. In the presented simulation the following values for the errors have been assumed:

$$\eta_{\omega} = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.25 \end{bmatrix}, \quad \eta_{\gamma} = \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 1.05 \end{bmatrix},$$

$$\delta_{\omega} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \delta_{\gamma} = \begin{bmatrix} 0.8 \\ -0.8 \\ 1.2 \end{bmatrix}.$$

Note that the errors given in (4.47)-(4.48), are very high compared with the standard real cases, however so high errors have been intentionally considered in order to emphasize the results achieved.

The observer has been tuned with $\mathbf{K}_x$ and $\mathbf{K}_s$ as suggested from Theorem 1 and Lemma 4 with $k_s = 0.9$ and $k_x = 4$. Moreover the quaternion components have been computed by means of Equation (4.2) with $k_{\xi} = 10$. Finally it has been assumed that $T_m = 0.8s$ and $T_M = 1.5s$ as shown in Figure 4.2b, where the evolution of the timer τ_r is given. From this Figure it is possible to see the displacement of the measurement times that is randomly distributed between $T_m = 0.8s$ and $T_M = 1.5s$.

Figure 4.2a show the simulated path e relative estimated positions for *Test.1* and *Test.2*, moreover Figure 4.2c shows the evolution along time of the same waveforms of Figure 4.2a, while Figure 4.2d shows the corresponding position estimation error. With

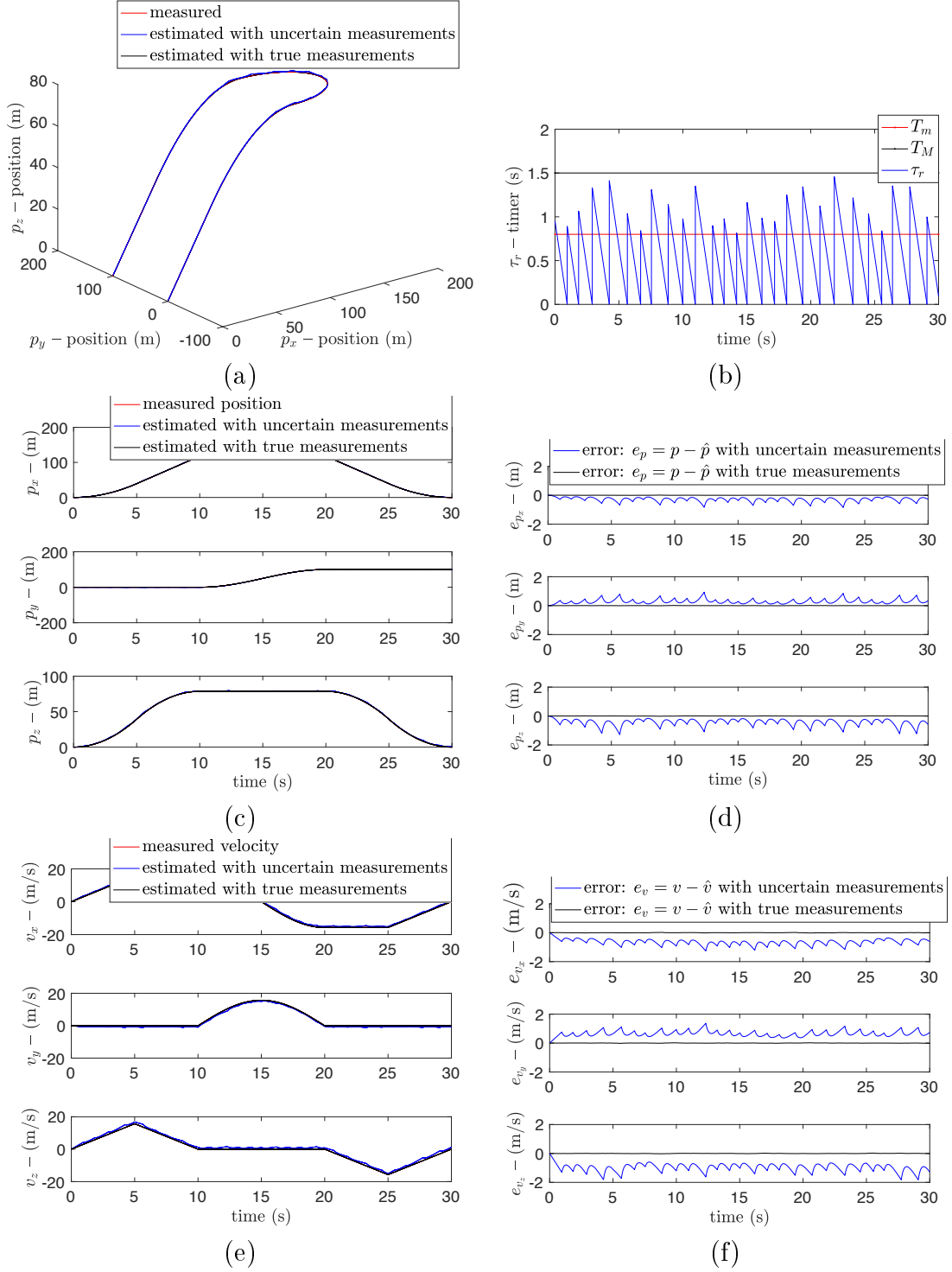


Figure 4.2: (a) Simulated path and relative estimated positions with and without error on the acceleration measurements. (b) Time evolution of the timer τ_r . (c)-(d) Real and estimated positions with and without error on the acceleration measurements and relative estimation error respectively. (e)-(f) Real and estimated velocities with and without error on the acceleration measurements and relative estimation error respectively.

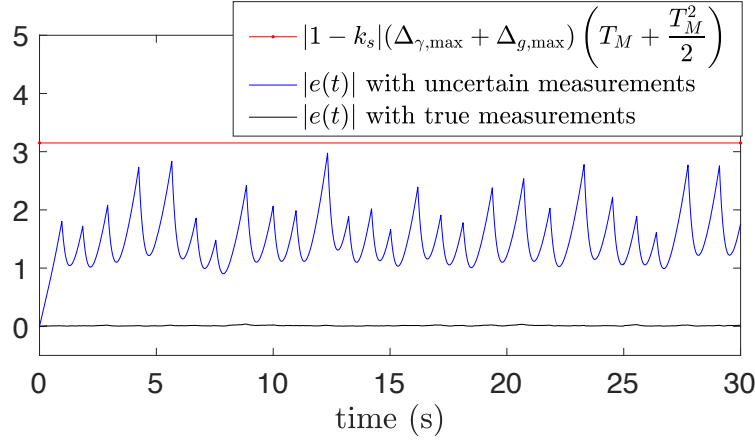


Figure 4.3: Norm of the estimation error $|e(t)|$ and relative upper bound computed as in Theorem 2.

regard to the velocity estimation, Figures 4.2e and 4.2f show the same kind of plots as for the position estimation. From these Figures it is evident the good performance of the proposed estimator. In particular, with regards to *Test.1*, where the measurements are only corrupted by zero-mean Gaussian white noise, the estimation error converges to zero as proven theoretically. When systematical errors are considered, as in *Test.2*, the estimation errors are bounded as proven in Theorem 2. In particular in order to validate the upper bound on the estimation error Figure 4.3 is given, where it is shown that the estimation error is always lower than the upper bound theoretically computed.

With regards to the quaternion filtering the results are shown in Figures 4.4-4.5. Figure 4.4 shows the measured and estimated components of the quaternion ξ_b^n for the above described *Test.1*, when the measurements of the angular speed are corrupted by zero-mean Gaussian white noise, moreover an initial condition different from the real one has been selected in order to highlight the transient behaviors of the filter. It is evident that the proposed solution correctly estimates the attitude of the system and the high frequency measurement noise is suitably filtered out. Figure 4.5 confirms the the remark given in Section III about the fact that the quaternion belongs to a linear space and does not lie in the set of the unit quaternions during the transient, where the norm is different to one. However after the first transient the variable $\hat{\xi}_b^n$ converges to the true measured quaternion and its norm converges to one.

Finally, Figures 4.6 and 4.7 show estimation errors of the quaternion for the above described *Test.2*, when the measurements of the angular velocity are affected by errors as described in (4.47). In particular Figure 4.6 shows the norm of the quaternion estimation error for different values of the gain k_ξ . From this figure it is evident that the error is

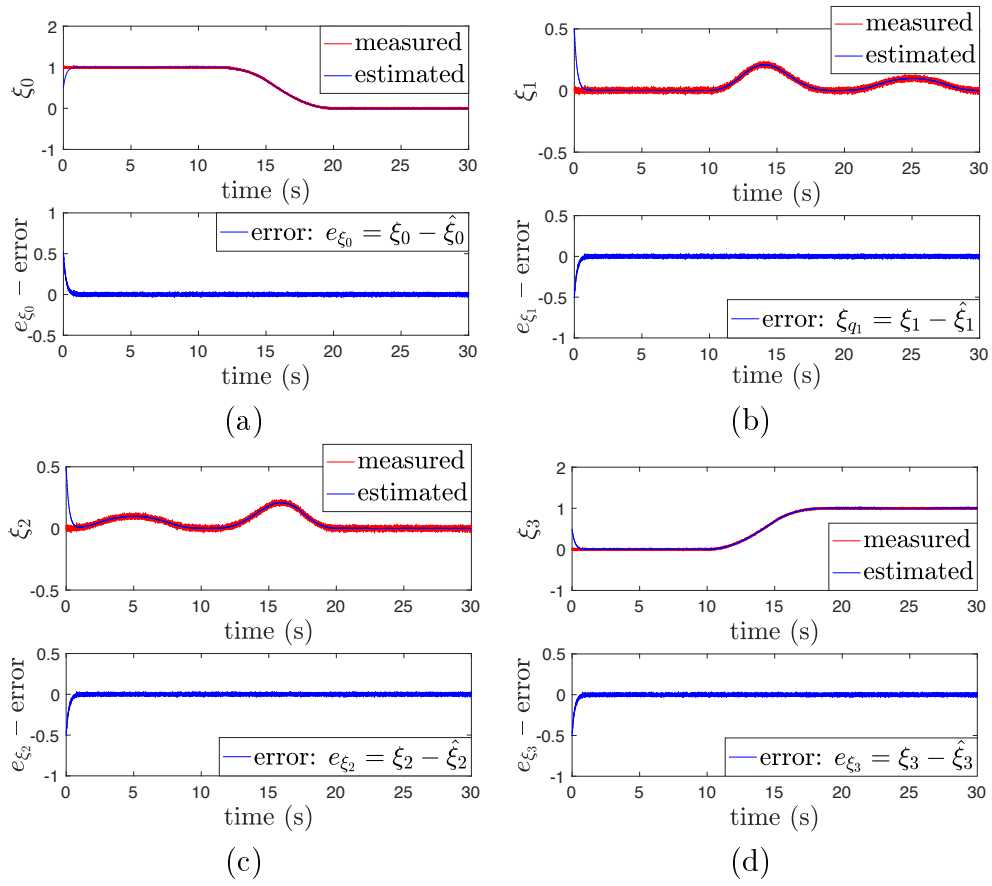


Figure 4.4: Measured and estimated components of ξ_b^n , and relative estimation error: ξ_0 in (a), ξ_1 in (b), ξ_2 in (c) and ξ_3 in (d).

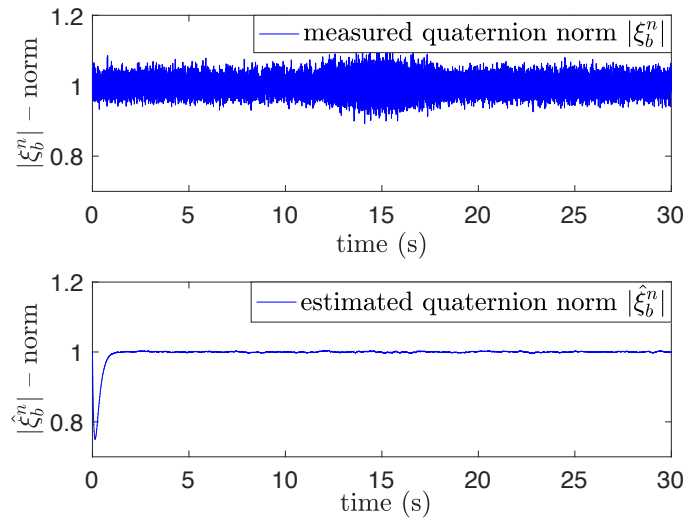


Figure 4.5: Measured and estimated quaternion norm.

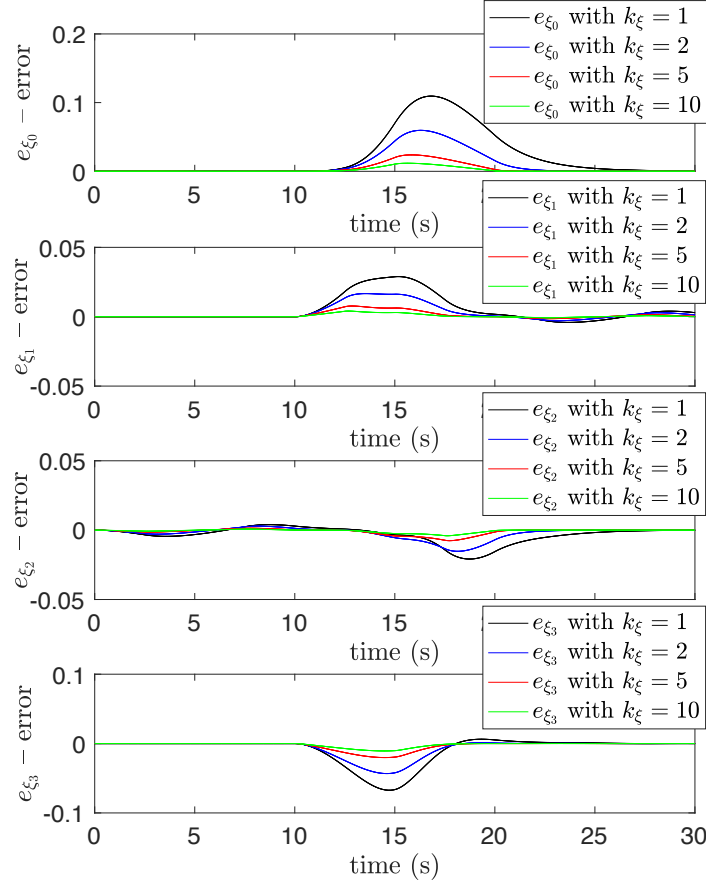


Figure 4.6: Norm of the quaternion estimation error for different values of the gain k_{ξ} .

bounded and it decreases for higher values of the observer gain and this fact confirm the result shown in Proposition 2. Obviously the gain k_{ξ} cannot be arbitrarily large in order to decrease the estimation error, otherwise the advantages of the filtering actions vanish. Moreover in order to validate Eq. (4.43), Figure 4.7 shows the norm of the quaternion estimation error $|e_{\xi}|$ and its relative upper bound $\frac{\|H(\Delta\omega)\|}{k_{\xi}}$ for $k_{\xi} = 1$. From this last figure it is evident that the estimation error never overcomes the bound theoretically computed.

4.6 Experimental results

The proposed HNLO has been evaluated experimentally with the test setup presented in Section 2.3.2. In particular, since the sampling time of the system is 20Hz while the GPS works at a lower frequency, it has been assumed $T_m = 0.85s$ and $T_M = 1.15s$.

The proposed observer, implemented on this equipment, consists of Equations (4.12)-(4.13) where $\mathbf{K}_x$ and $\mathbf{K}_s$ have been tuned as suggested from Theorem 1 and Lemma 4

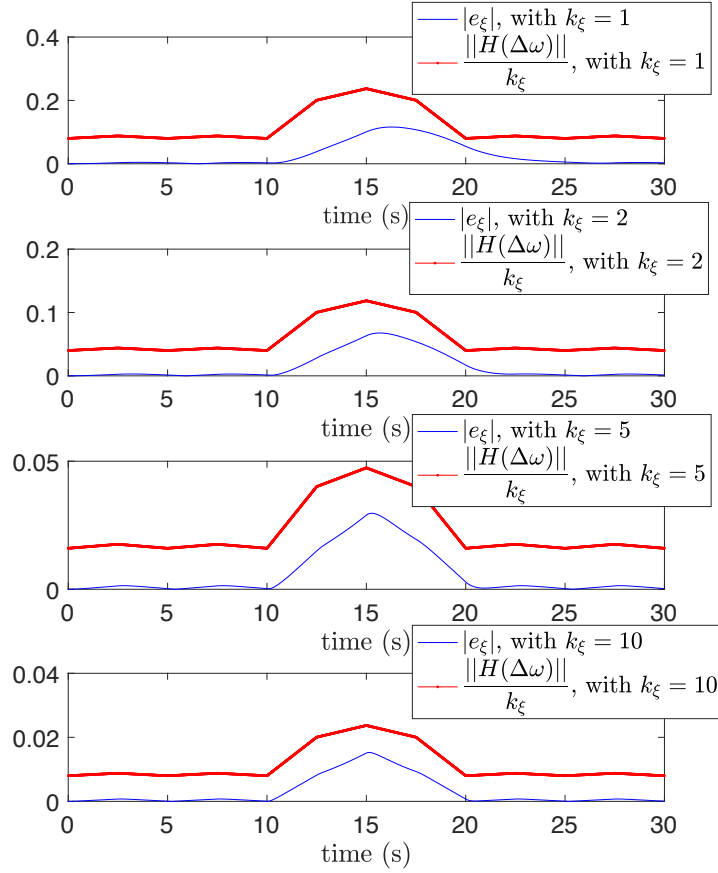


Figure 4.7: Norm of the quaternion estimation error and relative upper bound computed as in Proposition 2, for $k_\xi = 1$.

with $k_s = 0.8$ and $k_x = 4$. Moreover the quaternion components have been computed by means of Equation (4.2) with $k_\xi = 30$. The measured data given by the equipment have been suitably converted in order to unify the physical quantities, i.e. the geodetic position coordinates (latitude, longitude and altitude) has been converted in N-coordinates, and correspondently the speed given from the GPS has been converted in m/s along the same directions.

The experimental test consists of a path inside the university campus of Palermo. Figure 4.8 shows the photo of the chosen path. Figure 4.9 and 4.10 show the estimated and measured positions for the path in Figure 4.8. Starting from a different initial condition, it is evident that the observer converges to the measured value and it tracks correctly the path providing a good estimate also with sampled measurements. Actually the proposed observer merges properly the measurements, and use the integration of the model when the measurements are not available, exploiting all the information in order to provide an estimate as close as possible to the true value.

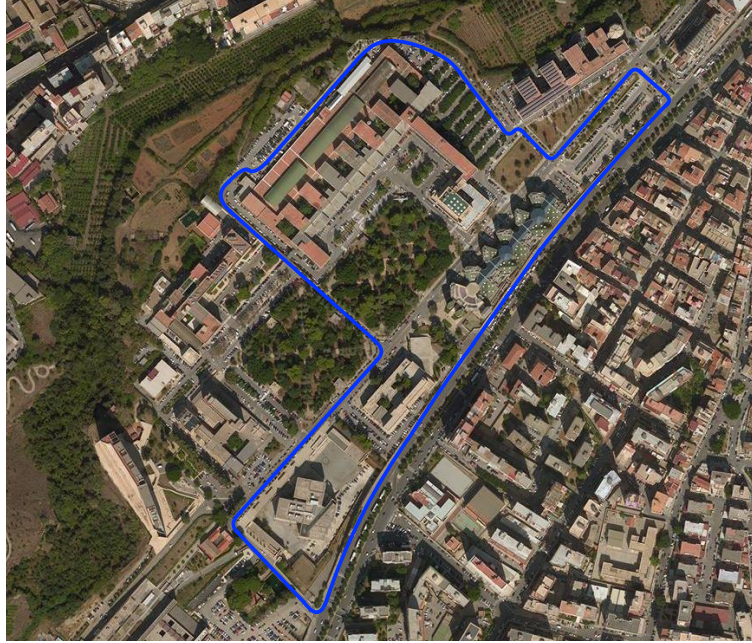


Figure 4.8: Path inside the university campus of Palermo.

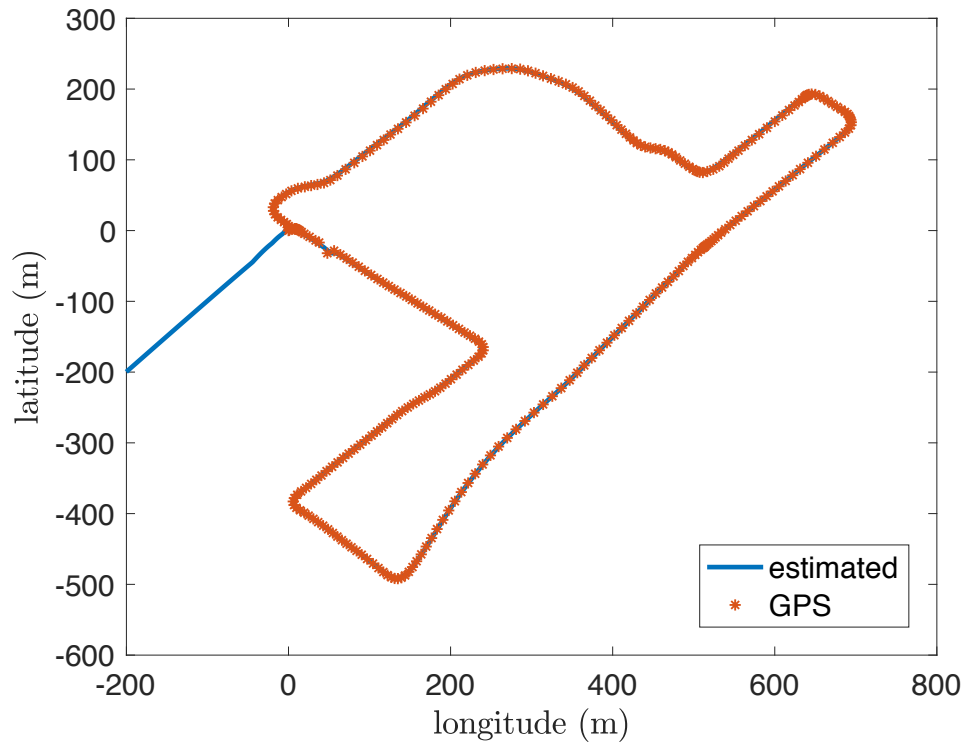


Figure 4.9: Estimated and measured positions.

Also the speed is correctly estimated as confirmed from Figure 4.11 and 4.12, where estimated and measured speeds along the two directions are shown. So the main goal

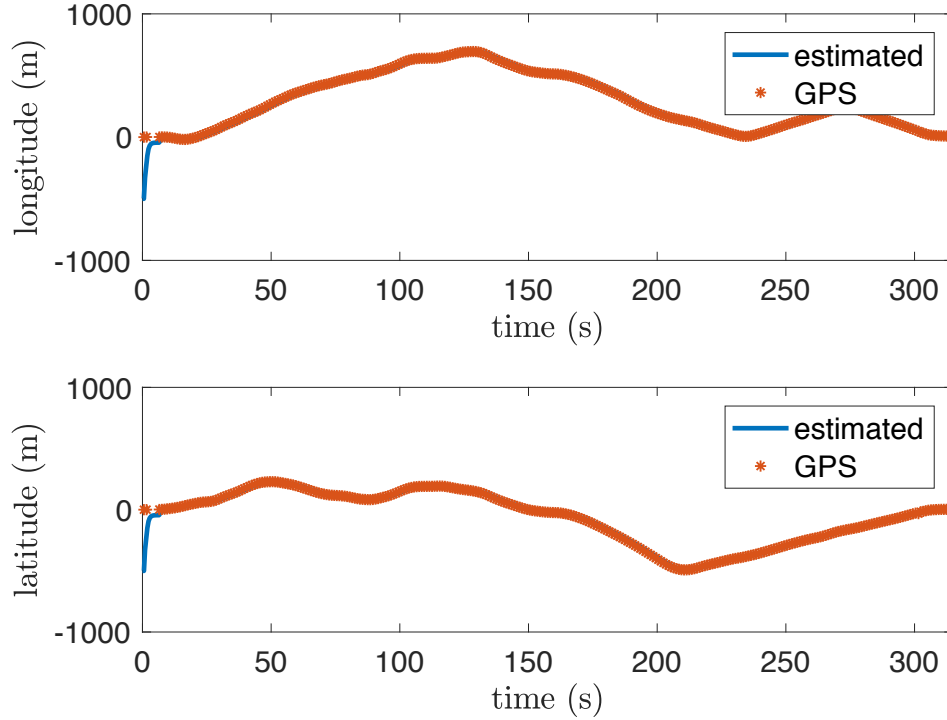


Figure 4.10: Estimated and measured positions.

of this work has been achieved since a continuous-time estimate, avoiding discontinuity in the estimated speed and position, has been provided taking advantage from all information. These results confirm the results already presented by simulation and similar considerations can be given, however the waveforms of estimation errors cannot be shown in the experimental results because during the experiments only the position measured by GPS is available, but the exact position (or real reference) is usually not available. This aspect enforces also the importance of the simulation results in order to validate the sensitivity analysis and the bounds of the estimation errors obtained in Section V.

In order to evaluate the effects of different choices of the observer gain, a further off-line test has been carried out. In particular using the same data acquired during the precedent test, the dynamics of the proposed observer has been evaluated for different values of the gains k_s and k_x . In particular Figures 4.14 and Figure 4.15 show particulars of the estimated trajectory for the following three cases: $k_s = 0.65$ and $k_x = 1$, $k_s = 0.8$ and $k_x = 4$, $k_s = 0.95$ and $k_x = 8$. The Figures show exactly the expected behavior for the proposed observer dynamics. Indeed in all cases the estimation error converge to zero exponentially, but with different rates and different filtering actions as expected by the theoretic results of Section IV. In particular, higher the gain k_s is and higher is the effect of the measurement for the correction of the estimate during jump, but contemporary

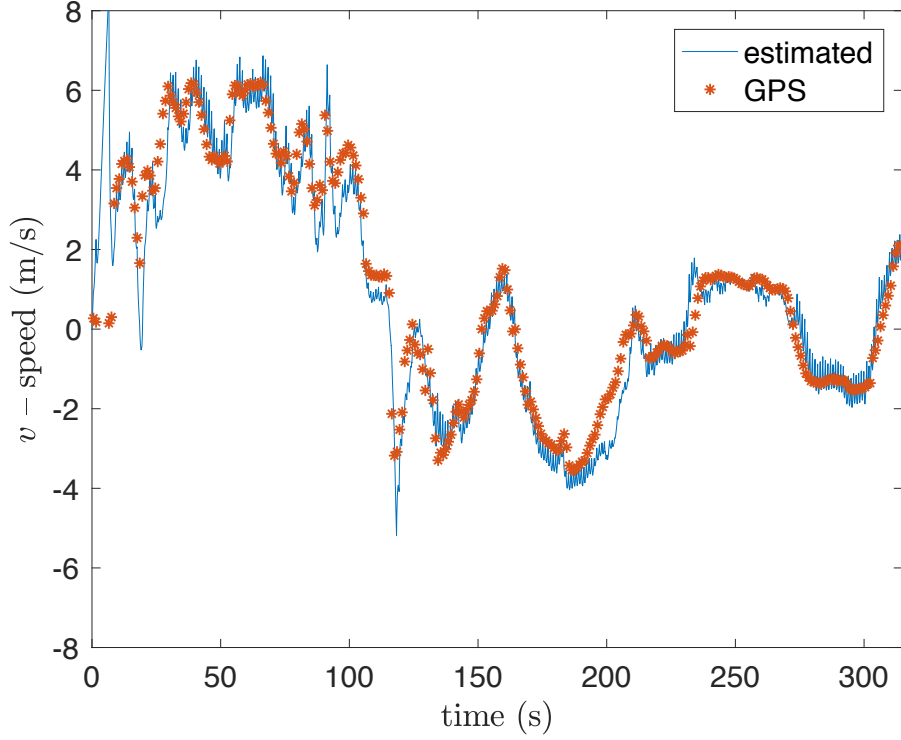


Figure 4.11: Estimated and measured speeds along longitude direction.

lower is the filtering action, so possibly uncertainties in the position measurement will be not adequately filtered out if a value of k_s close to one is chosen. Instead K_x drives the dynamics of the observer between two jumps, and it should be chosen in function of T_M such that the error e_x approaches to zero before the next jump as shown in Remark 1.

It should be emphasized that the shown estimation is not the result of a trivial interpolation, since an interpolation would require the knowledge of the future sample, but the trajectory after the last measurement evolves according the inertial model and it follows the real trajectory correctly. This confirm the effectiveness of the proposed sensor fusion procedure.

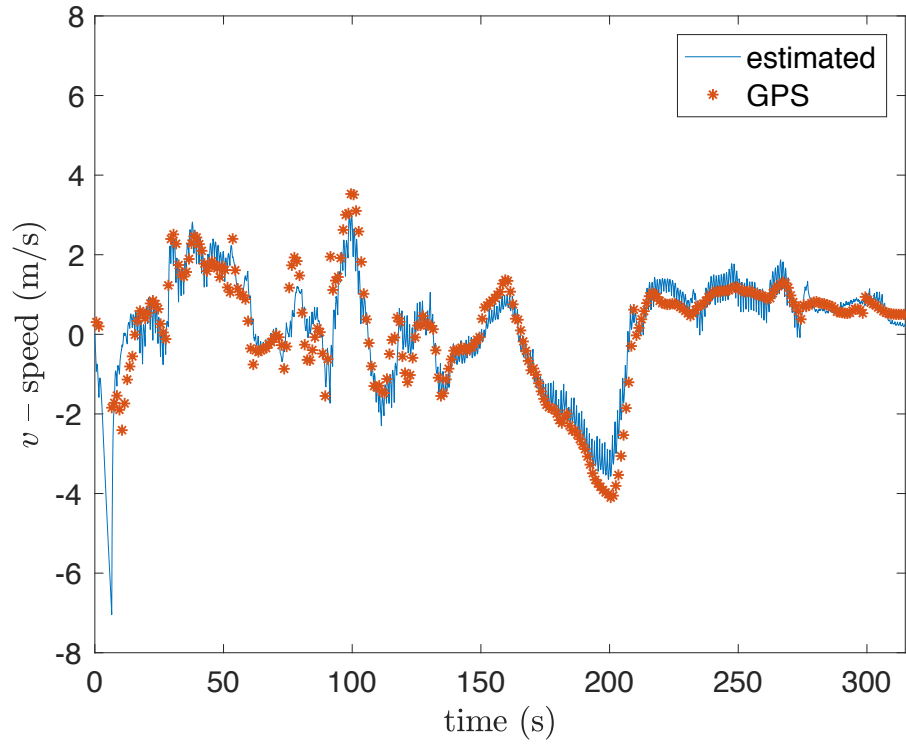
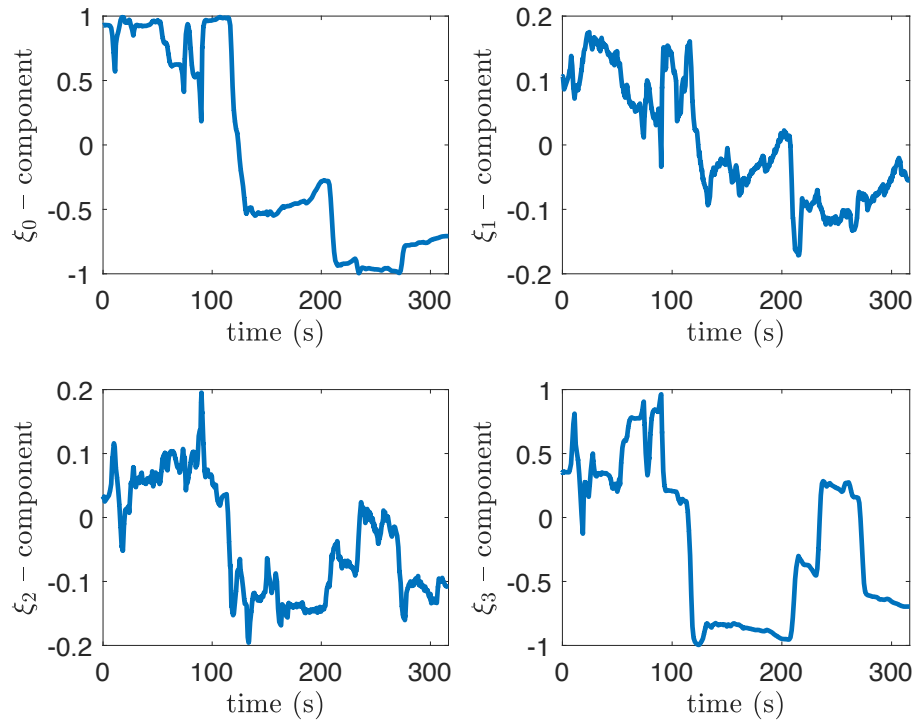


Figure 4.12: Estimated and measured speeds along latitude direction.

Figure 4.13: Components of the quaternion ξ_b^n .

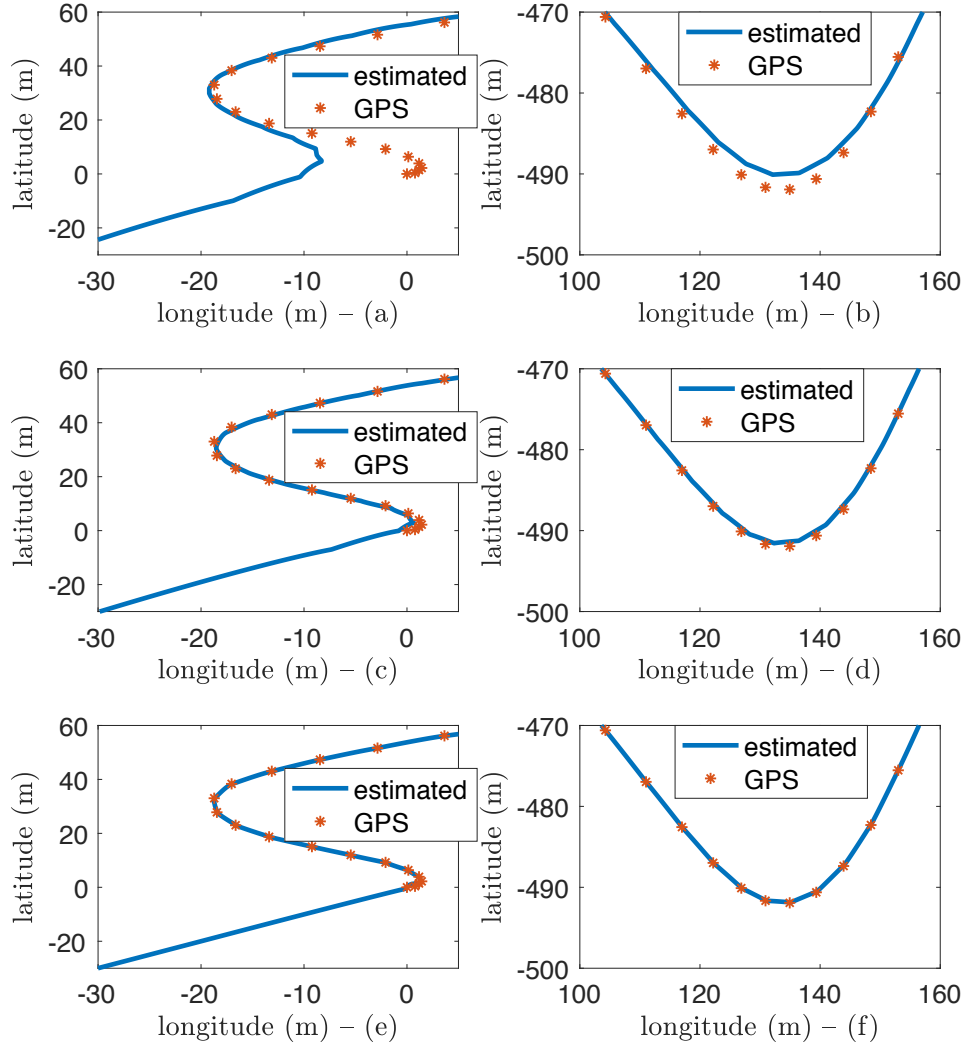


Figure 4.14: Particulars of the estimated and measured trajectory with: $k_s = 0.65$ and $k_x = 1$ in subplots (a)-(b), $k_s = 0.8$ and $k_x = 4$ in subplots (c)-(d), $k_s = 0.95$ and $k_x = 8$ in subplots (e)-(f).

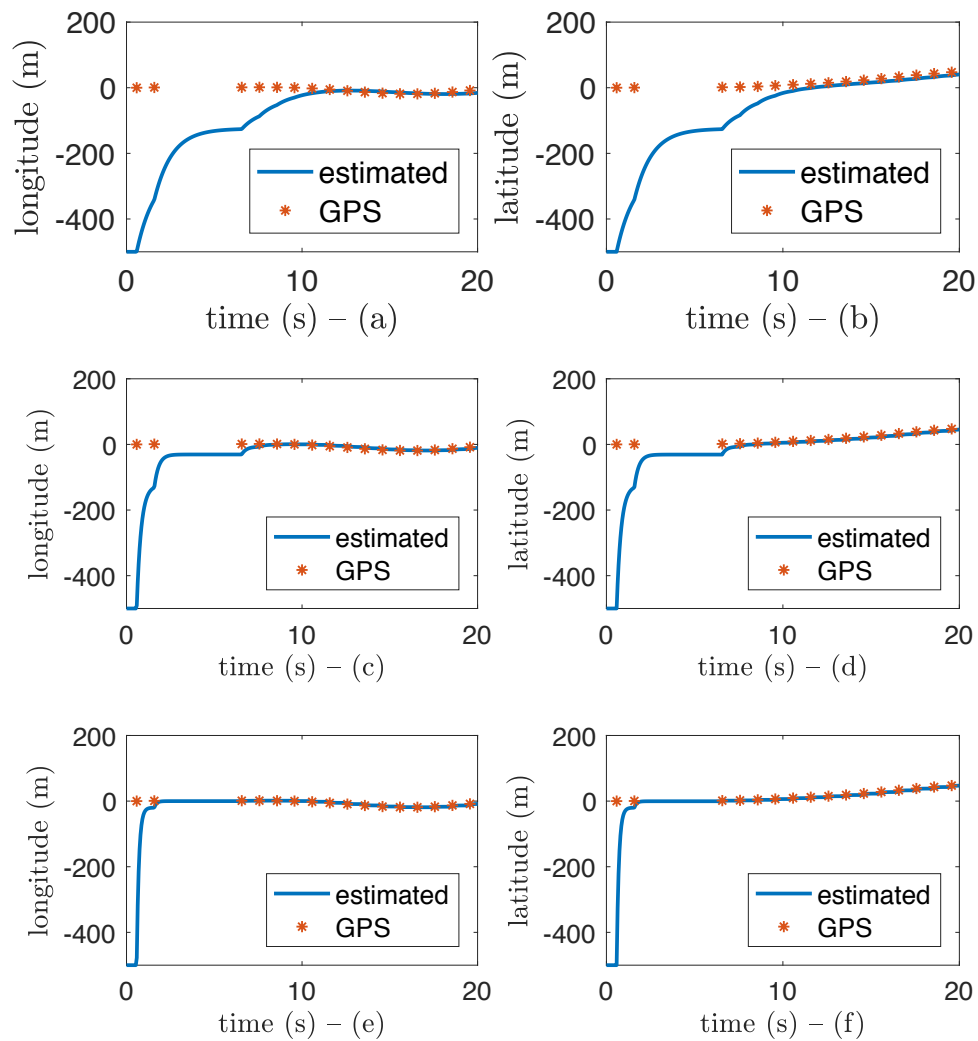


Figure 4.15: Initial transient of the estimated and measured latitude and longitude with: $k_s = 0.65$ and $k_x = 1$ in subplots (a)-(b), $k_s = 0.8$ and $k_x = 4$ in subplots (c)-(d), $k_s = 0.95$ and $k_x = 8$ in subplots (e)-(f).

Chapter 5

Alternative to Trilateration

OUTLINE. In this chapter a new scheme for range-based localization is proposed. The main goal is to estimate the position of a mobile point based on distance measurements from fixed devices, and on inertial measurements. Due to the non-linear nature of the problem, an analytic relation to compute the position starting from these measurements does not exist, and often trilateration methods are used, generally based on least-square algorithms. Here, a closed loop solution is proposed, suggested from the direct kinematics of parallel robot. In particular, inspired from the robotic context, a closed loop position observer, based on range measurement, is proposed. Looking at Fig. 5.1, if a point P , whose position has to be estimated, is identified as an end effector, and the distances of this point from the anchors as prismatic links, the system can be interpreted as a parallel robot. Therefore, if the position of the point has to be determined and the distances from other fixed points are known (or rather the length of the prismatic links is known), this is a classical direct kinematic problem for parallel robot, whose solution does not exist in a closed form. For this reason, it is possible to take advantage from the results presented in the robotic literature context, to solve the position estimation problem, avoiding the use of geometrical trilateration algorithms that represents the common way to deal with this kind of problems. The estimation error dynamics of the proposed observer shows a peculiar structure, allowing an intuitive gain tuning and ensuring global exponential stability of the error dynamics. Moreover, it is also shown a nice connection between observer gains and rate of convergence of the estimation error. Experimental results, will show the effectiveness of the proposed approach.

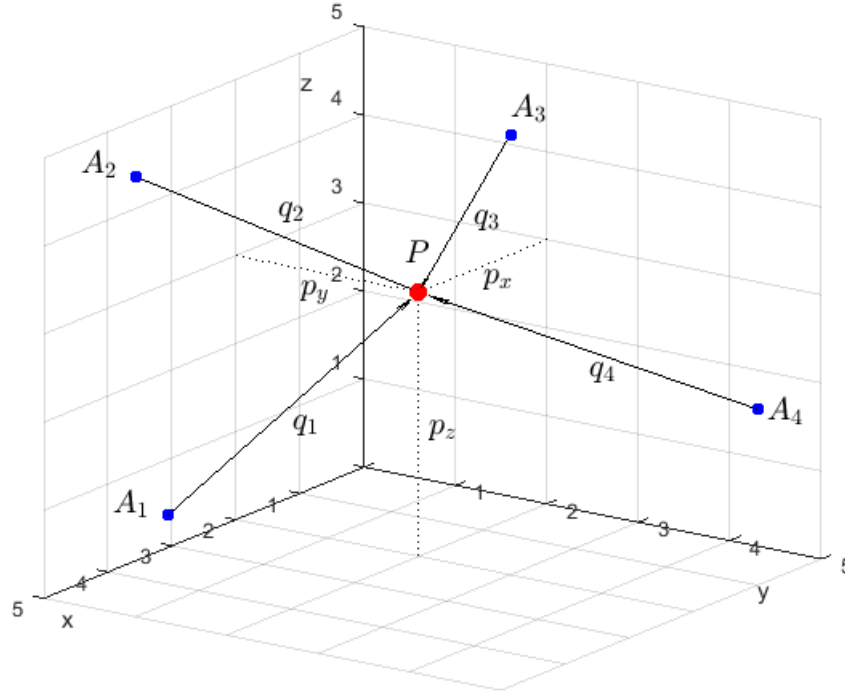


Figure 5.1: Scheme of the localization system.

5.1 Problem statement

The goal of this work is the position estimation of mobile vehicles in the 3-D space, starting from range measurements coming from fixed anchors whose positions are known. In particular, Fig. 5.1 schematizes a possible context where the position coordinates (p_x , p_y and p_z) of a mobile point P in a 3-D space have to be estimated, by measuring the distances, $q_1, \dots, q_n$, from n anchors located in fixed points, $A_1, \dots, A_n$. This problem is not new, and there is a huge number of works in literature dealing with it. However, as shown in Section 1.1.3, most of them calculate the best approximation of the mobile object position by recursive procedures, generally based on least-square techniques, by using a geometrical approach called trilateration. This is because, due to both the intrinsic non-linear nature of the problem and the disturbances affecting the distance measurements, it is difficult to determine a closed-form solution for the position of the point P .

In this Section a novel method to estimate the mobile object position P , that has never been applied in other past works at best Author's knowledge, is presented. In particular, the direct kinematics algorithm for parallel robot will be used. The inverse kinematics equations establish the relationship between the end-effector position and the joint variables of a parallel robot by means of a closed form solution. Note that for serial

robot it is exactly the opposite, i.e. direct kinematic (from the joint variables to the end-effector) has always a unique closed form solution, contrary to the inverse kinematic problem.

Looking at Fig. 5.1, if the point P is identified as an end effector, and the distances q_1 , q_2 , q_3 and q_4 as prismatic links, the system can be interpreted as a parallel robot. Therefore, if the position of the point P is known and we want to determine distances q_1 , q_2 , q_3 and q_4 , this is a classical inverse kinematic problem for parallel robot, whose solution always exists in a closed form. However, the goal is the opposite, or rather to determine the position of the point P starting from the measurements of the distances q_1 , q_2 , q_3 and q_4 . For this reason it is possible to take advantage from the results already presented in the robotic literature context (see, for example, [100]), where this problem has already been addressed, and to extend these results for the presented purpose.

A further important consideration is that the minimum number of anchors which allows to obtain the position of P without indeterminacy, is equal to four. Indeed, the use of three anchors causes indeterminacy in the position of P , because of it could be either over or under the anchors' plane, or rather the plane containing the three anchors. However, this indeterminacy can be excluded if it is known, a priory, that the whole motion of P occurs either under or over the anchors' plane.

5.2 Estimation algorithm

As already said in the precedent Section, looking at Fig. 5.1, if the point P is identified as an end effector, and the distances q_1 , q_2 , q_3 and q_4 as prismatic links, the system can be interpreted as a parallel robot whose direct kinematics equations are given by ([35, Chapter 3.7]):

$$\mathbf{p} = \mathcal{K}(\mathbf{q}). \quad (5.1)$$

However, in the case under study, it is known the structure of the inverse kinematics, given by:

$$\mathbf{q} = \mathcal{K}^{-1}(\mathbf{p}), \quad (5.2)$$

where the generic component q_i , $i=1, \dots, n$, of $\mathcal{K}^{-1}(\mathbf{p})$, is given by:

$$q_i = \sqrt{(x_i - p_x)^2 + (y_i - p_y)^2 + (z_i - p_z)^2} \quad \forall i \in \mathcal{S}, \quad (5.3)$$

where x_i , y_i , z_i denote the components of the position of the i^{th} anchor A_i referred to a fixed reference frame, p_x , p_y and p_z , denote the coordinate of the mobile point P , q_i is

the distance between P and the i^{th} anchor and $\mathcal{S} = \{1, 2, \dots, n\}$ denotes the discrete set of integer numbers, representing the number of anchors. In the case under study, $n \geq 3$.

The problem to solve is to estimate the coordinates of P , thus determining the direct kinematics (5.1). To this end, by computing the derivative of equation (5.2) with respect to time, it is obtained:

$$\dot{\mathbf{q}} = \mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p})\dot{\mathbf{p}}, \quad (5.4)$$

where $\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p}) \in R^{n \times 3}$ is the Jacobian matrix of the vectorial function $\mathcal{K}^{-1}(\mathbf{p})$, given by:

$$\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p}) = \frac{\partial \mathcal{K}^{-1}(\mathbf{p})}{\partial \mathbf{p}}. \quad (5.5)$$

Computing the derivative of equation (5.1), it is obtained:

$$\dot{\mathbf{p}} = \mathbf{J}_{\mathcal{K}}(\mathbf{q})\dot{\mathbf{q}}, \quad (5.6)$$

where:

$$\mathbf{J}_{\mathcal{K}}(\mathbf{q}) = \frac{\partial \mathcal{K}(\mathbf{q})}{\partial \mathbf{q}}. \quad (5.7)$$

From (5.6) and (5.4), it is obtained $\mathbf{J}_{\mathcal{K}}(\mathbf{q})\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p}) = \mathbf{I}_3$, and consequently $\mathbf{J}_{\mathcal{K}}(\mathbf{q})$ is the left pseudoinverse matrix of $\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p})$, given by:

$$\mathbf{J}_{\mathcal{K}}(\mathbf{q}) = (\mathbf{J}_{\mathcal{K}^{-1}}^T(\mathbf{p}) \mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p}))^{-1} \mathbf{J}_{\mathcal{K}^{-1}}^T(\mathbf{p}). \quad (5.8)$$

Now it is possible to use the inverse kinematics algorithm shown in [35, Chapter 3.7], but instead to apply the algorithm to robots, here it is applied in the different context of the localization. Additionally, differently than [35, Chapter 3.7], where a fixed end effector position is considered, here the case of a mobile point is addressed, by exploiting inertial measurements (usually provided by IMU) for improving the position estimation.

In the following the case when the anchor number n is equal to three ($n = 3$) is considered, with respect to the case $n \geq 4$, since for $n = 3$ it is possible to show some interesting properties of the estimation error dynamics, that allow to give some intuitive tuning hints for the estimator design.

5.2.1 Localization using three anchors

Before to give the main results of this subsection, the following Assumption is provided:

Assumption 3 *Given three anchors A_1 , A_2 and A_3 , identifying with Σ_A the plane con-*

taining these anchors, if an orthonormal reference frame x - y - z is fixed, whose x and y axis belong to Σ_A , the sign of the z component of the point P , with respect to the x - y - z reference frame, is assumed known.

Note that Assumption 3 is compliant with the consideration given in Section 5.1 about the minimum number of anchors which allows to obtain a unique solution.

The structure of the proposed closed loop control system for estimating $\mathbf{p}$ is given in Fig. 5.2, where the controller is described by the following equation:

$$\dot{\hat{\mathbf{p}}} = \mathbf{J}_{\mathcal{K}^{-1}}^{-1}(\hat{\mathbf{p}}) \left(\dot{\mathbf{q}} + \mathbf{K}_P(\mathbf{q} - \hat{\mathbf{q}}) + \mathbf{K}_I \int_0^t (\mathbf{q} - \hat{\mathbf{q}}) d\tau \right). \quad (5.9)$$

where $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$ are respectively the estimate of the position vector and the distance vector computed from it, whereas $\mathbf{p}$ and $\mathbf{q}$ are the true position and distance vectors. Note that the controller (5.9) is well defined because $\mathbf{J}_{\mathcal{K}^{-1}}^{-1}$ there exist since $\mathbf{J}_{\mathcal{K}^{-1}}$ is a square full-rank matrix except the case when the point P coincides with one of the anchors. However this last condition is not realistic from practical point of view, and therefore it will not be addressed.

One result of this work proves stability and convergence of the estimation error in the extended error space corresponding to the error variables:

$$\mathbf{e} = \begin{bmatrix} \mathbf{e}_q \\ \mathbf{e}_p \end{bmatrix} = \begin{bmatrix} \mathbf{q} - \hat{\mathbf{q}} \\ \mathbf{p} - \hat{\mathbf{p}} \end{bmatrix}. \quad (5.10)$$

Now it is possible to give the following theorem.

Theorem 3 *If Assumption 3 holds, the estimation errors (8.23) converge exponentially to zero along dynamics (5.9) and (5.6) if and only if there exists matrices $\mathbf{K}_P$ and $\mathbf{K}_I$ such that matrix $\mathbf{\Gamma}$, defined as:*

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}_I & -\mathbf{K}_P \end{bmatrix}, \quad (5.11)$$

is Hurwitz.

Proof: Premultiplying both members of (5.9) by $\mathbf{J}_{\mathcal{K}^{-1}}(\hat{\mathbf{p}})$, and considering equation (5.4), it is obtained:

$$\dot{\hat{\mathbf{q}}} = \dot{\mathbf{q}} + \mathbf{K}_P(\mathbf{q} - \hat{\mathbf{q}}) + \mathbf{K}_I \int_0^t (\mathbf{q} - \hat{\mathbf{q}}) d\tau \quad (5.12)$$

Since $\mathbf{e}_q = \mathbf{q} - \hat{\mathbf{q}}$, equation (5.12) can be rewritten as:

$$\dot{\mathbf{e}}_q + \mathbf{K}_P \mathbf{e}_q + \mathbf{K}_I \int_0^t (\mathbf{e}_q) d\tau = 0. \quad (5.13)$$

by deriving (5.13) it is obtained:

$$\ddot{\mathbf{e}}_q + \mathbf{K}_P \dot{\mathbf{e}}_q + \mathbf{K}_I \mathbf{e}_q = 0. \quad (5.14)$$

Therefore if matrix (5.11) is Hurwitz, then tracking error $\mathbf{e}_q$ converges exponentially to zero. In order to prove the exponential convergence of estimation error in the extended error space (8.23), it is necessary to show that also $\mathbf{e}_p$ converges exponentially to zero. Based on straightforward geometric considerations, in case of three anchors, Eq. (5.1) has two solutions, but only one can be chosen under Assumption 3. Therefore, the position error can be written as:

$$\mathbf{e}_p = \mathcal{K}(\mathbf{q}) - \mathcal{K}(\hat{\mathbf{q}}). \quad (5.15)$$

Since $\mathbf{e}_q$ converges exponentially to zero (as above proven), this implies that $\hat{\mathbf{q}} \rightarrow \mathbf{q}$ and therefore $\mathcal{K}(\mathbf{q}) - \mathcal{K}(\hat{\mathbf{q}}) \rightarrow 0$. This consideration leads to the exponential convergence of estimation error in the extended error space (8.23), because both $\mathbf{e}_q$ and $\mathbf{e}_p$ converge exponentially to zero. Note that, Assumption 3 can be removed if more than three anchors are used, because in this case Eq. (5.1) admits only one solution.

Finally, to prove also necessity, it is assumed that matrix Γ is not Hurwitz. This implies that at least one eigenvalue is with positive real part. This clearly generates a solution of (5.14) not converging to zero, thus proving that the exponential convergence of estimation error does not hold. $\square$

Remark 2 *Note that the role of the IMU in the algorithm is in the computation of the term $\dot{\mathbf{q}}$ in (5.9). Indeed it is computed applying (5.4) where $\dot{\mathbf{p}}$ is obtained by integration of the acceleration measured by the IMU. The reason of adding this term is because fast transients can be better tracked compared with the case when the inertial measurements are not considered. However, this term $\dot{\mathbf{q}}$ in (5.9) is usually affected by errors because IMU measurements are noisy and biased. Moreover, in the calculation of $\dot{\mathbf{q}}$, $\mathbf{J}_{\mathcal{K}^{-1}}(\hat{\mathbf{p}})$ is used instead of $\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p})$, therefore, this approximation represents a second source of error for the term $\dot{\mathbf{q}}$. However, as it will be shown in Section 5.2.2, due to the continuity property of the Jacobian this error is bounded. In this case, by following the same approach of Theorem 3 it is possible to show a practical stability of the estimation error when $\dot{\mathbf{q}}$ in (5.9) is not perfectly known. Indeed, assuming $\dot{\hat{\mathbf{q}}} := \dot{\mathbf{q}} + \Delta_{\dot{\mathbf{q}}}$, where $\dot{\mathbf{q}}$ is the measured*

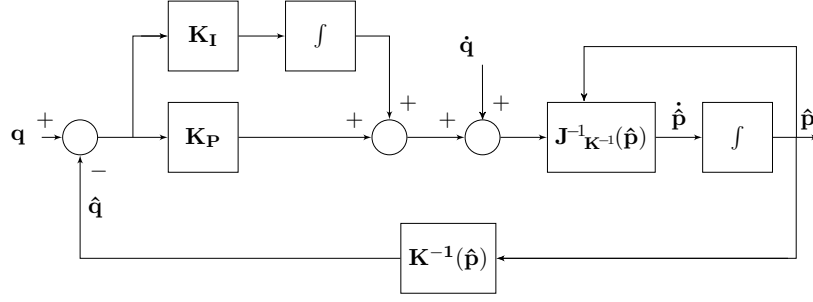


Figure 5.2: Block diagram of the proposed algorithm.

quantity and $\Delta_{\dot{\mathbf{q}}}$ is the related bounded error, and replacing $\dot{\hat{\mathbf{q}}}$ in (5.9) instead of $\dot{\mathbf{q}}$, then (5.14) can be rewritten as:

$$\ddot{\mathbf{e}}_q + \mathbf{K}_P \dot{\mathbf{e}}_q + \mathbf{K}_I \mathbf{e}_q + \Delta_{\dot{\mathbf{q}}} = 0. \quad (5.16)$$

Equation (5.16) leads to a steady state-error equal to:

$$\begin{bmatrix} \|\mathbf{e}_q\|_{ss} \\ \|\dot{\mathbf{e}}_q\|_{ss} \end{bmatrix} = \begin{bmatrix} \frac{\|\Delta_{\dot{\mathbf{q}}}\|}{\|\mathbf{K}_I\|} \\ 0 \end{bmatrix}. \quad (5.17)$$

Equation (5.17) highlights that the effect of possible measurement errors on $\dot{\mathbf{q}}$ can be suitably reduced by increasing $\mathbf{K}_I$.

Remark 3 Note that the proposed controller (5.9) can be simplified by neglecting the integral term, and considering the following simplified equation:

$$\dot{\hat{\mathbf{p}}} = \mathbf{J}_{\mathbf{K}^{-1}}^{-1}(\hat{\mathbf{p}}) (\dot{\mathbf{q}} + \mathbf{K}_P(\mathbf{q} - \hat{\mathbf{q}})). \quad (5.18)$$

However, as shown in Remark 2, the integral action gives an important contribution to reduction of the estimation error when $\dot{\mathbf{q}}$ is not perfectly known.

Note that the structure of the controller (5.9) is nonlinear, and it is suitably chosen so that the dynamics of the tracking error $\mathbf{e}_q$, defined as indicated in (8.23), becomes linear and time-invariant, as shown in (5.13) and (5.14). Therefore, it is not necessary a linearization procedure as it has been done in other past works [101].

In the following, an appealing result from the practical point of view is given. In particular, an explicit tuning rule is given to determine parameters $\mathbf{K}_P$ and $\mathbf{K}_I$ in order

to ensure a well defined convergence speed of the tracking error $\mathbf{e}_q$ and, consequently, of the estimation error $\mathbf{e}_p$.

Proposition 3 *Given scalars $\alpha > 0$ and $\beta > 2$, if $\mathbf{K}_P := k_p \otimes \mathbf{I}_3 = \beta\alpha \otimes \mathbf{I}_3$ and $\mathbf{K}_I := k_i \otimes \mathbf{I}_3 > \alpha^2(\beta - 1) \otimes \mathbf{I}_3$, there exist a symmetric, positive definite, matrix $\mathbf{P} := \mathbf{I}_3 \otimes \bar{\mathbf{P}}$ with $\bar{\mathbf{P}} \in \mathbb{R}^{2 \times 2}$, such that:*

$$|\bar{\mathbf{e}}_q(t)| \leq \sqrt{k} e^{-\alpha t} |\bar{\mathbf{e}}_q(0)|, \forall t \geq 0, \quad (5.19)$$

where $\bar{\mathbf{e}}_q := [\mathbf{e}_q^\top, \dot{\mathbf{e}}_q^\top]^\top$ and $k = \frac{\lambda_m(\mathbf{P})}{\lambda_M(\mathbf{P})}$.

Note that (5.19) induces an exponential bound with rate of convergence equal to α . Therefore α can be considered the rate of convergence for the tracking error $\bar{\mathbf{e}}_q$.

Proof. Consider the radial unbounded Lyapunov function defined as:

$$V(\bar{\mathbf{e}}_q) := \bar{\mathbf{e}}_q^\top \mathbf{P} \bar{\mathbf{e}}_q \quad (5.20)$$

where $\mathbf{P}$ is a symmetric positive definite matrix, whose derivative is:

$$\dot{V}(\bar{\mathbf{e}}_q) = \bar{\mathbf{e}}_q^\top (\mathbf{P}\mathbf{\Gamma} + \mathbf{\Gamma}^\top \mathbf{P}) \bar{\mathbf{e}}_q \quad (5.21)$$

where $\mathbf{\Gamma}$ is defined in (5.11). For [102, Definition 8.1] and [102, Theorem 8.2] the bound in (5.19) on $\bar{\mathbf{e}}_q$ can be inferred if matrix $\mathbf{P}$ in (5.21) is such that:

$$\mathbf{P}\mathbf{\Gamma} + \mathbf{\Gamma}^\top \mathbf{P} < -2\alpha \mathbf{P}. \quad (5.22)$$

In order to verify matrix inequality in (5.22) it needs to be:

$$\mathbf{P}\mathbf{\Gamma} + \mathbf{\Gamma}^\top \mathbf{P} + 2\alpha \mathbf{P} = \mathbf{P}(\mathbf{\Gamma} + \alpha \mathbf{I}_6) + (\mathbf{\Gamma} + \alpha \mathbf{I}_6)^\top \mathbf{P} < 0. \quad (5.23)$$

By defining $\mathbf{K}_P := k_p \otimes \mathbf{I}_3$ and $\mathbf{K}_I := k_i \otimes \mathbf{I}_3$ and applying the property of the Kronecker product, (5.23) can be rewritten as:

$$(\mathbf{I}_3 \otimes \bar{\mathbf{P}})(\mathbf{I}_3 \otimes \mathbf{\Gamma}_\alpha) + (\mathbf{I}_3 \otimes \mathbf{\Gamma}_\alpha)^\top (\mathbf{I}_3 \otimes \bar{\mathbf{P}}) = \mathbf{I}_3 \otimes (\bar{\mathbf{P}}\mathbf{\Gamma}_\alpha + \mathbf{\Gamma}_\alpha^\top \bar{\mathbf{P}}) < 0, \quad (5.24)$$

where $\mathbf{\Gamma}_\alpha = \begin{bmatrix} \alpha & 1 \\ -k_i & -k_p + \alpha \end{bmatrix} \in \mathbb{R}^{2 \times 2}$. Since $\mathbf{I}_3 > 0$, there always exist $\bar{\mathbf{P}}$ satisfying (5.24) if $\mathbf{\Gamma}_\alpha$ is Hurwitz [102, Theorem 8.3], or rather $\mathbf{\Gamma}_\alpha$ has eigenvalues with negative real part.

If k_p and k_i are defined as in the statement:

$$\begin{cases} k_p = \beta\alpha, \\ k_i > \alpha^2(\beta - 1), \end{cases} \quad (5.25)$$

with $\beta > 2$, then the polynomial defined as $\det(\lambda\mathbf{I}_2 - \mathbf{\Gamma}_\alpha) = \lambda^2 + \lambda(k_p - 2\alpha) + \alpha(\alpha - k_p) + k_i$ has roots with negative real part, which in turn implies that $\mathbf{\Gamma}_\alpha$ is Hurwitz and therefore there always exist $\bar{\mathbf{P}}$ satisfying (5.24). Finally, the existence of $\bar{\mathbf{P}}$ implies also the existence of $\mathbf{P} = \mathbf{I}_3 \otimes \bar{\mathbf{P}}$ such that (5.22) is satisfied. This concludes the proof. $\square$

Remark 4 *If the reader want to obtain the value of $\bar{\mathbf{P}}$ in order to derive the value of k in (5.19), he can solve the following numerical problem by means of YALMIP [103]:*

$$\begin{aligned} \min_{k, \bar{\mathbf{P}} = \bar{\mathbf{P}}^\top} k, \quad \text{subject to:} \\ \bar{\mathbf{P}}\mathbf{\Gamma}_\alpha + \mathbf{\Gamma}_\alpha^\top \bar{\mathbf{P}} \leq -2\alpha\bar{\mathbf{P}}, \\ \mathbf{I}_2 \leq \bar{\mathbf{P}} \leq k\mathbf{I}_2, \end{aligned} \quad (5.26)$$

where α is the desired speed of convergence and $\mathbf{\Gamma}_\alpha = \begin{bmatrix} \alpha^2(1-\beta) - \eta & \alpha(1-\beta) \\ \alpha(1-\beta) & 1 \end{bmatrix}$ with $\eta > 0$ and $\beta > 2$.

5.2.2 Localization using more than three anchors

If a number of anchors $n > 3$ is used, generalization of (5.9) allows to obtain an algorithmic solution based on the pseudo-inverse of the Jacobian matrix. Generalized formulation of the estimation algorithm leads to the following controller:

$$\dot{\hat{\mathbf{p}}} = \mathbf{J}_{\mathcal{K}^{-1}}^\#(\hat{\mathbf{p}}) \left(\dot{\mathbf{q}} + \mathbf{K}_P(\mathbf{q} - \hat{\mathbf{q}}) + \mathbf{K}_I \int_0^t (\mathbf{q} - \hat{\mathbf{q}}) d\tau \right), \quad (5.27)$$

where $\mathbf{J}_{\mathcal{K}^{-1}}^\#(\hat{\mathbf{p}})$ is the left pseudoinverse of the Jacobian matrix and it is defined as:

$$\mathbf{J}_{\mathcal{K}^{-1}}^\#(\hat{\mathbf{p}}) = (\mathbf{J}_{\mathcal{K}^{-1}}^\top(\hat{\mathbf{p}})\mathbf{J}_{\mathcal{K}^{-1}}(\hat{\mathbf{p}}))^{-1}\mathbf{J}_{\mathcal{K}^{-1}}^\top(\hat{\mathbf{p}}). \quad (5.28)$$

The theoretical results already given for the case when $n = 3$ can be extended to the case of $n > 3$ since the structure of the controller is the same. Here they are highlighted some properties of the Jacobian matrix of the vectorial function $\mathcal{K}^{-1}(\mathbf{p})$ already defined in (5.5)

whose explicit form is:

$$\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p}) = - \begin{bmatrix} j_{1x} & j_{1y} & j_{1z} \\ j_{2x} & j_{2y} & j_{2z} \\ \vdots & \vdots & \vdots \\ j_{nx} & j_{ny} & j_{nz} \end{bmatrix}, \in \mathbb{R}^{n \times 3} \quad (5.29)$$

where:

$$j_{ix} = \frac{x_i - p_x}{q_i}, \quad j_{iy} = \frac{y_i - p_y}{q_i}, \quad j_{iz} = \frac{z_i - p_z}{q_i}, \quad i = 1, 2, \dots, n.$$

The elements of $\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p})$ are continuous function of $\mathbf{p}$ except when $\mathbf{q}_i = \mathbf{0}$. This may happen if the point $\mathbf{p}$ coincide with one of the n anchors, anyway, as already discussed in section 5.2.1, this condition is not realistic from a practical point of view, and it will not be addressed. Therefore, it is considered: $\mathbf{p} \in \Omega$ with $\Omega = \mathbb{R}^3 \setminus \{\mathbf{A}_1 \cup \mathbf{A}_2 \cup \dots \cup \mathbf{A}_n\}$. The continuity of the elements of the Jacobian $\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p})$ in a generic point $\mathbf{p} \in \Omega$ implies that $\forall \epsilon > 0$ there exist a neighborhood of $\mathbf{p}$ of radius δ such that if $\|\hat{\mathbf{p}} - \mathbf{p}\| < \delta$, then $\|\mathbf{J}_{\mathcal{K}^{-1}}(\hat{\mathbf{p}}) - \mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p})\| < \epsilon$, which means that in that neighborhood $\mathbf{J}_{\mathcal{K}^{-1}}(\hat{\mathbf{p}})$ can be approximated by $\mathbf{J}_{\mathcal{K}^{-1}}(\mathbf{p})$ and vice versa. Note that the continuity property is also uniform because of the above mentioned elements of the Jacobian $\mathbf{J}_{\mathcal{K}^{-1}}(\hat{\mathbf{p}})$ are continuous functions in each point P of the space in which they are defined. The above described property of the Jacobian ensure that the proposed estimation solution is well posed.

Remark 5 *Note that, if a number of anchors greater than 3 is used, there is no more indetermination in the sign of the z component of $\mathbf{p}$ because more information are available for the estimation algorithm. Therefore it is possible to ignore Assumption 3 that it is no more necessary. The effectiveness of the control law (5.27) will be shown in the experimental result Section.*

5.3 Experimental Results

The experimental test carried on the environment described in Section 2.3.1 consists of a path with a rhombus shape like in Fig.5.3. In particular the robot is actuated so that the plate, mounted on its end effector, tracks a rhombus whose the major diagonal is oriented like the x -axis and measure $1.1m$, the minor diagonal measures $0.5m$. The starting point of the rhombus is located at $p_{1(x,y,z)} = [2.02, 2.2, 1.8]$, next point is $p_{2(x,y,z)} = [1.42, 1.95, 1.8]$, then $p_{3(x,y,z)} = [2.02, 1.70, 1.80]$, $p_{4(x,y,z)} = [2.62, 1.95, 1.8]$ and finally it

return to the initial position. Four fixed anchors are placed in $A_{1(x,y,z)} = [4.58, 4.58, 1.6]$, $A_{2(x,y,z)} = [4.58, 0, 2.2]$, $A_{3(x,y,z)} = [0, 0, 1]$ and $A_{4(x,y,z)} = [0, 4.58, 2.8]$. In order to highlight the convergence transient of the estimation error for the proposed algorithm, the initial condition has been set far from the real one and in particular in $p_{0(x,y,z)} = [0, 0, 1.8]$. Finally, to test only the closed loop localization algorithm proposed in this work, the motion imposed to the end effector is chosen so that the corresponding Euler angles are equal to zero. In this way, the results obtained are independent of attitude errors produced by the IMU because the attitude is known and the acceleration measurements referred to the body frame correspond to that ones referred to the inertial frame.

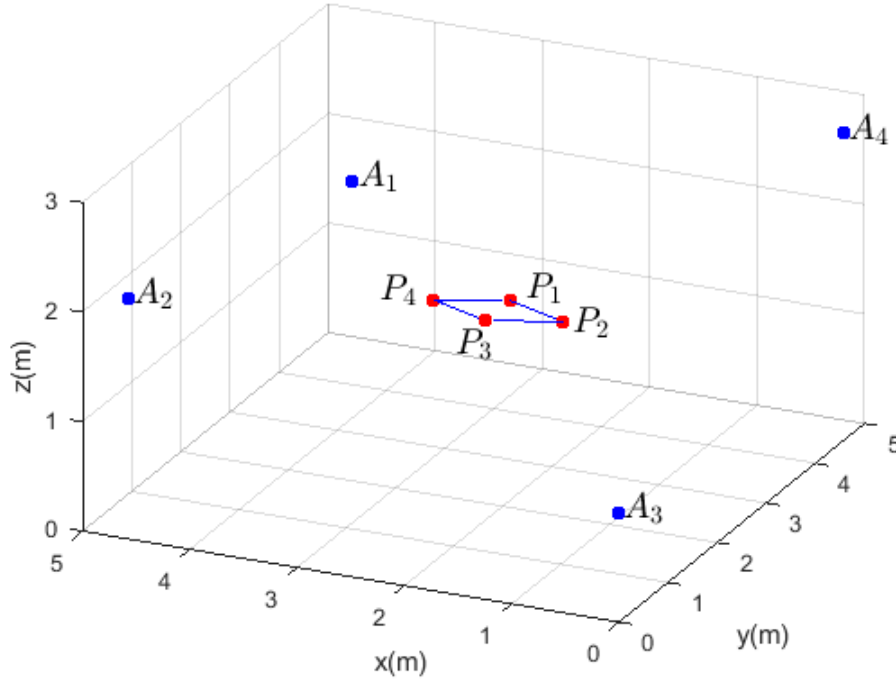


Figure 5.3: Scheme of the experimental test: (A_i) fixed beacon, (P_i) points of the path.

To prove the effectiveness of the proposed algorithm two tests have been carried out, the first one using 3 anchors whose results are showed in Sec.5.3.1 and the second one using 4 anchors whose results are showed in Sec.5.3.2. Each test was performed selecting $k_p = 2$ and $k_i = 1$, that correspond with a convergence rate $\alpha = 1$ according to Proposition 3. Also some tests with different value of α have been carried out in order to validate the exponential bound (5.19) of Proposition 3.

5.3.1 Experimental results using three anchors

In this first test only distance measurements from anchors A_1 , A_2 and A_3 have been used in order to prove the proposed estimation algorithm given in (5.18). In Figs. 5.4-5.5 the waveforms of the position component p_x and p_y are shown as well as the corresponding estimation error e_{px} and e_{py} . The measured and estimated distances from anchors q_1 , q_2 and q_3 are shown in Fig. 5.6 while in Fig. 5.7 there are the corresponding estimation errors e_{q1} , e_{q2} and e_{q3} . Finally in Fig. 5.8 it is shown a comparison of the three distance estimation errors waveforms e_{q1} , e_{q2} and e_{q3} obtained with three different values of α to prove the validity of the exponential bound (5.19). Starting from a different initial conditions, it is evident that the feedback variables, i.e. the estimated distances, converge to the reference values and the control system tracks correctly the path providing a good position estimate, indeed the estimated variables exponentially converge to the corresponding true variables. The steady state error has zero mean, both for positions and distances and it is contained in a band of ± 0.05 m for position and ± 0.1 m for distances coherently with the accuracy of the sensors. Moreover, the validity of the exponential bound (5.19) of Proposition 3 has been proven, as clearly shown in Fig. 5.8.

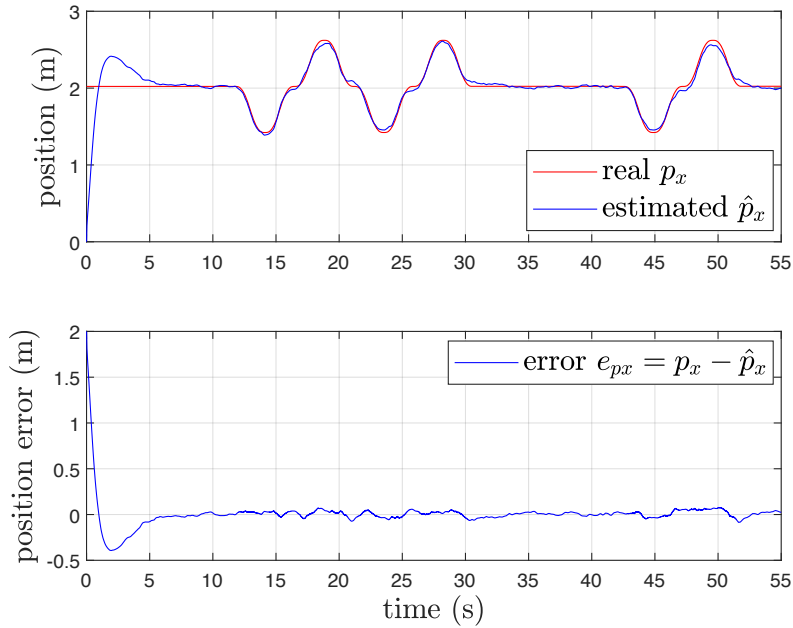


Figure 5.4: Real and estimated position component p_x . Test with 3 anchors.

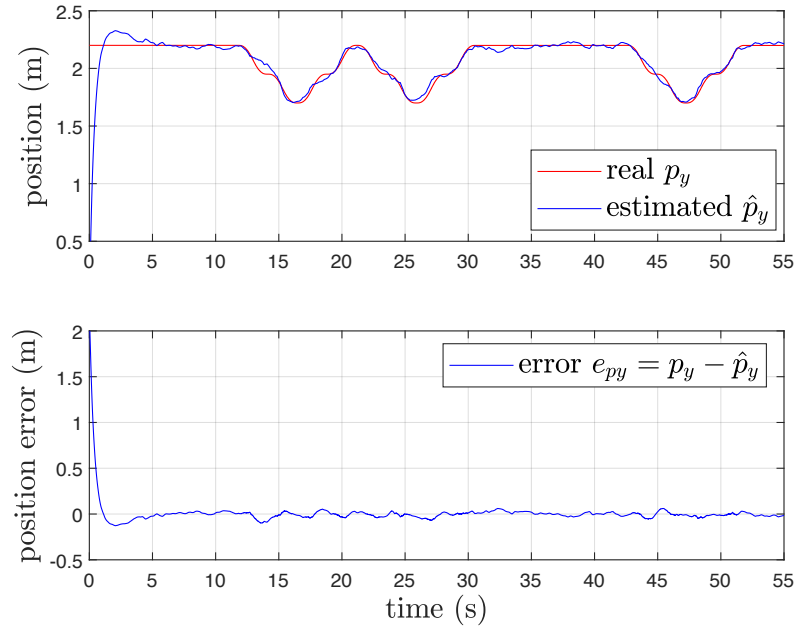


Figure 5.5: Real and estimated position component p_y . Test with 3 anchors.

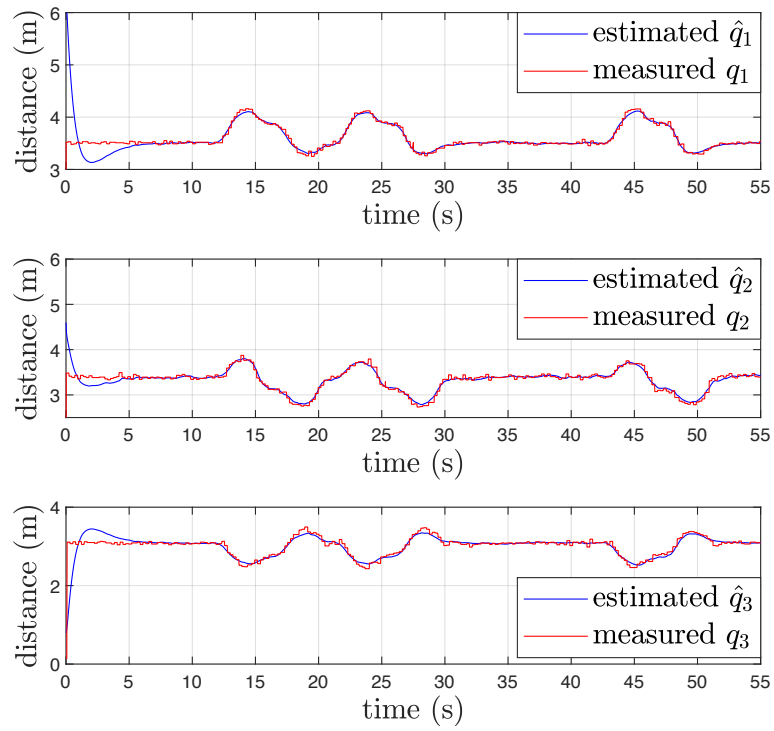


Figure 5.6: Measured and estimated distances q_1, q_2 and q_3 . Test with 3 anchors.

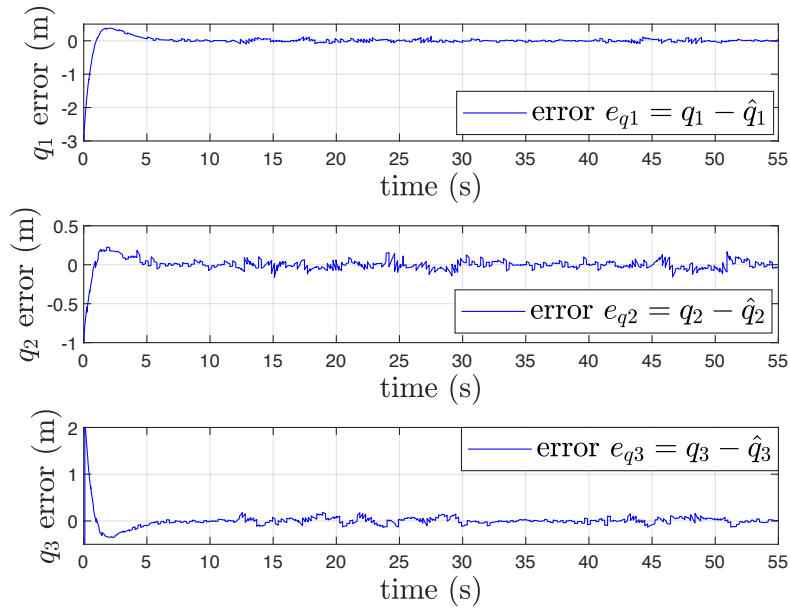
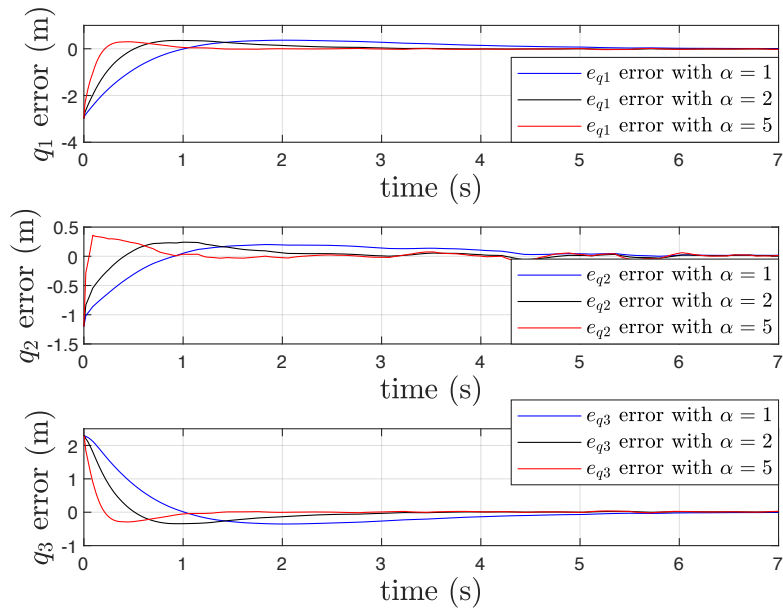


Figure 5.7: Distance errors. Test with 3 anchors.

Figure 5.8: Estimation error waveforms with different values of α . Test with 3 anchors.

5.3.2 Experimental results using four anchors

In this second test distance measurements from all anchors A_1 , A_2 , A_3 and A_4 have been used in order to prove the proposed estimation algorithm given in (5.27). In Figs. 5.9-5.10 the waveforms of the position component p_x and p_y are shown as well as the corresponding estimation error e_{px} and e_{py} .

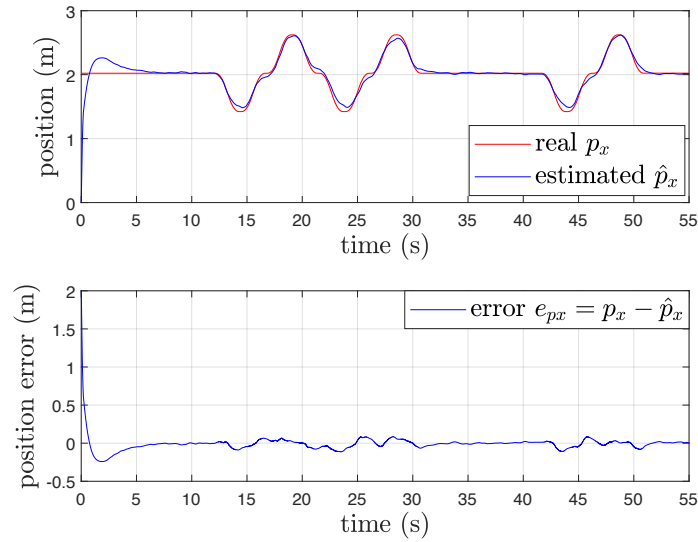


Figure 5.9: Real and estimated position component p_x . Test with 4 anchors.

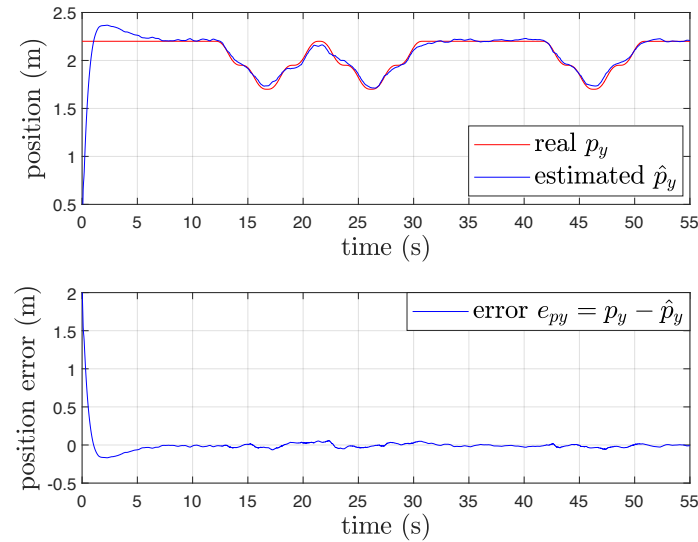


Figure 5.10: Real and estimated position component p_y . Test with 4 anchors.

The measured and estimated distances from anchors q_1 , q_2 , q_3 and q_4 are shown in Fig. 5.11 while in Fig. 5.12 there are the corresponding distance estimation errors e_{q1} , e_{q2} , e_{q3} and e_{q4} .

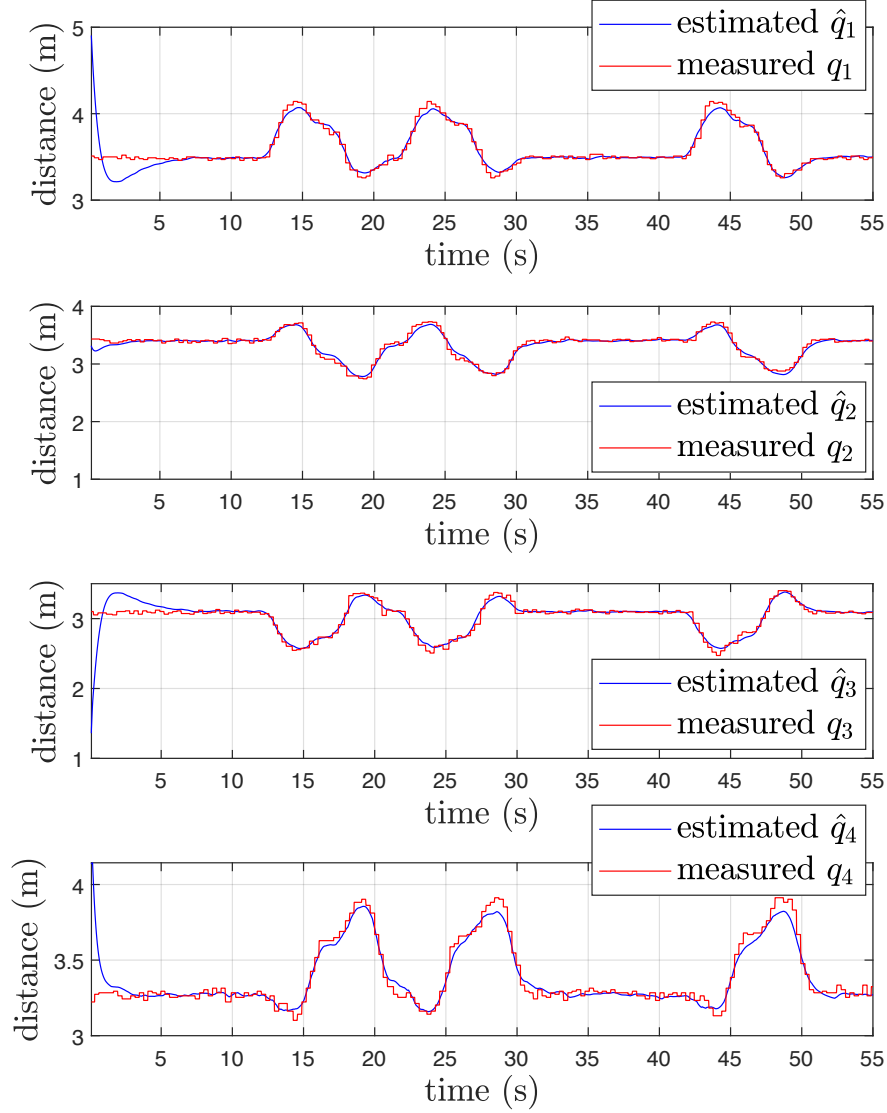


Figure 5.11: Measured and estimated distances q_1, q_2, q_3 and q_4 . Test with 4 anchors.

Finally, in Fig. 5.13 a comparison is shown among distance estimation errors obtained with three different values of α to prove the validity of the exponential bound (5.19).

In this case of four anchors, similar results of the case with three anchors have been obtained and therefore similar comments to those above given can be inferred. This shows the effectiveness of the proposed approach and the correctness of the theoretical results shown in Section 5.2.

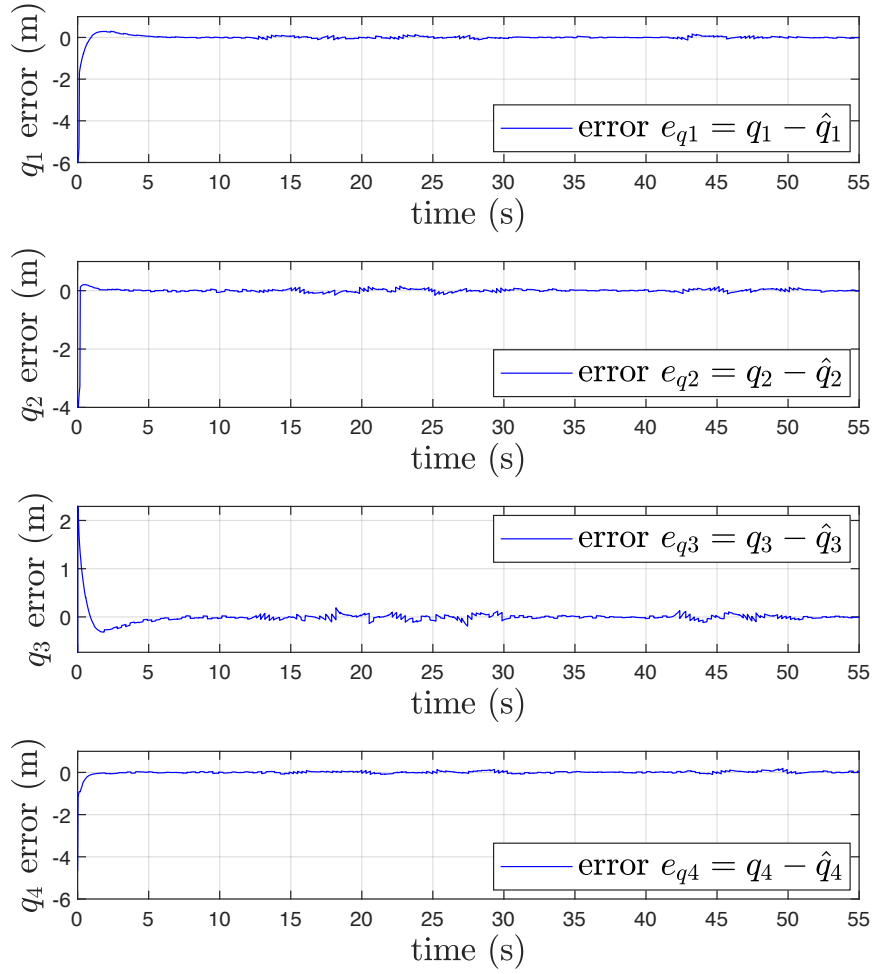


Figure 5.12: Distance errors. Test with 4 anchors.

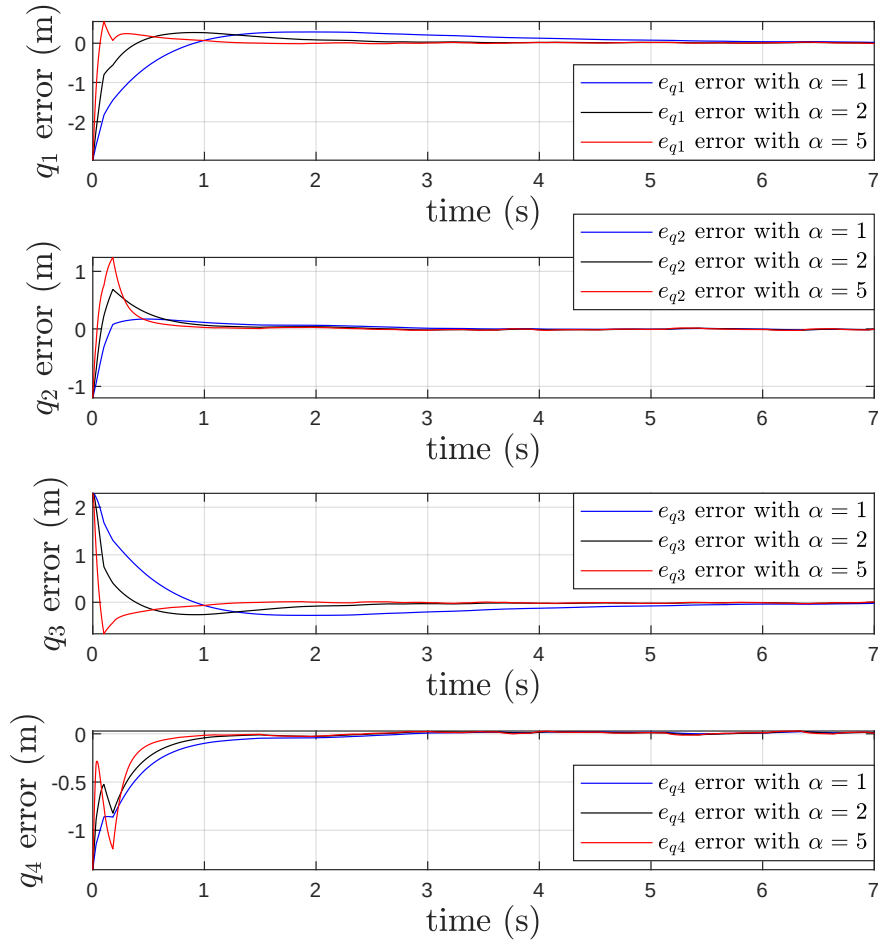


Figure 5.13: Estimation error waveforms with different values of α . Test with 4 anchors.

5.3.3 Comparison with general nonlinear least-square trilateration

In this subsection a comparison has been carried out between the proposed algorithm and the general Nonlinear Least-Squares (NLS) trilateration, based on Newton's method. In particular the same experiment shown in the previous section has been done, with the same path and under same conditions, and the results are given in Figs. 5.14-5.15 and Tab. 5.1. In particular, from Figs. 5.14-5.15 it is clear that the proposed algorithm exhibits a better filtering action and a lower estimation error. On the contrary the general NLS trilateration method presents a smaller settling time. However the speed of convergence of the proposed algorithm is sufficiently high, and it can be possibly increased as shown in Prop. 3. Obviously a higher convergence speed would results in a noisier estimate. In order to quantify exactly the performances of the two algorithms the following indexes have been defined and their values are shown in Tab. 5.1:

$$IAE_{ss} = \frac{1}{T_f - T_s} \int_{T_f}^{T_s} \sum_{k=x,y,z} |p_k(t) - \hat{p}_k(t)| dt \quad (5.30)$$

$$ITAE = \frac{1}{T_f} \int_0^{T_f} t \left(\sum_{k=x,y,z} |p_k(t) - \hat{p}_k(t)| \right) dt \quad (5.31)$$

where T_s is the settling time (in our case $T_s = 2.5s$), and T_f is the final time. In particular, the index IAE_{ss} represents the integral absolute error during steady state and the index $ITAE$ represents the integral time-weighted absolute error of the estimation error. As expected the two values of IAE_{ss} and $ITAE$ computed for the proposed algorithm are lower than the general NLS trilateration method (about -40% for IAE_{ss} and about -54% for $ITAE$).

In order to evaluate the performance with higher velocities and higher accelerations, a second test has been carried out, maintaining the same path, but with a higher velocity. The results of this second test are shown in Figs. 5.16-5.17 and Tab. 5.2. Also in this case the same considerations already given above hold, indeed it is evident a better filtering action of the proposed technique. However, looking at Tab. 5.2 the margins of improvements are a little bit reduced due to a faster dynamics of the general NLS trilateration. In any case, the performance indexes with the proposed method are better than general NLS trilateration, indeed the IAE_{ss} is almost 20% less and the $ITAE$ is almost 33% less.

With regards to the computational complexity, the scheduling time for both algorithms has been computed. In particular, using a Raspberry Pi3, the following scheduling times

Table 5.1: Performance indexes for low speed test

	Proposed algo- rithm	General NLS trilat- eration
IAE_{SS}	0.1062	0.1706
$ITAE$	1.34	2.86

Table 5.2: Performance indexes for higher speed test

	Proposed algo- rithm	General NLS trilat- eration
IAE_{SS}	0.1245	0.1553
$ITAE$	1.88	2.79

have been obtained:

- 43 μs for the general NLS trilateration;
- 97 μs for the proposed algorithm.

This means that the maximum execution frequency that can be obtained with general NLS trilateration is about 23kHz, while with the proposed algorithm is about 10kHz. Even if the proposed algorithm is heavier from the computational effort point of view, 10kHz is much more higher than the required frequency for this kind of applications. Note that the real bottleneck is given by the UWB measurement frequency that, in our case, is 12Hz. Therefore even if the two algorithms can be executed, potentially, with a very high frequency, both they have to deal with a low measurement frequency.

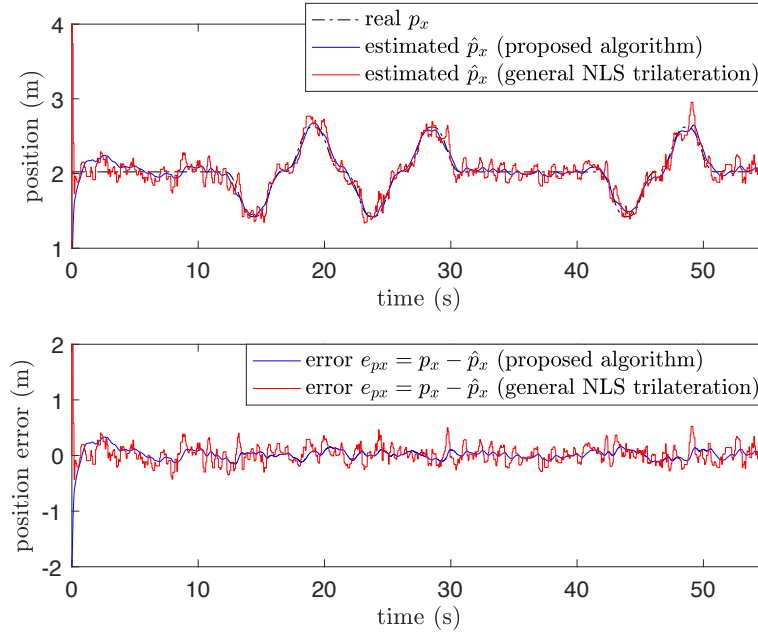


Figure 5.14: Real and estimated position component p_x using both the proposed localization algorithm and a general NLS trilateration.

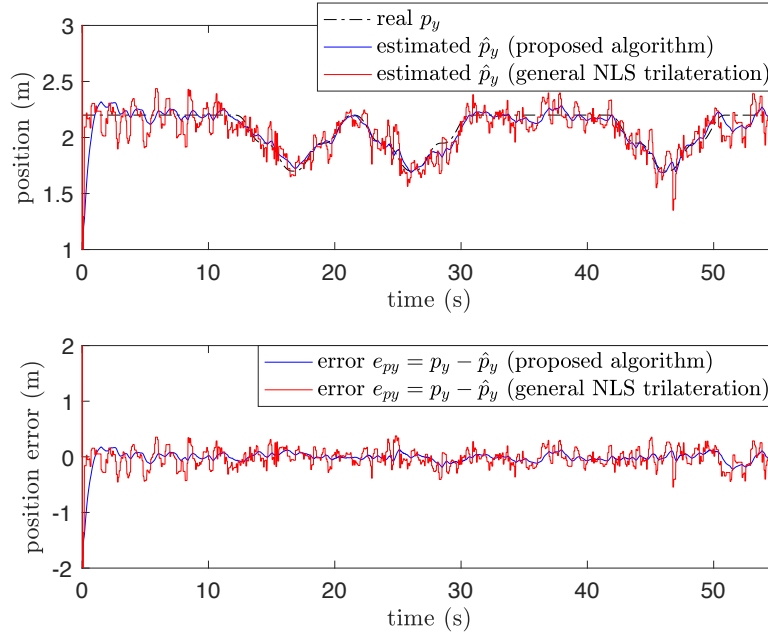


Figure 5.15: Real and estimated position component p_y using both the proposed localization algorithm and a general NLS trilateration.

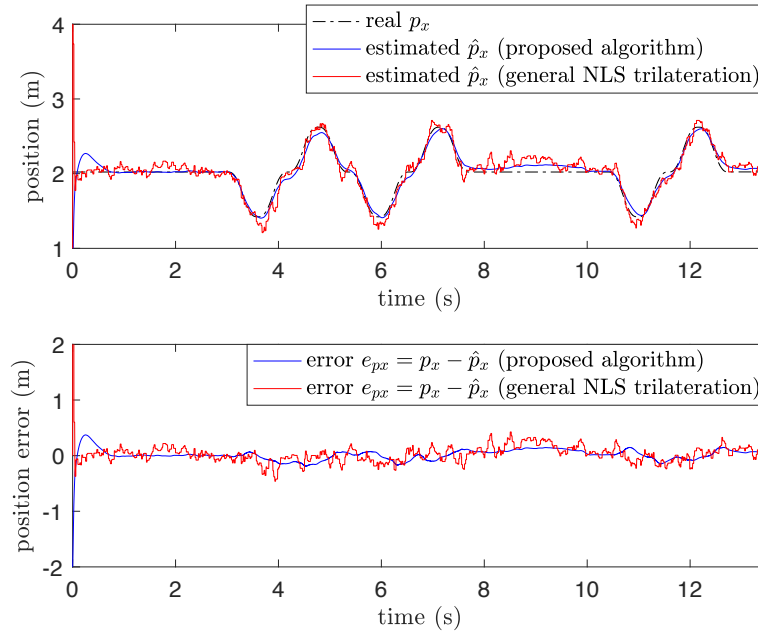


Figure 5.16: Real and estimated position component p_x using both the proposed localization algorithm and a general NLS trilateration (higher velocity).

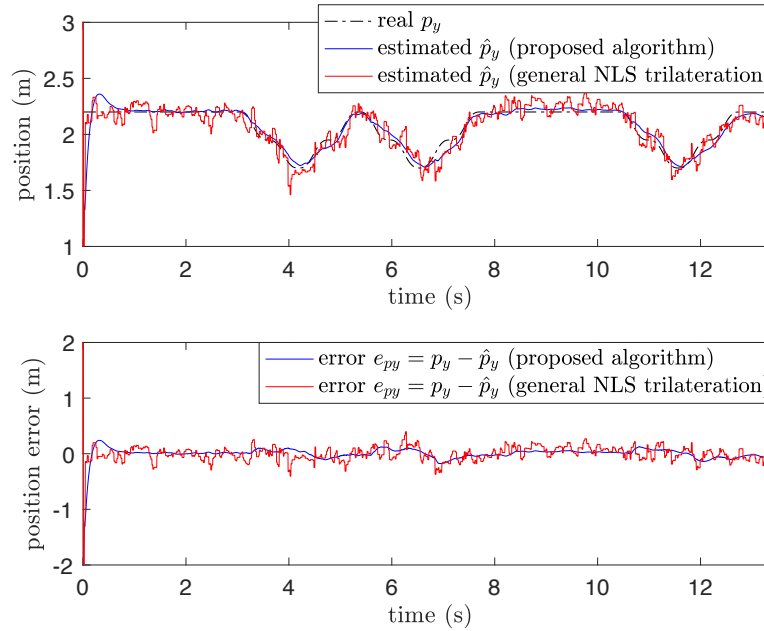


Figure 5.17: Real and estimated position component p_y using both the proposed localization algorithm and a general NLS trilateration (higher velocity).

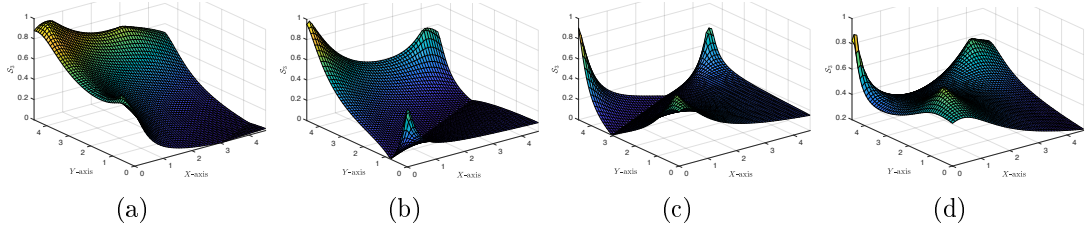


Figure 5.18: Mean sensitivity $\mathcal{S}$, using three anchors, for: $z = 0.6\text{m}$ (a), $z = 1.2\text{m}$ (b), $z = 1.8\text{m}$ (c) and $z = 2.4\text{m}$ (d).

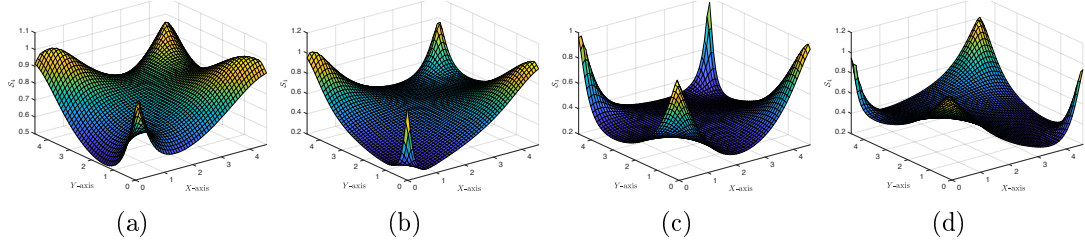


Figure 5.19: Mean sensitivity $\mathcal{S}$, using four anchors, for: $z = 0.6\text{m}$ (a), $z = 1.2\text{m}$ (b), $z = 1.8\text{m}$ (c) and $z = 2.4\text{m}$ (d).

5.3.4 Sensitivity analysis

Normally, the accuracy of the position estimate changes depending on different sensors and different positions. This is generally related with both the type of sensors and the algorithm used. In this section we don't investigate the accuracy of the estimate related with the particular choice of a sensor, because it is not the main focus of this work. Actually, it does not depend from the algorithm, but it depends from the experimental set-up. However, an analysis will be carried out in order to give some hints about the sensitivity of the proposed algorithm with respect to different positions and different number of anchors. In particular, the inverse on the norms of matrices $\mathbf{J}_{\mathcal{K}-1}^{-1}(\mathbf{p})$ (for the case with three anchors) and $\mathbf{J}_{\mathcal{K}-1}^{\#}(\mathbf{p})$ (for the case with four anchors) are used to define the *mean sensitivity* indexes of the algorithm as follows:

$$\mathcal{S}_3 = \|\mathbf{J}_{\mathcal{K}-1}^{-1}\|_2^{-1}, \quad \mathcal{S}_4 = \|\mathbf{J}_{\mathcal{K}-1}^{\#}\|_2^{-1}. \quad (5.32)$$

The indexes $\mathcal{S}_3$ and $\mathcal{S}_4$ can be effectively considered as a kind of "*sensitivity*" for the proposed algorithm because matrices $\mathbf{J}_{\mathcal{K}-1}^{-1}(\mathbf{p})$ and $\mathbf{J}_{\mathcal{K}-1}^{\#}(\mathbf{p})$ represent how the distance measurement errors are transferred to the position estimate derivative $\dot{\hat{\mathbf{p}}}$. Lower these indexes are, higher (on average) the position estimate derivative is. Consequently possible errors on the measurements result in high errors on the estimate if the indexes $\mathcal{S}_3$ and $\mathcal{S}_4$

are very small.

The values of $\mathcal{S}_3$ and $\mathcal{S}_4$ are computed and the results are shown in Figs. 5.18-5.19. In particular, Fig. 5.18 represents the case with three anchors, and it shows the value of $\mathcal{S}_3$, as defined in (5.32), for different values of x and y coordinates considering four values of z coordinate: $z = 0.6\text{m}$, $z = 1.2\text{m}$, $z = 1.8\text{m}$ and $z = 2.4\text{m}$. While Fig. 5.19 represents the same type of surfaces, but for the case with four anchors. From these figures is evident a lower sensitivity of the estimate when the point P is close to the anchors. Note that when the point P coincides with one of the anchors the indexes $\mathcal{S}_3$ and $\mathcal{S}_4$ are not defined (See comment after Eq. (5.9)). However, as already said, this last condition is not realistic from a practical point of view. Another important consideration, coming from the analysis of Fig. 5.18, is that for some points the value of $\mathcal{S}_3$ is zero. These points coincide with the intersection of the Σ_A plane containing the anchor A_1 , A_2 and A_3 , and the horizontal plane with a constant value of z . In this line there is a singularity as expected, and this is the reason why the Assumption 1 is necessary for the case of three anchors. Obviously there is not this problem if four anchors are used as shown in Fig. 5.19.

Chapter 6

Sporadic Position Measurements

OUTLINE. In this chapter a novel estimation strategy for inertial navigation in indoor/outdoor environments is proposed with a specific attention to the sporadic nature of the non-periodic measurements. In this context, the main challenge is the sensor fusion between the inertial and position measurements coming from various sensors implementing different technologies, operating at different sampling frequencies. Several works have been published to address this challenge. For example ([104]) presents one of the first nonlinear observers for GPS and IMU sensor data fusion. In ([105]), based on contraction theory, a deterministic nonlinear observer is proposed, which takes into account the sampled-data and sporadic nature of the measurements, but using additional velocity measurements (not used in this work). When these velocity measurements are available, a nonlinear observer for inertial navigation is proposed in [56], which uses a quaternion-based attitude representation. That work was then extended in [59] by introducing time-varying observer gains. Similar results relying on velocity measurements are given in [106], which addresses nonlinear position, velocity and attitude estimation for Unmanned Aerial Vehicles (UAVs).

The motivation for our work is that, especially when operating with beacons and non-periodic or unreliable communication channels, it is well known that the sampled position measurements are far from being available at a fixed frequency. To rigorously represent this scenario, we adopt the hybrid dynamical systems modeling framework of [60], which provides an effective and elegant notation, in addition to a number of useful well-posedness results. This hybrid framework has been already used in other works dealing with the design of observers with sporadic measurements [107, 108, 109, 110], due to its suitability for obtaining accurate models of systems with sampled measurements and non-periodic time-of-arrival. While those approaches can solve the estimation problem addressed here,

the explicit solution that we build in this work is more effective in terms of the tunability of the estimation error dynamics due to its peculiar cascaded structure.

6.1 Sporadic Position Measurements in an INS

Consider the strap-down model of an Inertial Navigation System (1.9)-(1.10) and here reported:

$$\dot{\mathbf{v}}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g}, \quad (6.1)$$

$$\dot{\mathbf{p}}^n = \mathbf{v}^n. \quad (6.2)$$

where $\mathbf{p}^n$, $\mathbf{v}^n$, $\mathbf{g}$, $\boldsymbol{\gamma}^b$, $\boldsymbol{\xi}_b^n$, $\mathbf{V}(\boldsymbol{\xi}_b^n)$ are the ones introduced in Section 1.2.2.

Model (6.1)-(6.2) is not robust because the speed and the position vectors are obtained by open-loop integration of the acceleration and this leads to drift problems. To solve these observability problems, a position measurement is used, provided by a Global Positioning System (GPS), for outdoor navigation, or beacons, for indoor navigation. Consequently, problems arise because these position measurements are available with a significantly lower sampling rate, as compared to the IMU signals. Moreover the time between two consecutive position measurements is not constant, thus providing a so-called sporadic position measurements situation [107, 110]. In order to formalize the above described setting, we assume in this work that no velocity measurement is available and that the only output of system (6.1)-(6.2) is represented by the vector $\mathbf{y} = \mathbf{p}^n$, only accessible at discrete instants of time, resulting in a sequence of 3-dimensional vectors $\mathbf{y}_k$, $k \in \mathbb{Z}_{\geq 1}$, defined as:

$$\mathbf{y}_k = \mathbf{p}_k^n := \mathbf{p}^n(t_k), \quad (6.3)$$

where t_k , $k \in \mathbb{Z}_{\geq 1}$, is a sequence of increasing non-negative real numbers that satisfies the following assumption:

Assumption 4 *There exist scalars T_m and T_M , with $0 < T_m \leq T_M$, such that:*

$$T_m \leq |t_{k+1} - t_k| \leq T_M, \quad \forall k \in \mathbb{Z}_{\geq 1}. \quad (6.4)$$

Assumption 4 essentially requires that the sporadic measurements $\mathbf{y}_k$ be available with intersample times that are lower and upper bounded by positive constants $T_m \leq T_M$ whose knowledge is not required by the estimation algorithm proposed here. As a consequence, Assumption 4 appears to be an extremely mild assumption ruling out the

defective cases of Zeno behaviors (because T_m is greater than zero) and the situation of sporadic measurements whose occurrence becomes increasingly rare as time grows.

Using the hybrid systems formalism of [60], it is possible to represent the sampled-data system associated with setting (6.1)–(6.3) as follows:

$$\begin{cases} \dot{\mathbf{v}}^n = \mathbf{V}(\boldsymbol{\xi}_b^n)\boldsymbol{\gamma}^b + \mathbf{g}, \\ \dot{\mathbf{p}}^n = \mathbf{v}^n, \\ \dot{\tau} = 1, \end{cases} \quad (\mathbf{v}^n, \mathbf{p}^n, \tau) \in \mathcal{C}, \quad (6.5a)$$

$$\begin{cases} \mathbf{v}^{n+} = \mathbf{v}^n, \\ \mathbf{p}^{n+} = \mathbf{p}^n, \\ \tau^+ = 0, \end{cases} \quad (\mathbf{v}^n, \mathbf{p}^n, \tau) \in \mathcal{D}, \quad (6.5b)$$

$$\mathbf{y} = \mathbf{p}^n, \quad (6.5c)$$

where the timer τ is an additional state variable keeping track of the elapsed time since the last measurement, while sets $\mathcal{C}$ and $\mathcal{D}$ are respectively the so-called *flow set* and *jump set*, defined as:

$$\mathcal{C} := \mathbb{R}^3 \times \mathbb{R}^3 \times [0, T_M], \quad (6.6)$$

$$\mathcal{D} := \mathbb{R}^3 \times \mathbb{R}^3 \times [T_m, T_M]. \quad (6.7)$$

The impulsive nature of the available measurement is represented by the extra property that $\mathbf{y} = \mathbf{p}^n$ is only available at the jump times.

6.2 Continuous-Discrete Observer Architecture

6.2.1 Observer dynamics

In this section, we propose an observer whose structure implicitly complies with the restriction specified in Assumption 4 about the available measurements. Our observer is capable of providing an asymptotic estimate of the plant state, for any sequence of sampling times t_k at which the sampled output is available, as long as these times satisfy Assumption 4 for some (not known) values of $0 < T_m \leq T_M$.

The hybrid structure of the proposed observer is the following one

$$\begin{cases} \dot{\hat{\mathbf{v}}}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g}, \\ \dot{\hat{\mathbf{p}}}^n = \hat{\mathbf{v}}^n, \\ \dot{\tau} = 1, \\ \dot{\tilde{\mathbf{p}}} = 0, \end{cases} \quad (\hat{\mathbf{v}}^n, \hat{\mathbf{p}}^n, \tau, \tilde{\mathbf{p}}) \in \mathcal{C} \times \mathbb{R}^3, \quad (6.8a)$$

$$\begin{cases} \hat{\mathbf{v}}^{n+} = \hat{\mathbf{v}}^n + (1 - \alpha_v) \left(\frac{\mathbf{p}^n - \hat{\mathbf{p}}^n - \tilde{\mathbf{p}}}{\tau} \right), \\ \hat{\mathbf{p}}^{n+} = \hat{\mathbf{p}}^n + (1 - \alpha_p)(\mathbf{p}^n - \hat{\mathbf{p}}^n), \\ \tau^+ = 0, \\ \tilde{\mathbf{p}}^+ = \alpha_p(\mathbf{p}^n - \hat{\mathbf{p}}^n) \end{cases} \quad (\hat{\mathbf{v}}^n, \hat{\mathbf{p}}^n, \tau, \tilde{\mathbf{p}}) \in \mathcal{D} \times \mathbb{R}^3, \quad (6.8b)$$

where $\tilde{\mathbf{p}}$ is an auxiliary “zero-order hold” state vector propagating the last position correction term across the intersample interval, and α_v and α_p are two positive gains that may be tuned to suitably adjust the estimation error transient. Note that the measurement $\mathbf{y} = \mathbf{p}^n$ is only used in the jump dynamics of our observer, which is then compliant with the measurement constraints imposed by Assumption 4.

As compared to the architecture used in [1], observer (6.8) considers a different jump map with an extra auxiliary state vector $\tilde{\mathbf{p}}$. This is necessary because [1] addresses a simpler problem where both the position and the speed are simultaneously measured. In the setting addressed here, where only the sampled position measurement is available, a more sophisticated dynamic correction is required.

6.2.2 Error Dynamics and Main Result

The main result of this work proves stability and convergence of the estimation error in the extended error space corresponding to the error variables

$$\mathbf{e} := \begin{bmatrix} \mathbf{e}_v \\ \mathbf{e}_p \end{bmatrix} := \begin{bmatrix} \mathbf{v}^n \\ \mathbf{p}^n \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{v}}^n \\ \hat{\mathbf{p}}^n \end{bmatrix} \quad (6.9a)$$

$$\tilde{\mathbf{e}}_v := \mathbf{e}_p - \tilde{\mathbf{p}} - \tau \mathbf{e}_v \quad (6.9b)$$

and provides design rules to select the tuning gains α_v and α_p in (6.8). In particular, with the error variables (6.9), the corresponding error dynamics issued from (6.5)-(6.8) is well represented by the cascaded structure schematically shown in Figure 7.1. In this figure, the upper subsystem, governed by the dynamics of $\tilde{\mathbf{e}}_v$, converges to zero in finite time at the

first measurement instant; the middle subsystem, governed by the velocity error dynamics $\mathbf{e}_v$, provides a constant continuous-time behavior in the inter-measurement periods and a discretely converging dynamics (tunable by adjusting α_v) at the measurement instants, perturbed by input $\tilde{\mathbf{e}}_v$. Finally, the third subsystem, governed by the position error dynamics $\mathbf{e}_p$, provides a constant continuous-time behavior perturbed by the velocity error $\mathbf{e}_v$ in the inter-measurement periods, and a discretely converging dynamics (tunable by adjusting α_p) at the measurement instants.

For clarity of exposition, we write the three dynamics represented in Figure 7.1 as three different systems, even though it is emphasized that the variable τ governing the jumps is the same one for all of them.

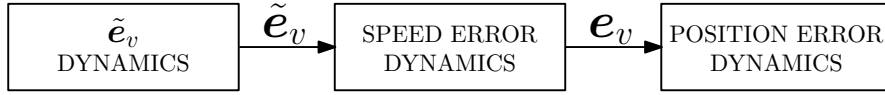


Figure 6.1: Block diagram of the speed and position error dynamics.

By considering (6.5) and (6.8), the hybrid position error dynamics can be written as:

$$\begin{cases} \dot{\mathbf{e}}_p = \mathbf{e}_v, \\ \dot{\tau} = 1, \end{cases} \quad (\mathbf{e}_p, \tau) \in \mathbb{R}^3 \times [0, T_M], \quad (6.10a)$$

$$\begin{cases} \mathbf{e}_p^+ = \alpha_p \mathbf{e}_p, \\ \tau^+ = 0, \end{cases} \quad (\mathbf{e}_p, \tau) \in \mathbb{R}^3 \times [T_m, T_M], \quad (6.10b)$$

while, the speed error dynamics can be written as:

$$\begin{cases} \dot{\mathbf{e}}_v = 0, \\ \dot{\tau} = 1, \end{cases} \quad (\mathbf{e}_v, \tau) \in \mathbb{R}^3 \times [0, T_M], \quad (6.11a)$$

$$\begin{cases} \mathbf{e}_v^+ = \alpha_v \mathbf{e}_v + \frac{\alpha_v - 1}{\tau} \tilde{\mathbf{e}}_v, \\ \tau^+ = 0, \end{cases} \quad (\mathbf{e}_v, \tau) \in \mathbb{R}^3 \times [T_m, T_M]. \quad (6.11b)$$

Finally, the dynamics of $\tilde{\mathbf{e}}_v$ is governed by

$$\begin{cases} \dot{\tilde{\mathbf{e}}}_v = 0, \\ \dot{\tau} = 1, \end{cases} \quad (\tilde{\mathbf{e}}_v, \tau) \in \mathbb{R}^3 \times [0, T_M], \quad (6.12a)$$

$$\begin{cases} \tilde{\mathbf{e}}_v^+ = 0, \\ \tau^+ = 0, \end{cases} \quad (\tilde{\mathbf{e}}_v, \tau) \in \mathbb{R}^3 \times [T_m, T_M]. \quad (6.12b)$$

Remark 6 *The cascaded interconnection represented in Figure 7.1 and whose dynamics correspond to (6.10), (6.11) and (6.12) is somewhat peculiar because: 1) the upper subsystem converges to zero in finite time (at the first jump); 2) the middle subsystem converges asymptotically to zero being perturbed by the upper subsystem across jumps; 3) the lower subsystem converges asymptotically to zero being perturbed by the upper subsystem along flows. This somewhat peculiar continuous-discrete cascaded interconnection naturally emerges from the continuous-discrete nature of the observer.*

Our main result provides necessary and sufficient conditions for the gains α_p and α_v in the observer dynamics (6.8) to ensure global exponential stability (GES) of the compact set (the attractor):

$$\mathcal{A} := \left\{ (\mathbf{e}, \tilde{\mathbf{e}}_v, \tau) : \mathbf{e} = 0, \tilde{\mathbf{e}}_v = 0, \tau \in [0, T_M] \right\}, \quad (6.13)$$

for the error dynamics (6.10), (6.11) and (6.12). The attractor in (6.13) corresponds to the set where the estimation error is zero, therefore our main result establishes necessary and sufficient conditions for exponential estimation of the unmeasured states $\mathbf{p}^n$ and $\mathbf{v}^n$.

Theorem 4 *Set $\mathcal{A}$ in (6.13) is globally exponentially stable for the error dynamics (6.10), (6.11) and (6.12) if and only if the gains α_p and α_v satisfy $|\alpha_p| < 1$ and $|\alpha_v| < 1$. In particular, those conditions hold if and only if there exist positive scalars κ and λ such that all solutions to (6.5)–(6.8) satisfy*

$$\begin{bmatrix} \mathbf{v}^n(t, j) - \hat{\mathbf{v}}^n(t, j) \\ \mathbf{p}^n(t, j) - \hat{\mathbf{p}}^n(t, j) \end{bmatrix} = \begin{bmatrix} \mathbf{e}_v(t, j) \\ \mathbf{e}_p(t, j) \end{bmatrix} \leq \kappa e^{-\lambda(t+j)} \left\| \begin{bmatrix} \mathbf{e}_v(0, 0) \\ \mathbf{e}_p(0, 0) \\ \tilde{\mathbf{e}}_v(0, 0) \end{bmatrix} \right\|. \quad (6.14)$$

While the proof of Theorem 4, in an enhanced version that take account also of measurement and process noise, is postponed to chapter 7, here we emphasize that the error dynamics (6.10), (6.11) and (6.12) satisfies the hybrid basic assumptions of [60, As. 6.5] (which in this single-valued case corresponds to checking that the flow and jump sets are closed and the flow and jump maps are continuous in these respective sets). Therefore, since the attractor $\mathcal{A}$ in (6.13) is compact, according to [60, Ch. 7], the established GAS property is uniform, namely the proposed observer (6.8) provides uniformly convergent asymptotic estimates of $\mathbf{p}^n$ and $\mathbf{v}^n$ under the stated assumptions on the gains. Finally, due to [60, Ch. 7], sufficiently small modeling and measurement errors do not destroy such a stability property, but provide a graceful performance degradation in a semiglobal practical sense.

6.3 Hardware-in-the-loop Results

We test observer (6.8) by running some hardware-in-the-loop experiments. To this end, the hybrid dynamics (6.8) has been implemented in a Raspberry-PI 3 with a sampling frequency of 100 Hz. While, by means of Matlab[®]-simulink[®], a suitable realistic scenario has been simulated resembling a possible path associated to the motion of a mobile vehicle. The corresponding trajectory is shown in Figures 6.2(a) and 6.2(b), which display the reference position and yaw angle, respectively. In the hardware-in-the-loop setup, sampled measurements of this trajectory, as well as accelerations and attitude angles are sent to the Raspberry. In particular, the IMU data corresponding to the accelerations and attitude angles are provided to the Raspberry periodically with a fixed frequency coinciding with its operating frequency (100 Hz), while the sporadic position measurements are available at random instants of time satisfying Assumption 4 for some small enough T_m and some large enough T_M . Even though these values are not important (the observer dynamics is independent of them), here we selected $T_m = 0.5\text{s}$ and $T_M = 1.5\text{s}$. The evolution of the timer variable τ is reported in Fig. 6.2(c), showing that the measurements occur randomly in the interval $[T_m, T_M]$, according to the described dynamics.

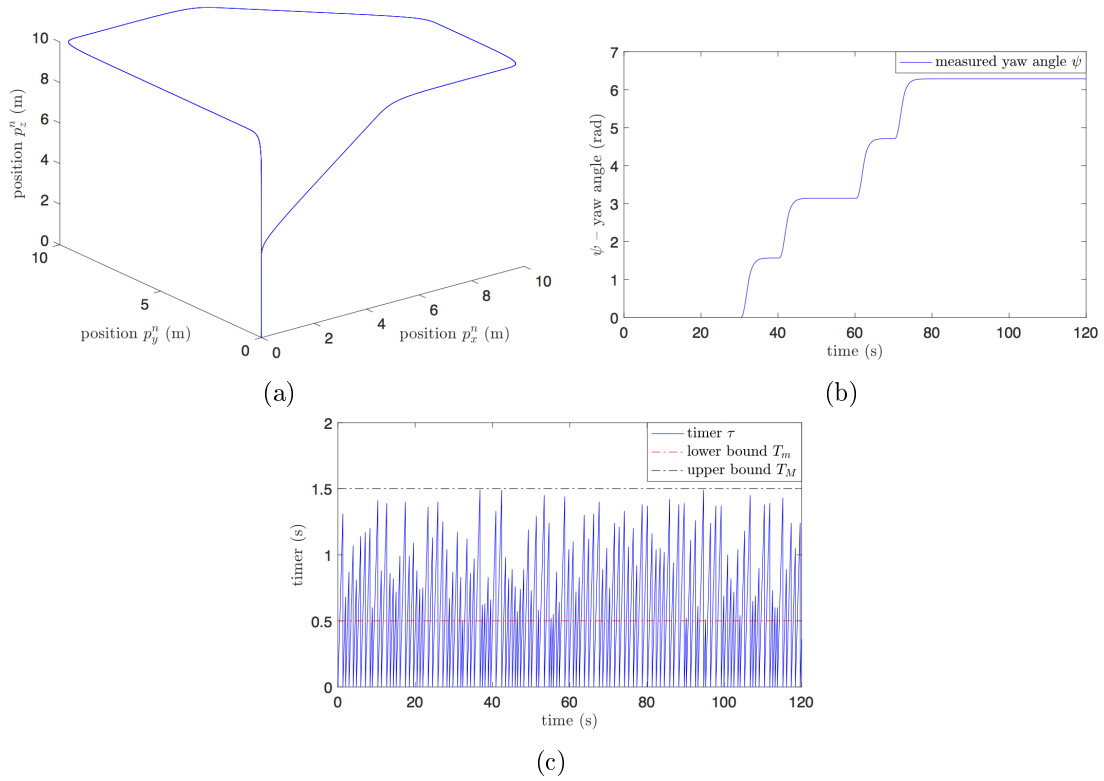


Figure 6.2: Reference position trajectory (a). Reference yaw angle (b). Evolution of the timer variable τ showing the sporadic nature of the sampled position measurements (c).

The parameters of the observer have been fixed at $\alpha_v = 0.9$ and $\alpha_p = 0.9$ and the initial conditions are $\mathbf{p}_0^n = [10 \ 10 \ 10]$ and $\mathbf{v}_0^n = [-0.85 \ -0.85 \ -0.85]$. Two tests have been performed: in *test 1* (noise-free case) the position trajectory of Fig. 6.2(a) has been exactly sampled, at the sporadic times discussed above, and fed to the observer. Similarly, the accelerations $\boldsymbol{\gamma}^b$ and the attitude angles $\boldsymbol{\xi}_b^n$ have been fed to the observer in a noise-free scenario. In *test 2* (noisy case) for the same values of the observer parameters and initial conditions, the measurements have been corrupted by noise. In particular, the accelerations have been corrupted by a Gaussian zero mean white noise having standard deviation equal to 0.04 m/s^2 , while the position has been corrupted by a Gaussian zero mean white noise having standard deviation equal to 0.5 m .

The results of the noise-free *test 1* are shown in Figures 6.3-6.5. In particular Figures 6.3 and 6.4 show the real and the estimated positions and velocities. It is apparent that the estimated variables exponentially converge to the corresponding state variables (all the errors exponentially converge to zero). The hybrid behavior of the estimate is clearly visible, especially during the initial transient. Indeed, in Fig. 6.5 the velocity error is constant during flowing, and decreases across jumps, as expected from the theoretical results Theorem 4, while the position error respects the bound given in Theorem 4. Figure 6.5 provides further insight by showing suitably zoomed (transient and steady-state) areas from Figure 6.5.

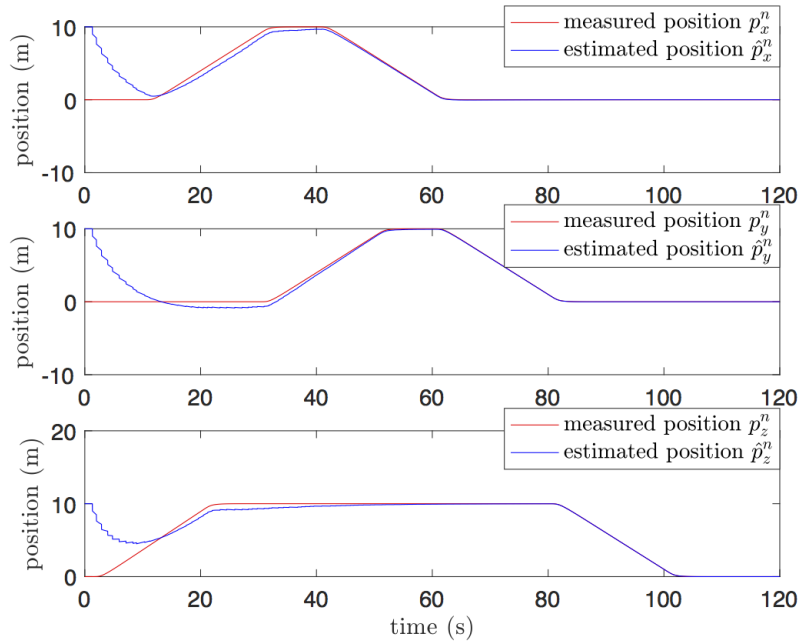


Figure 6.3: Actual and estimated positions in *test 1* (noise-free).

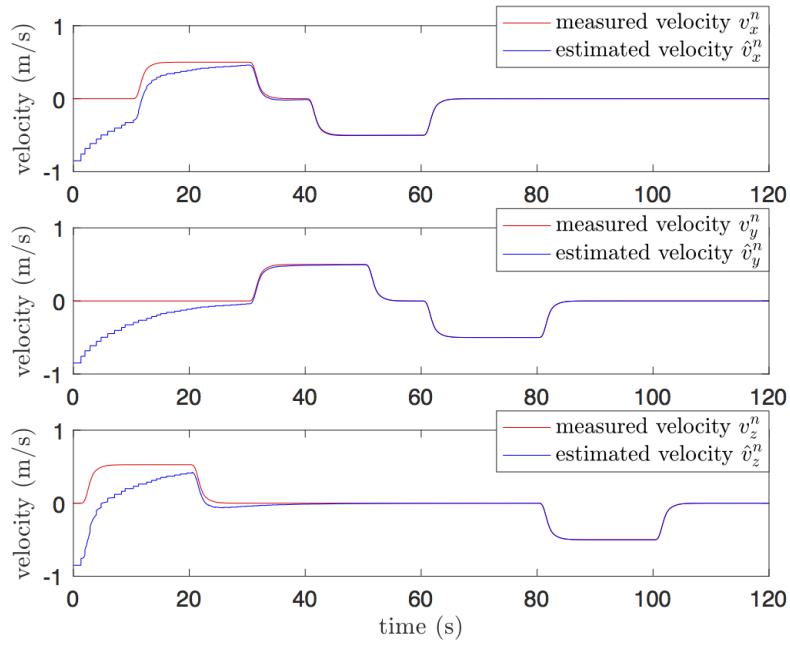


Figure 6.4: Actual and estimated velocities in *test 1* (noise-free).

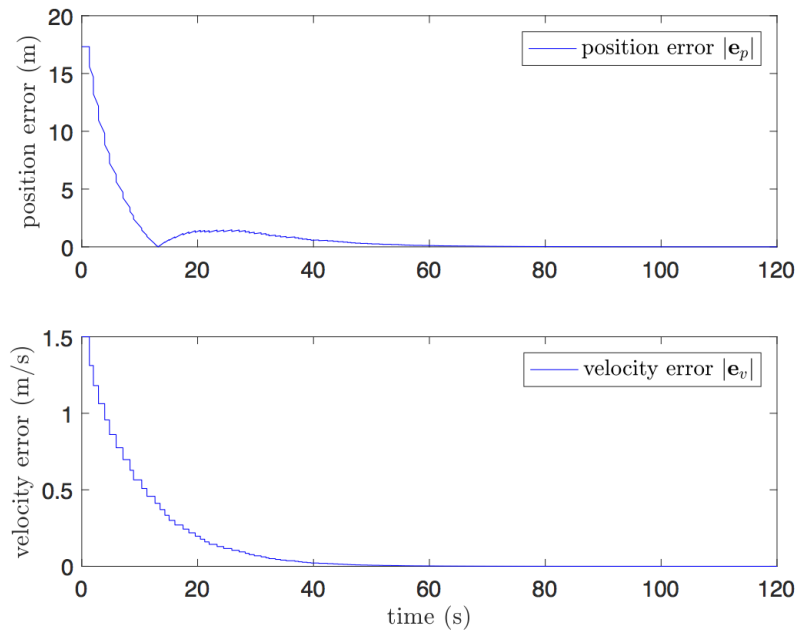


Figure 6.5: Position errors $\mathbf{e}_p$ (upper plot) and velocity errors $\mathbf{e}_v$ (bottom plot) in *test 1* (noise-free).

Similar tests have been provided for *test 2* (noisy case), and are reported in Figures 6.6-6.8. These noisy tests confirm that the observer is able to cope with noisy measurements, since both the position and the velocity are well estimated by a suitable filtering action of the measurement noise. Finally Figure 6.9 shows the position error $\mathbf{e}_p$ and the velocity error $\mathbf{e}_v$ for different values of α_p and α_v . As one may expect from the error dynamics characterized in Theorem 4, the closer α_p and α_v are to zero, the higher the convergence rate of the estimation error will be, but a higher convergence rate leads to a reduced filtering action on the measurement noise. The choice of α_p and α_v then should be carried out as a trade-off between these two goals. Note also that the convergence rate is influenced by the values of T_m and T_M . Indeed, if the measurements are accessible less frequently, then the convergence rate results to be slower.

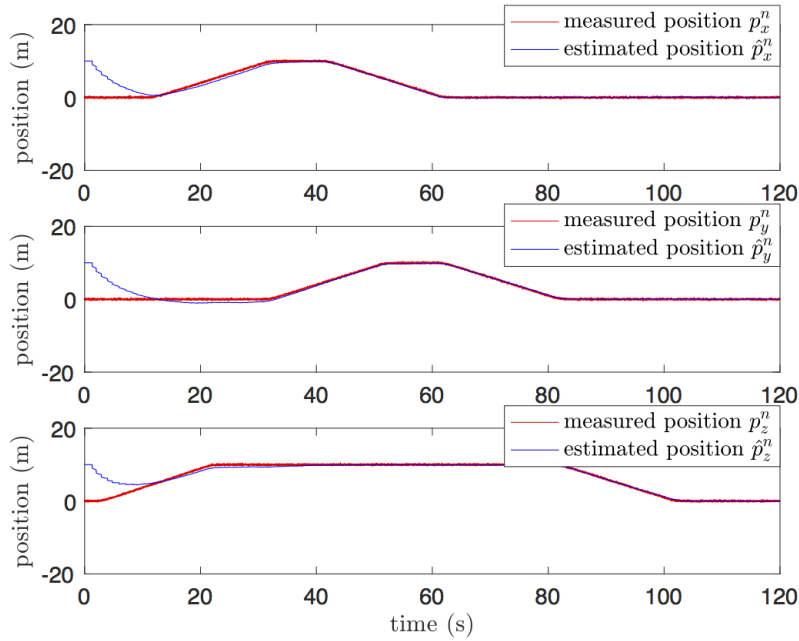


Figure 6.6: Actual and estimated positions in *test 2* (noisy).

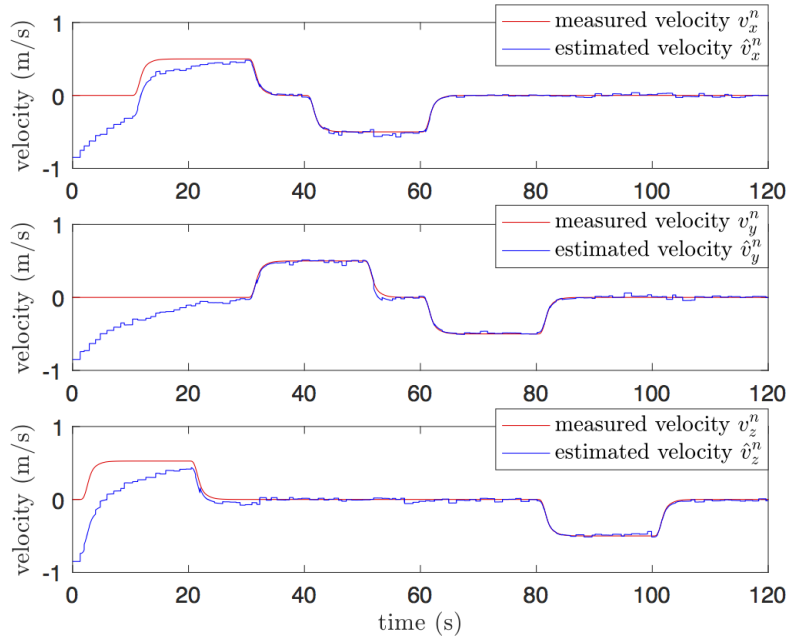


Figure 6.7: Actual and estimated velocities in *test 2* (noisy).

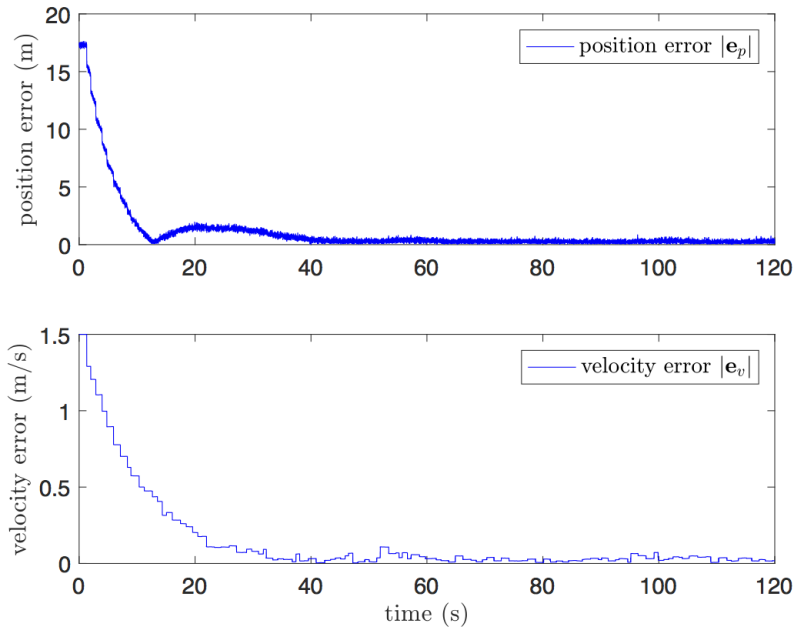


Figure 6.8: Position errors e_p (upper plot) and velocity errors e_v (bottom plot) in *test 2* (noisy).

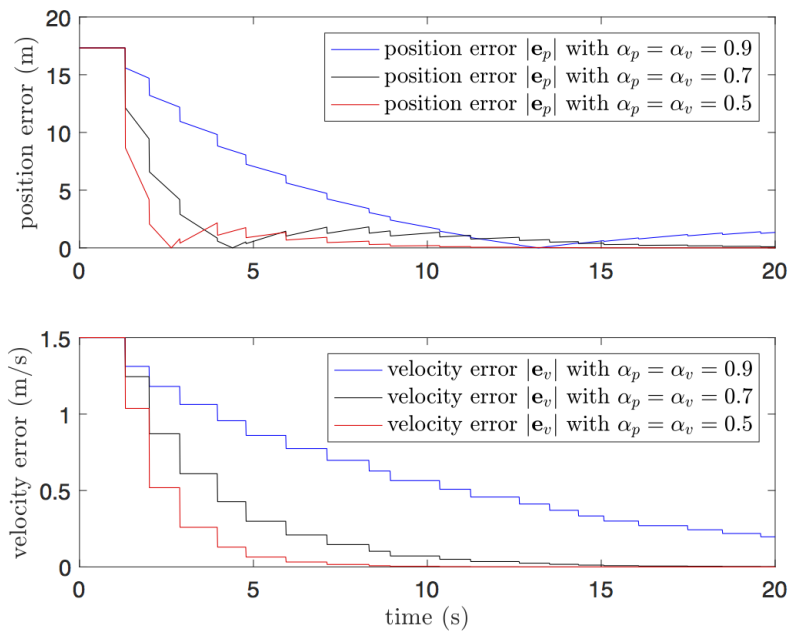


Figure 6.9: Waveforms of the position error e_p (upper plot) and of the velocity error e_p (bottom plot) for different values of α_p and α_v .

Chapter 7

Noisy Sporadic Position Estimation

OUTLINE. In this chapter we propose an asymptotic position and speed observer for inertial navigation in indoor/outdoor environments in the case where the position measurements are sporadic and affected by noise. We cast the problem in a hybrid dynamics framework where the continuous motion is affected by unknown continuous-time disturbances and the sporadic position measurements are affected by discrete-time noise. The estimation error dynamics with the proposed observer shows a peculiar cascaded interconnection of three subsystems, with perturbations occurring either on the jump or on the flow dynamics (depending on the specific subsystem under consideration). For this structure, we show global finite gain exponential ISS-based bounds depending on disturbance and noise.

The motivation for addressing sporadic position measurements is that, especially when operating with beacons and non-periodic or unreliable communication channels, it is well known that the sampled position measurements are far from being available at a fixed frequency. To rigorously represent this scenario, in Section 7.1 we adopt the hybrid dynamical systems modeling framework of [60], which provides an effective and elegant notation, in addition to a number of useful well-posedness results. Furthermore, the estimation error dynamics presents a peculiar and interesting cascaded structure discussed in Section 7.2.2.

Preliminary results in the direction of this work have been presented in Chapter 6. Here, as compared to those results we take into explicit account external perturbations characterized as process disturbances affecting the continuous-time dynamics and measurement noise affecting the sampled-data measurements, and shows that a suitable finite-gain input-to-state stability property holds between these previously neglected inputs and the estimation error. This generalizes the exponential bounds of Chapter 6, in addition to providing the proofs that were omitted in Chapter 6. Moreover, as compared to the

bounds given in Chapter 6, our main result stated in Section 7.2 better characterizes the impact of the observer tuning knobs (a position gain and a velocity gain) on the established exponential bounds. Finally, while only simulation results were given in Chapter 6, we provide here, in Section 7.3, an experimental validation that confirms the trends on the exponential bounds, as a function of the observer gains.

7.1 Noisy Sporadic Position Measurements in INS

Consider the strap-down model of an Inertial Navigation System (1.9)-(1.10) and here reported:

$$\dot{\mathbf{v}}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g} + \mathbf{d}, \quad (7.1)$$

$$\dot{\mathbf{p}}^n = \mathbf{v}^n. \quad (7.2)$$

where $\mathbf{p}^n$, $\mathbf{v}^n$, $\mathbf{g}$, $\boldsymbol{\gamma}^b$, $\boldsymbol{\xi}_b^n$ are the ones introduced in Section 1.2.2 and $\mathbf{V}(\boldsymbol{\xi}_b^n) \in SO(3)$ is the rotation matrix from B -frame to N -frame, as a function of the associated quaternion, corresponding to

$$\mathbf{V}(\boldsymbol{\xi}_b^n) := \mathbf{I} + 2\eta S(\boldsymbol{\epsilon}) + 2S(\boldsymbol{\epsilon})^2,$$

where the skew-symmetric matrix $S(\cdot)$ satisfies $S(u)v = u \times v$, $\forall u, v \in \mathbb{R}^3$. The inertial model (7.1), (7.2) is obtained under the assumption that the Earth angular velocity about the z -axis of the Earth-Centered Inertial (ECI) frame is small, so that it can be neglected [56, Remark 2].

The strap-down inertial navigation algorithm ([105, 56]) is not robust, because the speed and the position vectors are obtained by open-loop integration of the acceleration and this leads to drift problems. To solve these observability problems, a position measurement is used, provided by a Global Positioning System (GPS), for outdoor navigation, or beacons, for indoor navigation. Consequently, problems arise because these position measurements are available with a significantly lower sampling rate, as compared to the IMU signals. Moreover the time between two consecutive position measurements is not constant, thus providing a so-called sporadic position measurements situation [107, 110]. In order to formalize the above described setting, we assume in this work that no velocity measurement is available and that the only output of system (7.1)-(7.2) is represented by the vector $\mathbf{y} \in \mathbb{R}^3$, only accessible at discrete instants of time, resulting in a sequence of

3-dimensional vectors $\mathbf{y}_k$, $k \in \mathbb{Z}_{\geq 1}$, defined as:

$$\mathbf{y}_k = \mathbf{p}_k^n + \mathbf{n}_k := \mathbf{p}^n(t_k) + \mathbf{n}(k), \quad (7.3)$$

where $\mathbf{n}_k \in \mathbb{R}^3$ represents some measurement noise and t_k , $k \in \mathbb{Z}_{\geq 1}$, is a sequence of increasing non-negative real numbers that satisfies the following assumption:

Assumption 5 *There exist scalars T_m and T_M , with $0 < T_m \leq T_M$, such that:*

$$T_m \leq |t_{k+1} - t_k| \leq T_M, \quad \forall k \in \mathbb{Z}_{\geq 1}. \quad (7.4)$$

Assumption 5 essentially requires that the sporadic measurements $\mathbf{y}_k$ be available with intersample times that are lower and upper bounded by positive constants T_m and T_M , whose knowledge is not required by the estimation algorithm proposed here. As a consequence, Assumption 5 appears to be an extremely mild assumption ruling out the defective cases of Zeno behaviors (because T_m is greater than zero) and the situation of sporadic measurements whose occurrence becomes increasingly rare as time grows.

Assumption 6 *The measurement noise $\mathbf{n}$ and the process disturbance $\mathbf{d}$ are bounded. More precisely*

$$\|\mathbf{d}\| := \sup_{t \in \mathbb{R}_{\geq 0}} |\mathbf{d}(t)| < \infty, \quad \|\mathbf{n}\| := \sup_{k \in \mathbb{N}} |\mathbf{n}(k)| < \infty. \quad (7.5)$$

Note that we use the same symbol for continuous-time and discrete-times norms, because the exact set where the “sup” should be evaluated is inherited from the domain of the argument, namely $\|\mathbf{s}\| := \sup_{\tau \in \text{dom } \mathbf{s}} |\mathbf{s}(\tau)|$.

Using the hybrid systems formalism of [60], it is possible to represent the sampled-data system associated with setting (7.1)–(7.3) as follows:

$$\begin{cases} \dot{\mathbf{v}}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g} + \mathbf{d}, \\ \dot{\mathbf{p}}^n = \mathbf{v}^n, \\ \dot{\tau} = 1, \end{cases} \quad \tau \in [0, T_M], \quad (7.6a)$$

$$\begin{cases} \mathbf{v}^{n+} = \mathbf{v}^n, \\ \mathbf{p}^{n+} = \mathbf{p}^n, \\ \tau^+ = 0, \end{cases} \quad \tau \in [T_m, T_M], \quad (7.6b)$$

$$\mathbf{y} = \mathbf{p}^n + \mathbf{n}, \quad (7.6c)$$

where the timer τ is an additional state variable keeping track of the elapsed time since the last measurement, and rules the conditions under which the sampled measurements should occur. Using the proof technique of [111, Prop. 1], it is easy to show that the jump times of a solution satisfy Assumption 5 if and only if the domain of the solution satisfies the constraint in (7.6). These constraints can be written in terms of the so-called *flow set* $\mathcal{C}$ and *jump set* $\mathcal{D}$. More specifically:

$$(\mathbf{v}^n, \mathbf{p}^n, \tau) \in \mathcal{C} := \mathbb{R}^3 \times \mathbb{R}^3 \times [0, T_M], \quad (7.7)$$

$$(\mathbf{v}^n, \mathbf{p}^n, \tau) \in \mathcal{D} := \mathbb{R}^3 \times \mathbb{R}^3 \times [T_m, T_M]. \quad (7.8)$$

The impulsive nature of the available measurement is represented by the extra property that $\mathbf{y} = \mathbf{p}^n + \mathbf{n}$ is only available at the jump times.

7.2 Continuous-Discrete Observer Architecture

7.2.1 Observer dynamics

In this work, we propose an observer whose structure implicitly complies with the restriction specified in Assumption 5 about the available measurements. Our observer is capable of providing an asymptotic estimate of the plant state, for any sequence of sampling times t_k at which the sampled output is available, as long as these times satisfy Assumption 5 for some (not known) values of $0 < T_m \leq T_M$. These values are not used in the observer design.

The hybrid structure of the proposed observer is:

$$\begin{cases} \dot{\hat{\mathbf{v}}}^n = \mathbf{V}(\boldsymbol{\xi}_b^n) \boldsymbol{\gamma}^b + \mathbf{g}, \\ \dot{\hat{\mathbf{p}}}^n = \hat{\mathbf{v}}^n, \\ \dot{\tau} = 1, \\ \dot{\tilde{\mathbf{p}}} = 0, \end{cases} \quad \tau \in [0, T_M], \quad (7.9a)$$

$$\begin{cases} \hat{\mathbf{v}}^{n+} = \hat{\mathbf{v}}^n + (1 - \alpha_v) \left(\frac{\mathbf{y} - \hat{\mathbf{p}}^n - \tilde{\mathbf{p}}}{\tau} \right), \\ \hat{\mathbf{p}}^{n+} = \hat{\mathbf{p}}^n + (1 - \alpha_p)(\mathbf{y} - \hat{\mathbf{p}}^n), \\ \tau^+ = 0, \\ \tilde{\mathbf{p}}^+ = \alpha_p(\mathbf{y} - \hat{\mathbf{p}}^n), \end{cases} \quad \tau \in [T_m, T_M], \quad (7.9b)$$

where $\tilde{\mathbf{p}}$ is an auxiliary “zero-order hold” state vector propagating the last position correc-

tion term across the intersample interval, and α_v and α_p are two positive gains that may be tuned to suitably adjust the estimation error transient. Note that the measurement $\mathbf{y} = \mathbf{p}^n + \mathbf{n}$ is only used in the jump dynamics of our observer, which is then compliant with the measurement constraints imposed by Assumption 5.

As compared to the architecture used in Chapter 4, observer (7.9) considers a different jump map with an extra auxiliary state vector $\tilde{\mathbf{p}}$. This is necessary because in Chapter 4 a simpler problem is addressed where both the position and the speed are simultaneously measured. In the setting addressed here, where only the sampled position measurement is available, a more sophisticated dynamic correction is required. Alternative approaches for this estimation problem can be found in [107, 109], but they do not comprise the additional state $\tilde{\mathbf{p}}$, and their tuning is much more involved due the lack of an exponential decay guarantee related to the observer gains (α_p and α_v in our case). Due to this fact our solution is preferable because it exploits the peculiar architecture of the system to provide stronger guarantees.

7.2.2 Error Dynamics and Main Result

The main result of this work proves exponential input-to-state stability of the estimation error in the extended error space corresponding to the error variables

$$\mathbf{e} := \begin{bmatrix} \mathbf{e}_v \\ \mathbf{e}_p \end{bmatrix} := \begin{bmatrix} \mathbf{v}^n \\ \mathbf{p}^n \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{v}}^n \\ \hat{\mathbf{p}}^n \end{bmatrix}, \quad (7.10a)$$

$$\tilde{\mathbf{e}}_v := \mathbf{e}_p - \tilde{\mathbf{p}} - \tau \mathbf{e}_v, \quad (7.10b)$$

and provides design rules to select the tuning gains α_v and α_p in (7.9). With the error variables (7.10), the error dynamics issued from (7.6)-(7.9) comprises three subsystems. In Fig. 7.1, the measurement noise $\mathbf{n}$ impulsively affects all three subsystems at the measurement instants (jumps), while the process disturbance $\mathbf{d}$ affects the continuous dynamics of the two upper subsystems. Besides these external perturbations the overall dynamics shows a cascaded structure, where, in the absence of noise, the upper subsystem, governed by the dynamics of $\tilde{\mathbf{e}}_v$, converges to zero in finite time at the first measurement instant; the middle subsystem, governed by the velocity error $\mathbf{e}_v$, provides a constant continuous-time behavior in the inter-measurement periods and a discretely converging dynamics (tunable by adjusting α_v) at the measurement instants, perturbed by input $\tilde{\mathbf{e}}_v$. Finally, the third subsystem, governed by the position error $\mathbf{e}_p$, provides a constant continuous-time behavior perturbed by the velocity error $\mathbf{e}_v$ in the inter-measurement periods, and

a discretely converging dynamics (tunable by adjusting α_p) at the measurement instants.

By considering (7.6) and (7.9), we write the three dynamics represented in Fig. 7.1 as three different systems, even though the variable τ governing the jumps is the same one for all of them. The hybrid position estimation error dynamics can be written as:

$$\begin{cases} \dot{\mathbf{e}}_p = \mathbf{e}_v, \\ \dot{\tau} = 1, \end{cases} \quad \tau \in [0, T_M], \quad (7.11a)$$

$$\begin{cases} \mathbf{e}_p^+ = \alpha_p \mathbf{e}_p + (\alpha_p - 1) \mathbf{n}, \\ \tau^+ = 0, \end{cases} \quad \tau \in [T_m, T_M]. \quad (7.11b)$$

The speed estimation error dynamics can be written as:

$$\begin{cases} \dot{\mathbf{e}}_v = \mathbf{d}, \\ \dot{\tau} = 1, \end{cases} \quad \tau \in [0, T_M], \quad (7.12a)$$

$$\begin{cases} \mathbf{e}_v^+ = \alpha_v \mathbf{e}_v + \frac{\alpha_v - 1}{\tau} (\tilde{\mathbf{e}}_v + \mathbf{n}), \\ \tau^+ = 0, \end{cases} \quad \tau \in [T_m, T_M]. \quad (7.12b)$$

Finally, the dynamics of $\tilde{\mathbf{e}}_v$ is governed by:

$$\begin{cases} \dot{\tilde{\mathbf{e}}}_v = -\tau \mathbf{d}, \\ \dot{\tau} = 1, \end{cases} \quad \tau \in [0, T_M], \quad (7.13a)$$

$$\begin{cases} \tilde{\mathbf{e}}_v^+ = -\mathbf{n}, \\ \tau^+ = 0, \end{cases} \quad \tau \in [T_m, T_M]. \quad (7.13b)$$

We emphasize here that the estimation error dynamics (7.11)-(7.13) satisfies the hybrid basic assumptions of [60, Ass. 6.5] (which in this single-valued case corresponds to checking that the flow and jump sets are closed and the flow and jump maps are continuous in these respective sets).

With error dynamics (7.11)-(7.13) our main contribution is to prove an exponential ISS bound where we clarify the role of the tunable observer gains α_p and α_v in (7.9).

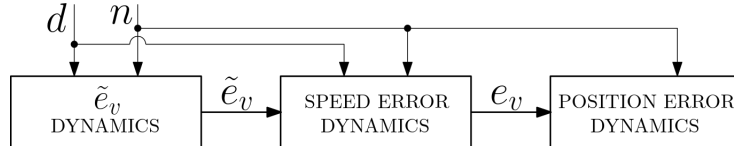


Figure 7.1: Block diagram of the speed and position estimation error dynamics.

Theorem 5 *Given plant (7.1)-(7.2) with outputs (7.3) available at measurement times t_k , $k \in \mathbb{N}$ satisfying Assumption 5, there exist $\kappa > 0$ and $\lambda > 0$ such that all solutions to (7.11)-(7.13) satisfy:*

$$\begin{bmatrix} \mathbf{e}_v(t, j) \\ \mathbf{e}_p(t, j) \end{bmatrix} \leq \kappa e^{-\lambda t} \left\| \begin{bmatrix} \mathbf{e}_v(0, 0) \\ \mathbf{e}_p(0, 0) \\ \tilde{\mathbf{e}}_v(0, 0) \end{bmatrix} \right\| + \kappa \|\mathbf{d}\| + \kappa \|\mathbf{n}\|, \quad (7.14)$$

if and only if the gains α_p and α_v satisfy $|\alpha_p| < 1$ and $|\alpha_v| < 1$. Moreover, there exist positive constants γ and η , independent of α_p and α_v , such that, for any $\alpha_p \in [0, 1)$ and $\alpha_v \in [0, 1)$ all solutions satisfy

$$\begin{aligned} |\mathbf{e}_p(t, j)| &\leq \frac{\alpha_p^{t/T_M}}{\alpha_p} |\mathbf{e}_p(0, 0)| \\ &\quad + \frac{T_M \alpha_v^{t/T_M}}{(1 - \alpha_p) \alpha_v} \left(|\mathbf{e}_p(0, 0)| + \frac{|\tilde{\mathbf{e}}_v(0, 0)|}{T_m} \right) \\ &\quad + \frac{T_M \gamma}{(1 - \alpha_p)(1 - \alpha_v)} \|\mathbf{d}\| + (\eta + 1) \|\mathbf{n}\|, \end{aligned} \quad (7.15)$$

$$\begin{aligned} |\mathbf{e}_v(t, j)| &\leq \frac{\alpha_v^{t/T_M}}{\alpha_v} \left(|\mathbf{e}_v(0, 0)| + \frac{|\tilde{\mathbf{e}}_v(0, 0)|}{T_M} \right) \\ &\quad + \frac{\gamma}{1 - \alpha_v} \|\mathbf{d}\| + \eta \|\mathbf{n}\|. \end{aligned} \quad (7.16)$$

Remark 7 *While the necessary and sufficient conditions of Theorem 5 provide a complete characterization of the estimation error dynamics stability, the case of practical interest is $\alpha_p \in [0, 1)$ and $\alpha_v \in [0, 1)$, because it avoids oscillatory behavior. This case is associated to bounds (7.16)-(7.15). These bounds provide insight about the tuning on the proposed observer, since they show the relations among the ISS bound of the estimation error and the tuning parameters α_p and α_v (as well as the minimum and maximum sampling time T_m and T_M). In particular, first note that the terms $\frac{\alpha^{t/T_M}}{\alpha}$ provide exponential decrease proportional to $\lambda = -\frac{\log(\alpha)}{T_M} > 0$ (positive because $\alpha < 1$) because $\frac{\alpha^{t/T_M}}{\alpha} = \frac{1}{\alpha} e^{-\lambda t}$. Then, choosing a smaller α_p or α_v increases the speed of convergence, but also increases the overshoot, due to the term α at the denominator. Finally, we observe that a further negative effect occurs with α too large: in addition to slowing down the transient it undesirably increases the gain from $\|\mathbf{d}\|$ and the cascade gain from $|\mathbf{e}_v|$ to $|\mathbf{e}_p|$. In our experiments we select $\alpha_v = \alpha_p = 0.5$, which provides a desirable trade off.*

7.2.3 Proof of Theorem 5

The proof of Theorem 5 is established by characterizing the input-to-state stability (ISS) properties [112] of the three subsystems of Figure 7.1, corresponding to dynamics (7.11), (7.12) and (7.13). In our setting, we may simplify the definitions of [112] because our jump and flow sets only depend on the timer τ and our solutions satisfy a dwell time condition. We may then introduce

$$\begin{cases} \dot{x} = f(x, u), \\ \dot{\tau} = 1, \\ x^+ = g(x, u), \\ \tau^+ = 0, \end{cases} \quad \begin{matrix} \tau \in [0, T_M] \\ \\ \tau \in [T_m, T_M], \end{matrix} \quad (7.17)$$

whose solution pairs $((x, \tau), u)$ share the same hybrid time domain $\text{dom}(x) = \text{dom}(u)$. We say that system (7.17) is finite gain-exponentially input-to-state stable with respect to u if there exist positive scalars κ , λ and γ such that any solution pair of (7.17) satisfies

$$|x(t, j)| \leq \kappa e^{-\lambda t} |x(0, 0)| + \gamma \|u\|, \quad \forall (t, j) \in \text{dom } x. \quad (7.18)$$

To the end of proving Theorem 5, we first prove three technical lemmas, each of them establishing suitable properties of the three subsystems (7.11), (7.12) and (7.13).

Lemma 5 *If α_p satisfies $|\alpha_p| < 1$, then the position estimation error dynamics (7.11) is finite-gain exponentially ISS with respect to $\mathbf{e}_v$ and $\mathbf{n}$. Moreover, when $\alpha_p \in [0, 1)$ all solutions satisfy*

$$|\mathbf{e}_p(t, j)| \leq \frac{\alpha_p^{t/T_M}}{\alpha_p} |\mathbf{e}_p(0, 0)| + \frac{T_M}{1 - \alpha_p} \|\mathbf{e}_v\| + \|\mathbf{n}\|, \quad (7.19)$$

for all $(t, j) \in \text{dom } \mathbf{e}_p$.

Proof 1 *Consider any state-input solution pair $((\mathbf{e}_p, \tau), (\mathbf{e}_v, \mathbf{n}))$ of (7.11), and any $(t, k) \in \text{dom}(\mathbf{e}_p)$. Since $\dot{\tau} = 1$ and the flow set requires $\tau \leq T_M$, we have that $t - t_k \leq T_M$, therefore*

$$\begin{aligned} |\mathbf{e}_p(t, k)| &\leq |\mathbf{e}_p(t_k, k)| + (t - t_k) \|\mathbf{e}_v\|_\infty \\ &\leq |\mathbf{e}_p(t_k, k)| + T_M \|\mathbf{e}_v\|_\infty, \quad \forall t \in [t_k, t_{k+1}]. \end{aligned} \quad (7.20)$$

In particular, when selecting $(t, j - 1) \in \text{dom}(\mathbf{e}_p)$ such that $t = t_j$, we may apply (7.20)

with $(t, k) = (t_j, j - 1)$ to get

$$|\mathbf{e}_p(t_j, j - 1)| \leq |\mathbf{e}_p(t_{j-1}, j - 1)| + T_M \|\mathbf{e}_v\|_\infty. \quad (7.21)$$

We continue the proof for the case $\alpha_p > 0$ because similar steps, with less strict exponential bounds, can be followed for $\alpha_p < 0$. Moreover, at each jump time t_j , $j \geq 1$, we may use the jump equation in (7.11), combined with (7.21), to get

$$\begin{aligned} |\mathbf{e}_p(t_j, j)| &\leq \alpha_p |\mathbf{e}_p(t_j, j - 1)| + |\alpha_p - 1| \|\mathbf{n}\|_\infty \\ &\leq \alpha_p |\mathbf{e}_p(t_{j-1}, j - 1)| + \alpha_p T_M \|\mathbf{e}_v\|_\infty + (1 - \alpha_p) \|\mathbf{n}\|_\infty. \end{aligned}$$

By iterating recursively the above inequalities, we obtain for any $(t, j) \in \text{dom}(\mathbf{e}_p)$:

$$\begin{aligned} |\mathbf{e}_p(t, j)| &\leq |\mathbf{e}_p(t_j, j)| + T_M \|\mathbf{e}_v\|_\infty \\ &\leq \alpha_p^j |\mathbf{e}_p(t_0, 0)| + \sum_{i=0}^j \alpha_p^i T_M \|\mathbf{e}_v\|_\infty + \sum_{i=1}^j \alpha_p^i (1 - \alpha_p) \|\mathbf{n}\|_\infty \\ &\leq |\alpha_p|^j |\mathbf{e}_p(0, 0)| + \left(\sum_{i=0}^{\infty} \alpha_p^i \right) \left(T_M \|\mathbf{e}_v\|_\infty + (1 - \alpha_p) \|\mathbf{n}\|_\infty \right) \\ &= \alpha_p^j |\mathbf{e}_p(0, 0)| + \frac{T_M}{1 - \alpha_p} \|\mathbf{e}_v\|_\infty + \|\mathbf{n}\|_\infty, \end{aligned} \quad (7.22)$$

where we used $t_0 = 0$ and the sum in the next to last line converges because $0 < \alpha_p < 1$ by assumption.

Due to the fact that solutions to (7.11) are forced to jump when $\tau = T_M$, it holds for $(t, j) \in \text{dom}(\mathbf{e}_p)$ that $j \geq \frac{t}{T_M} - 1$. As a consequence we have $\alpha_p^j \geq \frac{\alpha_p^{t/T_M}}{\alpha_p}$, where we considered the hypothesis $0 < \alpha_p < 1$. By substituting this last inequality in (7.22), the ISS bound (7.19) is proven.

Lemma 6 *If α_v satisfies $|\alpha_v| < 1$, then the velocity estimation error dynamics (7.12) is finite-gain exponentially ISS with respect to $\tilde{\mathbf{e}}_v$, $\mathbf{d}$ and $\mathbf{n}$. Moreover, when $\alpha_v \in [0, 1)$ all solutions satisfy*

$$|\mathbf{e}_v(t, j)| \leq \frac{\alpha_v^{t/T_M}}{\alpha_v} |\mathbf{e}_v(0, 0)| + \frac{T_M}{1 - \alpha_v} \|\mathbf{d}\| + \frac{1}{T_m} \left(\|\tilde{\mathbf{e}}_v\| + \|\mathbf{n}\| \right), \quad (7.23)$$

for all $(t, j) \in \text{dom } \mathbf{e}_v$.

Proof 2 Consider any state-input solution pair $((\mathbf{e}_v, \tau), (\tilde{\mathbf{e}}_v, \mathbf{d}, \mathbf{n}))$ to (7.12), and any $(t, k) \in \text{dom}(\mathbf{e}_v)$. Since $\dot{\mathbf{e}}_v = \mathbf{d}$ and $\dot{\tau} = 1$ along flows, and the flow set requires $\tau \leq T_M$,

we have that $t - t_k \leq T_M$, therefore we have:

$$\begin{aligned} |\mathbf{e}_v(t, k)| &\leq |\mathbf{e}_v(t_k, k)| + (t - t_k) \|\mathbf{d}\|_\infty \\ &\leq |\mathbf{e}_v(t_k, k)| + T_M \|\mathbf{d}\|_\infty. \end{aligned} \quad (7.24)$$

Similar to Lemma 5 we focus on the case $\alpha_v > 0$, because $\alpha_v < 0$ is associated with equivalent derivations. Moreover, at each jump time t_j , $j \geq 1$, we may use the jump equation in (7.12), combined with (7.24), and the fact that jumps cannot occur unless $\tau \geq T_m$, to get

$$\begin{aligned} |\mathbf{e}_v(t_j, j)| &\leq \alpha_v |\mathbf{e}_v(t_j, j-1)| + \frac{1 - \alpha_v}{T_m} (\|\tilde{\mathbf{e}}_v\|_\infty + \|\mathbf{n}\|_\infty) \\ &\leq \alpha_v |\mathbf{e}_v(t_{j-1}, j-1)| + \alpha_v T_M \|\mathbf{d}\|_\infty + \frac{1 - \alpha_v}{T_m} (\|\tilde{\mathbf{e}}_v\|_\infty + \|\mathbf{n}\|_\infty). \end{aligned}$$

By iterating the above inequalities, we obtain:

$$\begin{aligned} |\mathbf{e}_v(t, j)| &\leq |\mathbf{e}_v(t_j, j)| + T_M \|\mathbf{d}\|_\infty \\ &\leq \alpha_v^j |\mathbf{e}_v(t_0, 0)| + \sum_{i=0}^j \alpha_v^i T_M \|\mathbf{d}\|_\infty + \sum_{i=1}^j \alpha_v^i \frac{1 - \alpha_v}{T_m} (\|\tilde{\mathbf{e}}_v\|_\infty + \|\mathbf{n}\|_\infty) \\ &\leq \alpha_v^j |\mathbf{e}_v(0, 0)| + \left(\sum_{i=0}^{\infty} \alpha_v^i \right) \left(T_M \|\mathbf{d}\|_\infty + \frac{1 - \alpha_v}{T_m} (\|\tilde{\mathbf{e}}_v\|_\infty + \|\mathbf{n}\|_\infty) \right) \\ &= \alpha_v^j |\mathbf{e}_v(0, 0)| + \frac{T_M}{1 - \alpha_v} \|\mathbf{d}\|_\infty + \frac{1}{T_m} (\|\tilde{\mathbf{e}}_v\|_\infty + \|\mathbf{n}\|_\infty) \end{aligned}$$

where we used $t_0=0$ and the sum in the next to last line converges because $|\alpha_v| < 1$ by assumption. The proof is completed like in the end of the proof of Lemma 5 to obtain bound (7.23).

Lemma 7 *The $\tilde{\mathbf{e}}_v$ dynamics (7.13) is finite-gain exponentially ISS with respect to $\mathbf{d}$ and $\mathbf{n}$. In particular, for any $\tilde{\alpha} \in (0, 1)$ all solutions satisfy:*

$$|\tilde{\mathbf{e}}_v(t, j)| \leq \frac{\tilde{\alpha}^{t/T_M}}{\tilde{\alpha}} |\tilde{\mathbf{e}}_v(0, 0)| + T_M^2 \|\mathbf{d}\| + \|\mathbf{n}\|, \quad (7.25)$$

for all $(t, j) \in \text{dom } \tilde{\mathbf{e}}_v$.

Proof 3 *Let us first concentrate on the (possibly zero-length) flowing interval $t_1 - t_0$, to*

get

$$\begin{aligned} |\tilde{\mathbf{e}}_v(t, 0)| &\leq |\tilde{\mathbf{e}}_v(0, 0)| + \int_{t_0}^t \tau(s, 0) |\mathbf{d}(s, 0)| ds \\ &\leq \frac{\tilde{\alpha}^{t/T_M}}{\tilde{\alpha}} |\tilde{\mathbf{e}}_v(0, 0)| + T_M^2 \|\mathbf{d}\|, \forall t \in [t_0, t_1] \end{aligned} \quad (7.26)$$

where the first term comes from $t - t_0 = t \leq T_M$, namely $\frac{t}{T_M} - 1 \leq 0$ and then $\tilde{\alpha}^{\frac{t}{T_M} - 1} \geq 1$, while the second term comes from $t_1 - t_0 \leq T_M$ and $|\tau| \leq T_M$.

Furthermore, from (7.13), at each jump time (t_j, j) , $j \geq 1$, we have $|\mathbf{e}_v(t_j, j)| \leq \|\mathbf{n}\|$, which may be used to follow similar steps to those in (7.26) to get

$$|\tilde{\mathbf{e}}_v(t, j)| \leq \|\mathbf{n}\| + T_M^2 \|\mathbf{d}\|, \quad \forall (t, j) \in \text{dom } \tilde{\mathbf{e}}_v, j \geq 1.$$

Combining the bound above with (7.26), we obtain bound (7.25), as to be proven.

Proof of Theorem 5 Let us first prove necessity of $|\alpha_p| < 1$ and $|\alpha_v| < 1$. Assume that either of them does not hold and focus on the corresponding subsystem (7.11) or (7.12) taking a zero initial condition for the preceding subsystem in the cascade connection of Figure 7.1, and considering the case when $\mathbf{d} = \mathbf{n} = 0$. The corresponding dynamics coincides with (7.17) with $f(x, u) = 0$ and $g(x, u) = \alpha x$, where $|\alpha| \geq 1$. This clearly generates non converging solutions, thus proving that an exponential bound cannot hold. The sufficiency follows by simply concatenating the exponential.

Now we prove the second statement with $\alpha_p \in [0, 1)$ and $\alpha_v \in [0, 1)$. Concatenating (7.23) and (7.25) with $\tilde{\alpha} = \alpha_v$, we get

$$|\mathbf{e}_v(t, j)| \leq \frac{\alpha_v^{t/T_M}}{\alpha_v} \left(|\mathbf{e}_v(0, 0)| + \frac{1}{T_m} |\tilde{\mathbf{e}}_v(0, 0)| \right) + \left(\frac{T_M}{1 - \alpha_v} + \frac{T_M^2}{T_m} \right) \|\mathbf{d}\| + \frac{2}{T_m} \|\mathbf{n}\|,$$

which implies (7.16) with $\gamma = T_M \left(1 + \frac{T_M}{T_m} \right)$ and $\eta = \frac{2}{T_m}$ (ensuring $(\frac{T_M}{1 - \alpha_v} + \frac{T_M^2}{T_m}) \leq \frac{\gamma}{1 - \alpha_v}$ for any $\alpha_v \in [0, 1)$).

Once (7.16) is proven, we can plug it into (7.19) to get (7.15), which complete the proof. $\square$

7.3 Experimental validation

7.3.1 Experimental Results

The setup, designed to verify experimentally the proposed observer, is the one introduced in Section 2.3.1. The advantage of using a robot for holding the mobile beacon is that the encoder measurements at the robot joints provides a convenient ground truth information, available at high sampling frequency. The experimental test consists in actuating the robot in such a way that the plate secured at the end effector tracks a rhombus. Figure 7.2 shows the trajectory of the mobile beacon.

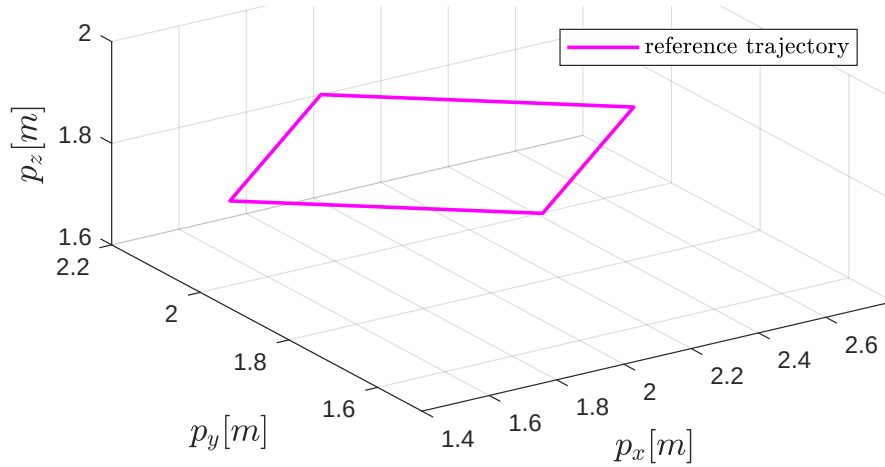


Figure 7.2: Reference position trajectory represented in the $\mathbf{p}^n = [p_x \ p_y \ p_z]^\top$ coordinates.

In order to assess the effectiveness of the proposed observer, two tests have been carried out, both of them based on the same experimental curve. In the first test (*Test 1*) the proposed observer exploits sampled versions of the accurate position measurements provided by the robot internal controller. These position measurements are based on the encoder information (the beacons are ignored in this first test), therefore they are characterized by low noise levels. In the second test (*Test 2*) the position measurements are provided by the beacons. In this case the measurements are certainly affected by a higher noise level, which allows validating the ISS bounds established in the theoretical results. For both tests we forced a random position sampling in order to obtain the sporadic measurements compliant with Assumption 5. In particular, even if the measurements were available at a higher rate, we used only some of them, at random times, ensuring that the elapsed time between two consecutive measurements was between $T_m = 0.3s$ and $T_M = 0.7s$, as shown in Fig. 7.3. Finally, for both tests we tuned the observer gains as

$$\alpha_p = \alpha_v = 0.5.$$

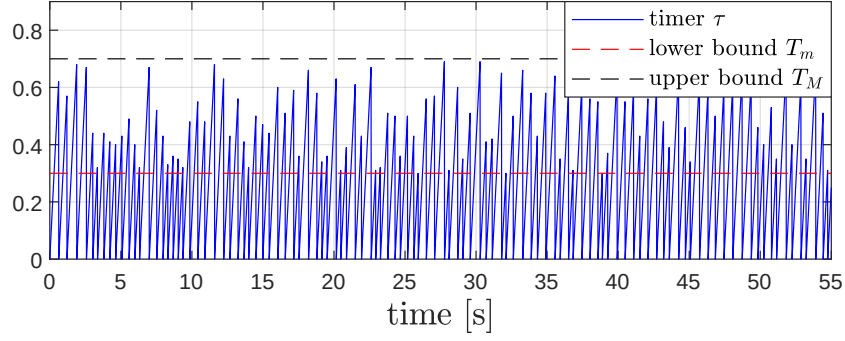


Figure 7.3: Evolution of the timer variable τ illustrating the interval between each pair of consecutive sampled position measurements.

The results of both tests are given in Figures 7.4 and 7.5, showing in black the accurate measurement based on the robot encoders (the ground truth) and the estimated positions and velocities in the x and y directions, as obtained from observer (7.9). It is apparent that the estimated variables exponentially converge to the corresponding state variables.

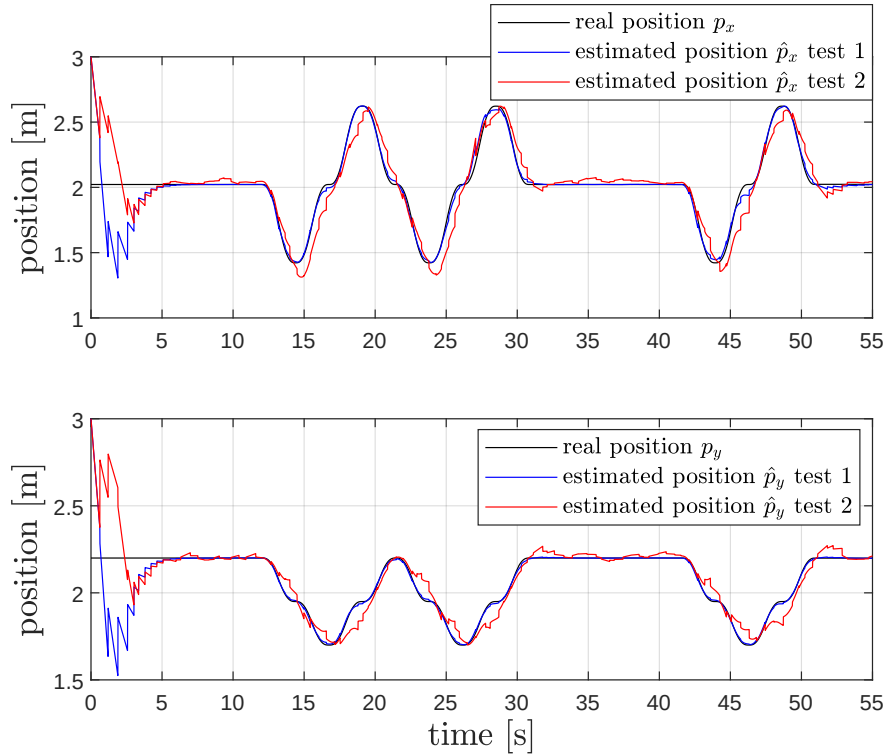


Figure 7.4: Real and estimated positions for *Test 1* (blue color) and *Test 2* (red color).

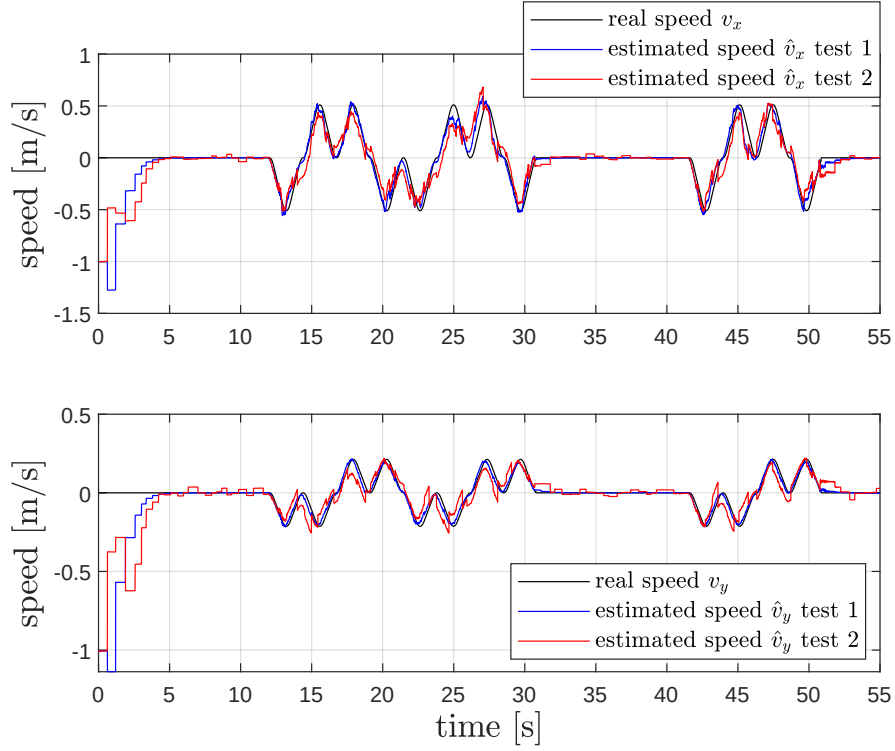


Figure 7.5: Real and estimated velocities for *Test 1* (blue color) and *Test 2* (red color).

To better highlight the behavior of the proposed observer, a second pair of tests have been performed from a different initial condition associated with a larger initial estimation error, and the observer behavior has been assessed again by repeating the scenarios of *Test 1* and *Test 2*, described above.

Figure 7.6 shows in logarithmic scale the position and speed estimation errors for *Test 1* (blue) and *Test 2* (red). From this figure the hybrid nature of the estimation is clearly visible, especially during the initial transient. Indeed, the velocity estimation error is constant during flowing, and decreases across jumps, as expected from the theoretical results, while the position estimation error verifies the ISS bound (7.19) proven in Lemma 5.

The comparison between the blue and red curves of Figure 7.6 reveals that *Test 2* is associated to significantly larger steady-state estimation errors. Indeed, in *Test 2*, the observer (7.9) operates based on the noisy beacon measurements.

As one may reasonably expect, the estimation error is significantly reduced in the intervals $t \in [5, 10]$ and $t \in [31, 41]$ when the robot does not move. These noisy tests confirm that the observer is able to cope with noisy measurements, since both the position and the velocity are well estimated through a desirable filtering action. Moreover, the

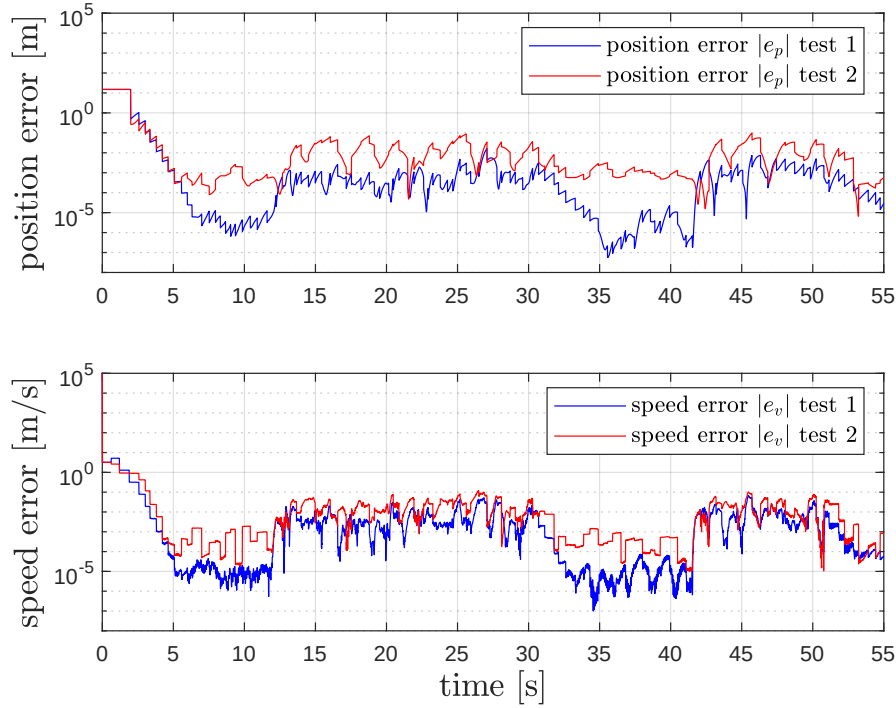
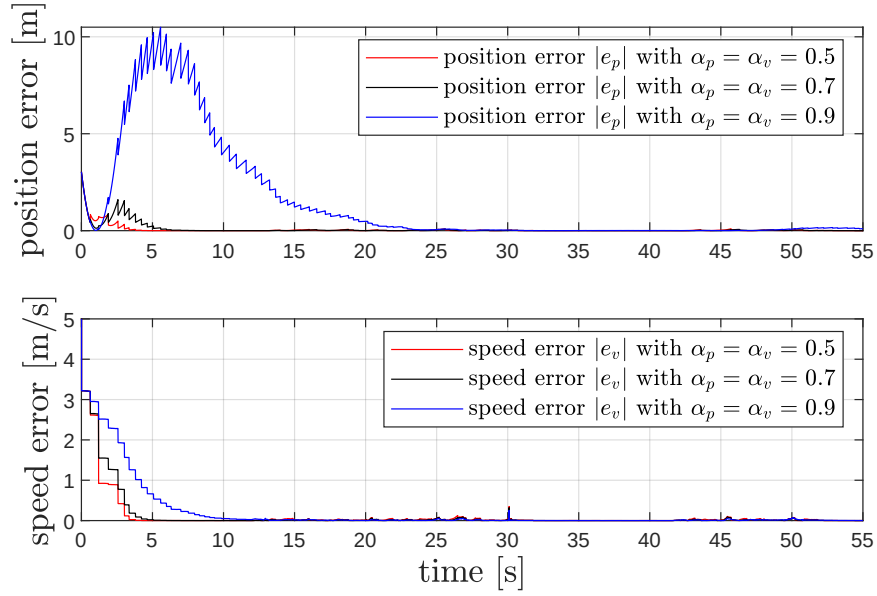


Figure 7.6: Position errors $\mathbf{e}_p$ (upper plot) and velocity errors $\mathbf{e}_v$ (bottom plot) in *Test 1* (blue color) and *Test 2* (red color).

limited effects of the measurement noise on the estimation error confirms the desirable features of the proposed observer.

Finally, Figure 7.7 shows the position estimation error $\mathbf{e}_p$ and the velocity estimation error $\mathbf{e}_v$ for different values of α_p and α_v during *Test 2* (when using the beacon measurements). As one may expect from bounds (7.15)-(7.16), characterized in Theorem 5, the closer α_p and α_v are to zero, the higher the convergence rate of the estimation error. But the higher convergence rate comes at the price of a reduced filtering action on the measurement noise, as confirmed by the numbers reported in Table 7.1, representing the Integral Absolute Error (IAE) indexes computed on the steady-state response: $IAE(\mathbf{e}) := \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |\mathbf{e}(t)| dt$, with $t_1 = 35\text{s}$ and $t_2 = 55\text{s}$. The choice of α_p and α_v should then be carried out as a trade-off between these two goals. Note that the convergence rate is influenced by the values of T_m and T_M even through the observer is independent of them. Instead, the increased overshoot predicted by (7.15)-(7.16) and commented in Remark 7 cannot be appreciated in the experimental tests.

<i>Test 1</i>	$\alpha_p = \alpha_v = 0.5$	$\alpha_p = \alpha_v = 0.7$	$\alpha_p = \alpha_v = 0.9$
IAE_{e_p}	0.034	0.03	0.027
IAE_{e_v}	0.077	0.075	0.070
<i>Test 2</i>	$\alpha_p = \alpha_v = 0.5$	$\alpha_p = \alpha_v = 0.7$	$\alpha_p = \alpha_v = 0.9$
IAE_{e_p}	0.138	0.107	0.068
IAE_{e_v}	0.162	0.154	0.135

Table 7.1: Steady-state IAE indexes for different values of α_p and α_v .Figure 7.7: Waveforms of the position error e_p (upper plot) and of the velocity error e_v (bottom plot) for different values of α_p and α_v .

Chapter 8

Quadcopter Application

OUTLINE. In this chapter, since UAVs (Unmanned Aerial Vehicles) can be used for both in indoor and outdoor scenario and so they represents a good RTLS test application, a robust motion control law for autonomous quadcopters (starting from an input-output linearizable model) is presented to cope with autonomous flight. In particular, the suggested technique can be considered as a robust feedback linearization, where the nonlinear state-feedback terms which contains the aerodynamic forces and moments and other unknown disturbances, are online estimated by means of extended state observers. Thereby, the control system is made robust against unmodelled dynamics and endogenous as well as exogenous disturbances. The desired closed loop dynamics is obtained by means of pole-assignment. In order to have a feasible control action, i.e. the forces produced by the motors belong to an admissible set of forces, suitable reference signals are generated by means of differentiators supplied by the desired trajectory. The proposed control algorithm is tested by means of simulation experiments on a Rapsberry-PI board by means of the hardware-in-the-loop method, showing the effectiveness of the proposed approach. Moreover, it is compared with the standard feedback linearization control method, where the above nonlinear terms are computed using nominal parameters, and the aerodynamical disturbances are neglected. The comparison shows that the control algorithm based on the online estimation of the above nonlinear state-feedback terms gives the better static and dynamic behaviour over the standard feedback linearization control method.

In the last years, the control of unmanned mobile vehicles has received great attention. This interest is motivated by many applications where the autonomous control is required, such as the tracking of a trajectory for the accomplishment of a particular task. On this subject, a huge number of works propose valid control solutions, especially in the field of the aerial vehicles. Most of these solutions have been summarized in several papers and

books (see for example [113, 114, 115, 116] and references therein).

In this work the attention is focused on the control of autonomous quadcopter vehicles. With regard to this field, many problems have been addressed in literature. The Feedback Linearization (FL) via dynamic feedback has been shown in [117], starting from the classical mathematical model of the vehicle with twelve state variables, which is not input-output linearizable. From this model, using the method of the input dynamic feedback and neglecting the exogenous disturbances, a model with fourteen state variables is obtained, which results input-output linearizable. The model linearized by means of FL appears decoupled in four SISO models having the three components of the position vector referred to an inertial reference frame and the yaw angle as outputs, and an auxiliary input vector with four components. A state feedback controller is then designed for each of the above four models using the pole assignment technique. Simulation results confirm the feasibility of the controller based on the linearized and decoupled model. In [118], the input-output linearized and decoupled model is completed by adding to the auxiliary input the vector of the four exogenous equivalent disturbances projected on this input space. In this way, four linear decoupled and time-invariant models are obtained, each of them differs from the Brunovsky canonical form for the presence of an unknown equivalent disturbance added to a component of the auxiliary input vector. The four components of the equivalent disturbance vector are computed from the state variables estimated by means of a high order sliding mode state observer, and then, they are conveniently filtered by means of a filter not indicated in the work, and the filtered values are used to compensate for exogenous disturbances. Consequently, the determination of these components is open loop type. The remaining attitude variables, and the angular velocities in BODY frame, not estimated by the sliding observer, are computed using the estimated variables and the above mentioned linearizable model without exogenous disturbances. The last ones variables and those estimated are used for the implementation of the control law. In [119] PID-type classical control techniques and optimal linear quadratic control are employed using linear approximated models only for attitude control. An adaptive version of optimal controller is also discussed. In [120], the Authors deals with the on-line planning of a trajectory for an helicopter with an engine OFF flight condition, and robust tracking of the planned trajectory. The planning approach is model-based and feasible and optimal trajectories can be generated. Regarding the controller design, the μ -synthesis approach is used using a low order, linear and time-invariant plant with a multiplicative uncertainty, in order to compensate for high frequency unmodelled dynamics. In [121], the flight control problem is solved using an inner loop for controlling the attitude, and an outer loop for controlling translational motion. Both control loops use adaptive control

techniques with the aim of minimizing the effects of modelling errors. The method of pseudo-control hedging is employed for decoupling the dynamics of the two control loops. For the sake of completeness, it is useful to observe that, in the literature, methods of autonomous flight, which involve video cameras and vision algorithms, are discussed. For example, in [122] it is described a vision controlled micro UAV that is capable to flight autonomously using only a video camera and an IMU on board. In [123] a control scheme is proposed for attitude control of a quadrotor aimed at the compensation of time delays in the inputs of the quadrotor model. The scheme consists of a nominal controller, which forces the plant to pursue the desired behaviour in absence of delays and a compensator, which gives a signal added to that of the nominal controller for compensating delays. It is shown that robust stability and behaviour can be obtained.

This work deals with the motion control of autonomous quadcopters, able to track a desired trajectory ensuring a high dynamic performance and the robustness of the whole system. The robust motion control is designed so as to satisfy the following requirements: a) decoupling of the linear motions along the three axes of the inertial frame, and the yaw angle; b) robustness against endogenous and exogenous disturbances; c) assignment of the desired dynamics to each of the above decoupled motions; d) the control actions, i.e. the forces to be produced by the motors, have to belong to the admissible set of forces, i.e. the set of forces that the motor-propeller are able to generate. In order to satisfy the above requirements, a combination of feedback linearization and active disturbance rejection approaches is utilized. More precisely, starting from the fourteen order nonlinear model illustrated in [118], defining a convenient nonlinear equivalent disturbance vector which contains both the endogenous disturbances and the exogenous ones, assuming known the equivalent disturbance, a linear model is obtained consisting of four linear and decoupled models, each of them having the Brunovsky canonical form, forced only by means of a component of the auxiliary input. This goal has been achieved by applying the nonlinear control theory shown in [124]-[125]. Differently from the procedure used in [118], each component of the nonlinear equivalent disturbance vector is on line estimated by means of an Extended State Observer (ESO) and this guarantees robustness the proposed control law. In this way, the requirements of a) and b) are satisfied. Note that the applied technique is also different from the μ -synthesis method used in [123, 120], which is a robust linear control technique, and it is also different with respect to the other techniques shown in [119, 121]. In order to satisfy the requirements c), pole placement control techniques are used. Finally in order to satisfy the requirement d), the desired trajectory is suitably generated so as the output variables can track their references with bounded forces, or rather with forces that the motor-propeller systems can give.

The validation of the control law, is carried out by means of several simulations, carried out by using the standard mathematical model of the quadrotor. In order to increase accuracy of the simulations, the controller has been discretized and it has been executed with a fixed sampling time in a Rapsberry-PI board by means of the hardware-in-the-loop method. Moreover, the quantization level of the measured signals has been taken into consideration, as well as the fact that the data acquired by the IMU and the GPS, i.e. position and velocity of the quadrotor center of mass, are available with different rates is also considered. This is bargained for by using the hybrid nonlinear observer illustrated in [1]. Beside to this, the robustness of the designed control law is another aspect which increases the effectiveness of the controller, and the feasibility of its experimental implementation. To the best of the authors knowledge, the control architecture proposed in this work, for quadcopter control, is not described in the literature.

Table 8.1: List of symbols for quadcopter application

SYMBOLS	
x, y, z	coordinates of the center of gravity referred to inertial frame NED, (m);
u, v, w	velocities referred to inertial frame, (m/s);
ϕ, θ, ψ	roll, pitch and yaw angles, (rad);
p, q, r	angular speeds in the body frame referred to the ϕ, θ and ψ angles respectively, (rad/s);
m	mass of the vehicle, (kg);
g	gravity acceleration, (m/s ²);
a_x, a_y, a_z	aerodynamical forces, (N);
a_ϕ, a_θ, a_ψ	aerodynamical moments, (Nm);
I_x, I_y, I_z	inertia coefficients, (Nms ²);
d	distance from the center of mass to the rotors, (m);
f_i	force generated by the i^{th} rotor, (N);
μ	force to moment scaling factor, (m);
$s_\alpha, c_\alpha, t_\alpha$	$\sin(\alpha), \cos(\alpha), \tan(\alpha)$;

8.1 Preliminaries on the mathematical model of the quadcopter

The mathematical model of the quadcopter considered in this work is the state space model obtained in [113], considered also in [117] and [118]. The state variables are the

coordinates of the center of gravity (x, y, z) of the vehicle referred to an inertial frame NED, the speeds (u, v, w) referred to the same frame, the Euler angles (roll ϕ , pitch θ and yaw ψ) referred to the body frame and the angular speeds (p, q, r) referred to the same body frame. Since this work is aimed at determining a control law useful for trajectory tracking, the output of the model is chosen as a vector consisting of the coordinate of the center of gravity (x, y, z) and the yaw angle ψ . So, this model is given by the following equations (cf. [117, 118]):

Cinematic Model

$$\begin{aligned}
 \dot{x} &= u, \\
 \dot{y} &= v, \\
 \dot{z} &= w, \\
 \dot{\psi} &= \frac{qs_\phi + rc_\phi}{c_\theta}, \\
 \dot{\theta} &= qc_\phi - rs_\phi, \\
 \dot{\phi} &= p + t_\phi(qs_\phi + rc_\phi).
 \end{aligned} \tag{8.1}$$

Dynamical Model

$$\begin{aligned}
 \dot{u} &= \frac{a_x - (c_\phi c_\psi s_\theta + s_\phi s_\psi)u_1}{m}, \\
 \dot{v} &= \frac{a_y - (c_\phi s_\psi s_\theta + s_\phi c_\psi)u_1}{m}, \\
 \dot{w} &= g + \frac{a_z - c_\theta c_\phi u_1}{m}, \\
 \dot{p} &= \frac{(I_y - I_z)qr + a_p + du_2}{I_x}, \\
 \dot{q} &= \frac{(I_z - I_x)pr + a_q + du_3}{I_y}, \\
 \dot{r} &= \frac{(I_x - I_y)qq + a_r + u_4}{I_z}.
 \end{aligned} \tag{8.2}$$

The symbols are defined in Table 8.1. Model (8.1)-(8.2) has four input variables, denoted by u_i , $i = 1, \dots, 4$, having a well precise physical meaning: u_1 represents the resultant of the external forces generated, along the z-body axis, by the four propellers; u_2 represents the moment generated around the x-body axis by the above mentioned external forces scaled by d (cf. Fig. 8.1); u_3 represents the moment generated by the external forces around the y-body axis scaled by d (cf. Fig. 8.1); u_4 represents the moment generated by the external forces around the z-body axis. In order to determine u_4 , it is assumed that

the propellers are in the x-y plane, and the angular speed of a propeller is positive when its rotation is counter clockwise with respect to positive direction of the z-body axis. The propellers $T1$ and $T3$ rotate counter clockwise at angular speeds ω_{T1} and ω_{T3} , whereas $T2$ and $T4$ rotate clockwise at speeds ω_{T2} and ω_{T4} . Considering, for example, the propeller $T1$, its counter clockwise rotation produces a clockwise moment $m_{T1} = c_d \omega_{T1}^2$ around the z-body axis, where c_d is the drag coefficient, besides a force $f_1 = c_f \omega_{T1}^2$, where c_t is the thrust coefficient. It follows that $m_{T1} = \mu f_1$, where $\mu = \frac{c_d}{c_t}$. Then, the expressions of u_i , $i = 1, \dots, 4$ in terms of the above forces are given by:

$$\begin{aligned} u_1 &= f_1 + f_2 + f_3 + f_4, \\ u_2 &= f_2 - f_4, \\ u_3 &= f_1 - f_3, \\ u_4 &= \mu(-f_1 + f_2 - f_3 + f_4), \end{aligned} \tag{8.3}$$

The above equations allow to obtain the forces f_i starting from the inputs u_i . Since for assigned u_i $i = 1, \dots, 4$, there exist a unique set of forces that produces the inputs themselves, it is convenient to obtain, firstly, the control law in terms of u_i , and then to obtain the forces f_i solving the equations (8.3).

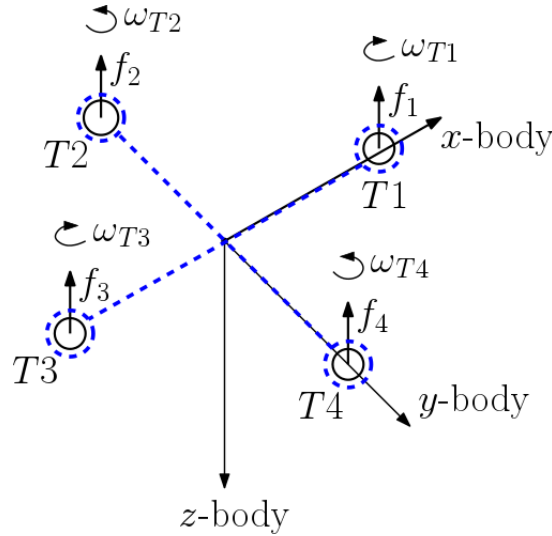


Figure 8.1: Scheme of the quadcopter: $T1-T4$: the four propellers; f_1-f_4 : forces generated by the propellers $T1-T4$; $\omega_{T1}-\omega_{T4}$: angular speeds of the propellers.

Finally, the output is given by:

$$\mathbf{y} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \end{bmatrix}^T = \begin{bmatrix} x & y & z & \psi \end{bmatrix}^T.$$

Assumption 7 *In order to avoid singularity problems, the following constraint is assumed for model (8.1)-(8.2):*

$$\theta \in (-\pi/2, \pi/2). \quad (8.4)$$

Note that this constraint is satisfied in a correct operation of a quadcopter vehicle, and unlikely the quadcopter reaches the boundary of this domain where singularity problems can occur ($\theta = \pm\pi/2$, see fourth equation of (8.1)).

In [117] it is shown that the model (8.1),(8.2) has a vector relative degree equal to $\{r_1, r_2, r_3, r_4\}$, with $r_1 = r_2 = r_3 = r_4 = 2$. Consequently, this model cannot be input-output linearized using a static nonlinear feedback. In order to cope with this drawback, the input dynamic extension technique ([124, 125]) can be employed. This technique introduces new inputs given by the time derivatives of the actual ones. In this way the extended state model becomes input-output linearizable. If the sum of the relative degrees (total relative degree) is equal to the order of the extended model, there is no zero-dynamics and the stability of the whole system is ensured by a suitable choice of the controller for the linearized model. In [117] it is shown that the vector relative degree can be increased deriving two times the input u_1 , putting $u_1 = \zeta$, $\dot{u}_1 = \xi$ and $\ddot{u}_1 = \tau_1$. In this way, putting $u_2 = \tau_2$, $u_3 = \tau_3$ and $u_4 = \tau_4$ the extended model of the quadcopter is given by the Cinematic Model (8.1) and the extended dynamic model given by:

$$\begin{aligned} \dot{u} &= \frac{a_x - (c_\phi c_\psi s_\theta + s_\phi s_\psi)\zeta}{m}, \\ \dot{v} &= \frac{a_y - (c_\phi s_\psi s_\theta + s_\phi c_\psi)\zeta}{m}, \\ \dot{w} &= g + \frac{a_z - c_\theta c_\phi \zeta}{m}, \\ \dot{\zeta} &= \xi, \\ \dot{\xi} &= \tau_1, \\ \dot{p} &= \frac{(I_y - I_z)qr + a_p + d\tau_2}{I_x}, \\ \dot{q} &= \frac{(I_z - I_x)pr + a_q + d\tau_3}{I_y}, \\ \dot{r} &= \frac{(I_x - I_y)qq + a_r + \tau_4}{I_z}. \end{aligned} \quad (8.5)$$

The vector relative degree of model (8.1),(8.5) is given by $\{\hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4\}$, with $\hat{r}_1 = \hat{r}_2 = \hat{r}_3 = 4$ and $\hat{r}_4 = 2$. Consequently, the model in question is input-output

linearizable because the total relative degree is equal to the order of the model itself. In order to obtain the linearized model, the following procedure is followed. By deriving the outputs as many times as the corresponding relative degree is, the following model is obtained:

$$y_i^{(\hat{r}_i)} = b_i(\mathbf{x}) + \sum_{j=1}^{n_u} \delta_{i,j}(\mathbf{x})\tau_j, \quad i = 1, \dots, 4, \quad (8.6)$$

where $y_i^{(\hat{r}_i)}$ $n_u = 4$ is the number of the inputs, $b_i(\mathbf{x})$ is the equivalent disturbance acting on the output y_i , and $\mathbf{x}$ is the state vector of model (8.1),(8.5), given by:

$$\mathbf{x} = [x \ y \ z \ \phi \ \theta \ \psi \ u \ v \ w \ \zeta \ \xi \ p \ q \ r]^T.$$

These Equations, in matrix form, can be written as follows:

$$\begin{bmatrix} y_1^{(\hat{r}_1)} & y_2^{(\hat{r}_2)} & y_3^{(\hat{r}_3)} & y_4^{(\hat{r}_4)} \end{bmatrix}^T = \mathbf{b}(\mathbf{x}) + \mathbf{\Delta}(\mathbf{x})\boldsymbol{\tau}, \quad (8.7)$$

where:

$$\begin{aligned} \boldsymbol{\tau} &= [\tau_1 \ \tau_2 \ \tau_3 \ \tau_4]^T, \\ \mathbf{b}(\mathbf{x}) &= [b_1(\mathbf{x}) \ b_2(\mathbf{x}) \ b_3(\mathbf{x}) \ b_4(\mathbf{x})]^T, \\ \mathbf{\Delta}(\mathbf{x}) &= \begin{bmatrix} \delta_{11}(\mathbf{x}) & \delta_{12}(\mathbf{x}) & \delta_{13}(\mathbf{x}) & \delta_{14}(\mathbf{x}) \\ \delta_{21}(\mathbf{x}) & \delta_{22}(\mathbf{x}) & \delta_{23}(\mathbf{x}) & \delta_{24}(\mathbf{x}) \\ \delta_{31}(\mathbf{x}) & \delta_{32}(\mathbf{x}) & \delta_{33}(\mathbf{x}) & \delta_{34}(\mathbf{x}) \\ \delta_{41}(\mathbf{x}) & \delta_{42}(\mathbf{x}) & \delta_{43}(\mathbf{x}) & \delta_{44}(\mathbf{x}) \end{bmatrix}. \end{aligned}$$

The terms $\mathbf{b}(\mathbf{x})$ and $\mathbf{\Delta}(\mathbf{x})$, named respectively disturbance vector and control gain matrix, are given in the Appendix.

The determinant of matrix $\mathbf{\Delta}(\mathbf{x})$ is given by:

$$\det [\mathbf{\Delta}(\mathbf{x})] = -\frac{d^3 \zeta^2 c_\phi}{m^3 I_x I_y I_z c_\theta}. \quad (8.8)$$

It follows that, under the Assumption 7, $\det [\mathbf{\Delta}(\mathbf{x})] \neq 0$ providing that $\zeta \neq 0$.

8.2 Design of the control algorithm

Now, the motion control law for model (8.7) has to be designed so as to satisfy the following requirements:

- a) decoupling of the motions along the three axes of the inertial frame and that of the orientation along the yaw angle;
- b) robustness against endogenous and exogenous disturbances;
- c) each of the above decoupled motions have to occurs according to a given dynamics;
- d) the control actions, i.e. the forces to be produced by the motor-propellers, have to belong the the set of forces that the propellers can generate.

As it will be seen later, items a) and b) are obtained if a good knowledge is gained on the disturbance vector and on the control gain matrix. Item c) will be satisfied by using pole assignment techniques. Item d) will be gained by choosing suitably the dynamics of the whole closed loop control system.

8.2.1 Control architecture

The structure of (8.7) is such that if the control gain matrix, $\Delta(\mathbf{x})$, and the disturbance vector, $\mathbf{b}(\mathbf{x})$, are known, then the control law given by:

$$\boldsymbol{\tau} = [\Delta(\mathbf{x})]^{-1} (-\mathbf{b}(\mathbf{x}) + \boldsymbol{\nu}), \quad (8.9)$$

where $\boldsymbol{\nu}$ in an auxiliary control vector, allows to obtain a closed loop control system given by:

$$\begin{bmatrix} y_1^{(\hat{r}_1)} & y_2^{(\hat{r}_2)} & y_3^{(\hat{r}_3)} & y_4^{(\hat{r}_4)} \end{bmatrix}^T = \boldsymbol{\nu}. \quad (8.10)$$

It is worthwhile to observe that model (8.10) consists of four chains of integrators of orders $\hat{r}_1$, $\hat{r}_2$, $\hat{r}_3$, and $\hat{r}_4$. Moreover, it is possible to assign to each chain the desired closed loop dynamics, acting on the relative component of the auxiliary control $\boldsymbol{\nu}$.

However in order to apply the control law (8.9), a good knowledge of $\Delta(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$ needs. With reference to the control gain matrix, its elements (see Appendix A) depend on few mechanical parameters of the system (m , I_x , I_y , I_z , d) which can be assumed known, and from state variables (Euler angles and ζ) which can be obtained by the IMU on board. Moreover, since $\zeta \neq 0$ during flight, the matrix in question is always invertible. The situation is different when we consider the disturbance vector $\mathbf{b}(\mathbf{x})$. In fact, as shown in the Appendix A, the elements of this vector are function of the Euler angle and angular speeds in the body frame, together with the second derivatives of the aerodynamical forces and the aerodynamic moments acting on the vehicle. These forces and moments are obviously unknown. As a result, $\mathbf{b}(\mathbf{x})$ is unknown even if the parameters

of the quadcopter model are known, and cannot be computed on-line and compensated as previously hypothesized. It follows that the control law based on the feedback linearization approach does not give the desired results.

Motivated from these considerations, it is proposed a control law based on the disturbance vector estimation, $\hat{\mathbf{b}}(\mathbf{x})$. Replacing in (8.9) $\mathbf{b}(\mathbf{x})$ with $\hat{\mathbf{b}}(\mathbf{x})$, the control law becomes:

$$\boldsymbol{\tau} = [\boldsymbol{\Delta}(\mathbf{x})]^{-1} \left(-\hat{\mathbf{b}}(\mathbf{x}) + \boldsymbol{\nu} \right). \quad (8.11)$$

If the estimation of the disturbance vector is sufficiently accurate (i.e. $\hat{\mathbf{b}}(\mathbf{x}) \rightarrow \mathbf{b}(\mathbf{x})$), then the closed loop control system works according to (8.10).

The control law previously described appears, practically, as a feedback linearization control in which the nonlinear term, representing endogenous and exogenous disturbances, is on-line estimated instead of analytically computed. This technique is called Active Disturbance Rejection Control (ADRC) [126]. The nonlinear disturbance $\mathbf{b}(\mathbf{x})$ is estimated by means of Extended State Observers (ESOs) [127], together with the output variables and their derivatives up to the useful order for implementing the controller. To the best of the authors' knowledge, the ADRC, as proposed in this work, has never been applied for quadcopter control.

8.2.2 Design of the extended state observers

In the present work, we will obtain the estimation of the disturbance vector by constructing four Extended State Observers (ESOs) designed from model (8.7).

In particular, this model can be divided in four independent sub-models, as follows:

$$y_i^{(\hat{r}_i)} = b_i(\mathbf{x}) + \sum_{j=1}^{n_u} \delta_{i,j}(\mathbf{x}) \tau_j, \quad i = 1, \dots, 4. \quad (8.12)$$

With reference to model (8.12), let us define the following extended state vector:

$$\hat{\mathbf{x}}_i = \begin{bmatrix} \hat{x}_{i,1} \\ \hat{x}_{i,2} \\ \vdots \\ \hat{x}_{i,\hat{r}_i} \\ \hat{x}_{i,\hat{r}_i+1} \end{bmatrix} = \begin{bmatrix} y_i \\ y_i^{(1)} \\ \vdots \\ y_i^{(\hat{r}_i-1)} \\ b_i(\mathbf{x}) \end{bmatrix}, \quad i = 1, \dots, 4. \quad (8.13)$$

Then it is possible to define the following extended state model:

$$\begin{aligned}
 \dot{\hat{x}}_{i,1} &= \hat{x}_{i,2}, \\
 \dot{\hat{x}}_{i,2} &= \hat{x}_{i,3}, \\
 &\vdots \\
 \dot{\hat{x}}_{i,\hat{r}_i} &= \hat{x}_{i,\hat{r}_i+1} + \sum_{j=1}^{n_u} \delta_{i,j}(\mathbf{x}) \tau_j, \\
 \dot{\hat{x}}_{i,\hat{r}_i+1} &= \dot{b}_i(\mathbf{x}),
 \end{aligned} \quad i = 1, \dots, 4. \tag{8.14}$$

An ESO for models (8.14) has the following structure ([127]):

$$\begin{aligned}
 \dot{z}_{i,1} &= z_{i,2} + \epsilon^{\hat{r}_i-1} g_{i,1} \left(\frac{x_{i,1} - z_{i,1}}{\epsilon^{\hat{r}_i}} \right), \\
 \dot{z}_{i,2} &= z_{i,3} + \epsilon^{\hat{r}_i-2} g_{i,2} \left(\frac{x_{i,1} - z_{i,1}}{\epsilon^{\hat{r}_i}} \right), \\
 &\vdots \\
 \dot{z}_{i,\hat{r}_i} &= z_{i,\hat{r}_i+1} + g_{i,\hat{r}_i} \left(\frac{x_{i,1} - z_{i,1}}{\epsilon^{\hat{r}_i}} \right) + \sum_{j=1}^{n_u} \delta_{i,j}(\mathbf{x}) \tau_j, \\
 \dot{z}_{i,\hat{r}_i+1} &= \epsilon^{-1} g_{i,\hat{r}_i+1} \left(\frac{x_{i,1} - z_{i,1}}{\epsilon^{\hat{r}_i}} \right),
 \end{aligned} \tag{8.15}$$

for $i = 1, \dots, 4$. The functions $g_{i,j}$ depend from the error $\frac{x_{i,1} - z_{i,1}}{\epsilon^{\hat{r}_i}}$. In this work, a Linear ESO (LESO) is considered, so $g_{i,j}$ are chosen such that:

$$g_{i,j} \left(\frac{x_{i,1} - z_{i,1}}{\epsilon^{\hat{r}_i}} \right) = \beta_{i,j} \frac{x_{i,1} - z_{i,1}}{\epsilon^{\hat{r}_i}}, \tag{8.16}$$

where $\beta_{i,j}$, $i = 1, \dots, 4$, and $j = 1, \dots, \hat{r}_i + 1$, are constants. Defining, now, the following error variables:

$$\eta_{i,j} = \frac{x_{i,j} - z_{i,j}}{\epsilon^{\hat{r}_i+1-j}}, \quad i = 1, \dots, 4, \quad j = 1, \dots, \hat{r}_i + 1, \tag{8.17}$$

and by taking into account (8.15)-(8.16) the following error dynamics are obtained:

$$\begin{aligned}
 \epsilon \dot{\eta}_{i,1} &= \eta_{i,2} - \beta_{i,1} \eta_{i,1}, \\
 \epsilon \dot{\eta}_{i,2} &= \eta_{i,3} - \beta_{i,2} \eta_{i,1}, \\
 &\vdots \\
 \epsilon \dot{\eta}_{i,\hat{r}_i+1} &= -\epsilon \dot{b}_i(\hat{\mathbf{x}}) - \beta_{i,\hat{r}_i+1} \eta_{i,1},
 \end{aligned} \quad i = 1, \dots, 4. \tag{8.18}$$

Equations (8.18) can be expressed in matrix form as follows:

$$\epsilon \dot{\boldsymbol{\eta}}_i = \mathbf{E} \boldsymbol{\eta}_i + \epsilon \mathbf{h} \dot{b}_i(\mathbf{x}), \quad i = 1, \dots, 4. \quad (8.19)$$

where:

$$\mathbf{E} = \begin{bmatrix} -\beta_{i,1} & 1 & 0 & \cdots & 0 \\ -\beta_{i,2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\beta_{i,\hat{r}_i} & 0 & 0 & \cdots & 1 \\ -\beta_{i,\hat{r}_i+1} & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -1 \end{bmatrix}.$$

In [127] it is shown that for a given $\epsilon > 0$, choosing $\beta_{i,j}$ so that $\mathbf{E}$ is a Hurwitz matrix and assuming that $|\dot{b}_i(\mathbf{x})| < M$, for some $M > 0$, then $\boldsymbol{\eta}_i$ converges within a ball around the origin. Moreover, for $\epsilon \rightarrow 0$, the ball shrinks to the origin itself. This implies that if the equivalent disturbance is bounded, then the design of the observers requires the choice of $\beta_{i,j}$ so that $\mathbf{E}$ is a Hurwitz matrix, and the final error can be made as small as possible by acting on ϵ . Note that the LESO (8.15)-(8.16), can estimate the equivalent disturbance $b_i(\mathbf{x})$, from $z_{i,\hat{r}_i+1} = b_i(\mathbf{x})$. To do this, it requires only the knowledge of the input $\boldsymbol{\tau}$, of the matrix $\boldsymbol{\Delta}(\mathbf{x})$, and of the variable $x_{i,1}(t)$. No other signals or functions are needed. Moreover, besides the equivalent disturbance, the ESO can estimate the state variable $x_{i,1}(t)$ and its derivatives up to that of order $\hat{r}_i - 1$.

8.2.3 Design of the controller

With reference to the design of the controller, the starting point is the knowledge of a good estimate of the disturbance vector. Then, it is possible to apply the control law (8.11) to model (8.7), thus obtaining model (8.10).

This model can be divided into four sub-models, as follows:

$$y_i^{(\hat{r}_i)} = \nu_i, \quad i = 1, \dots, 4. \quad (8.20)$$

The state-space models associated to (8.20) are:

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2}, \\ \dot{x}_{i,2} &= x_{i,3}, \\ &\vdots \\ \dot{x}_{i,\hat{r}_i} &= \nu_i, \end{aligned} \quad i = 1, \dots, 4. \quad (8.21)$$

which is the Brunovsky canonical form. For this model, the control law is obtained so as to assign the desired dynamics to the corresponding closed loop system. Note that this fact is important because it shows that it is possible to obtain four independent sub-models, and these sub-models can be controlled by four independent inputs ν_i , $i = 1, 2, \dots, 4$ as shown in (8.20). Therefore it allows to simplify the controller design, because, instead of considering a MIMO system, it is possible to consider four decoupled SISO systems, independent of each other. The method applied in this work is the pole-assignment for all the four sub-models.

By denoting with $y_{i,d}^{(k)}$, $k = 0, 1, \dots, \hat{r}_i$, the k^{th} derivative of the i^{th} desired output, and with $y_i^{(k)}$, $k = 0, 1, \dots, \hat{r}_i - 1$, the k^{th} derivative of the i^{th} output, then the control law given by:

$$\nu_i = y_{i,d}^{(\hat{r}_i)} - \sum_{j=0}^{\hat{r}_i-1} \gamma_{i,j} \left(y_i^{(j)} - y_{i,d}^{(j)} \right), \quad (8.22)$$

leads to:

$$e_i^{(\hat{r}_i)} + \sum_{j=0}^{\hat{r}_i-1} \gamma_{i,j} e_i^{(j)} = 0, \quad (8.23)$$

where $e_i^{(j)} = y_i^{(j)} - y_{i,d}^{(j)}$ is the j^{th} derivative of the tracking error. Equation (8.23) is an homogenous differential equation of degree $\hat{r}_i$. This implies that if the coefficients $\gamma_{i,j}$ are chosen such that polynomials:

$$s^{\hat{r}_i} + \gamma_{i,\hat{r}_i-1} s^{\hat{r}_i-1} + \dots + \gamma_{i,1} s + \gamma_{i,0}, \quad i = 1, \dots, 4, \quad (8.24)$$

are Hurwitz, then $\lim_{t \rightarrow +\infty} e_i = 0$, i.e. the tracking error is leaded to zero asymptotically.

8.3 Differentiators design

This Section deals with Linear and Nonlinear Differentiators (LD and ND, respectively). Starting from the desired motion in the output space $\{y_{1,d}, y_{2,d}, y_{3,d}, y_{4,d}\}$ these dynamical systems are aimed at obtaining smooth reference signals so that the desired motion belongs to the space of the feasible motions. Obviously, in the quadcopter case, this space depends on both its physical characteristics, and its range of thrusts generated by the motors. In the following, three five-order LDs are designed for positioning the center of gravity of the quadcopter, and one third-order ND for its orientation. The three LDs have the same structure and parameters.

According to [128], the structure of a ND of order n is given by:

$$\begin{aligned}\dot{\chi}_1 &= \chi_2, \\ \dot{\chi}_2 &= \chi_3, \\ &\vdots \\ \dot{\chi}_n &= -p_1 f_{al}(\chi_1 - y_d, \alpha, \delta) - p_2 f_{al}(\chi_2, \alpha, \delta) \\ &\quad - \cdots - p_n f_{al}(\chi_n, \alpha, \delta),\end{aligned}\tag{8.25}$$

where:

$$f_{al}(\rho, \alpha, \delta) = \begin{cases} |\rho|^\alpha \text{sgn}(\rho), & |\rho| > \delta, \\ \rho/\delta^{1-\alpha}, & |\rho| \leq \delta, \end{cases}\tag{8.26}$$

and the coefficients p_i , $i = 1, \dots, n$, are chosen so as the polynomial:

$$s^n + p_n s^{n-1} + \cdots + p_2 s + p_1\tag{8.27}$$

is Hurwitz. Model (8.25) allows to estimate the derivatives of y_d up to that of order $n-1$.

Equation (8.26) shows that $f_{al}(\rho, \alpha, \delta) = \rho$ for $\alpha = 1$, and, with this choice, (8.25) describes an LD, given by:

$$\dot{\chi} = \mathbf{A}\chi + \mathbf{h}p_1 y_d,\tag{8.28}$$

with:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -p_1 & -p_2 & -p_3 & \cdots & -p_n \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

The model (8.28) is asymptotically stable, and its transfer function, from $Y_d(s)$ to $\chi_1(s)$, is given by:

$$W(s) = \frac{p_1}{s^n + p_n s^{n-1} + \cdots + p_2 s + p_1}.\tag{8.29}$$

The tracking error $E(s) = (1 - W(s)) Y_d(s)$ results:

$$E(s) = \frac{s(s^{n-1} p_n s^{n-2} + \cdots + p_2)}{s^n + p_n s^{n-1} + \cdots + p_2 s + p_1} Y_d(s).\tag{8.30}$$

It follows that the LD tracks constant desired signals with null steady-state error, and ramp desired signals with steady-state error equal to p_2/p_1 . The derivatives of these

signals, at the steady-state, are null or constant, respectively, and are reproduced correctly. During transients, these derivatives are smooth signals. In order to design the LD, it is sufficient to choose the parameters p_i , $i = 1, \dots, n$, so as $s^n + p_n s^{n-1} + \dots + p_2 s + p_1$ is Hurwitz. For designing ND, besides the choice of these parameters, it is necessary to choose $\alpha \neq 0$, and a suitable value of δ .

8.4 Testing of the control law

In order to test the control laws designed in the previous Sections, simulations have been carried out by means of hardware-in-the-loop method. In particular, the control law has been implemented in a Rapsberry-PI with a sampling frequency of 100 Hz, as well as the reference trajectories and the hybrid nonlinear observer described below in order to perform the data fusion between GPS, working at 10 Hz, and IMU, whose measurements occur at the same frequency of the controller. The quadcopter has been simulated in matlab-simulink environments, implementing equations (8.1)-(8.3) and considering the following parameters ([129]):

$$\begin{aligned} I_x = I_y = 0.0073 \text{ Nms}^2, \quad I_z = 0.013 \text{ Nms}^2, \\ d = 0.23 \text{ m}, \quad m = 0.65 \text{ kg}, \quad \mu = 0.05 \text{ m}. \end{aligned} \quad (8.31)$$

The aerodynamic forces are chosen as $a_x = 1$, $a_y = a_z = 0.4N$, but they act on the system after 10, 20 and 30 s, respectively. The aerodynamic moments are chosen as $a_p = a_q = a_r = 0.1 \sin(0.1t)$, and act after 10, 20 and 30 s, respectively. At $t = 60s$ all the aerodynamical forces and moments are removed. The waveforms of the reference position vector and yaw are shown in Fig 8.2-8.3.

The extended state observers for the positioning, that estimate the equivalent disturbances along the axes x , y , z , have been designed so that the eigenvalues of the corresponding dynamical matrix $\mathbf{E}$ of the observer are:

$$\text{eig}(\mathbf{E}) = \{-28.28 \pm 28.28i, -169.68, -226.24, -282.8\}.$$

With reference to the extended state observer for the orientation ψ , it has been designed so as the eigenvalues of the corresponding dynamical matrix $\mathbf{E}$ of the observer are:

$$\text{eig}(\mathbf{E}) = \{-14.14 \pm 14.14i, -42.42\}.$$

With reference to the controllers for positioning and orientation (yaw), they have been designed by means of the pole assignment technique so that the polynomial (8.24) is equal to $(s + 2.5)^4$, for positioning, and $(s + 2)^2$, for orientation.

The differentiators are designed in order to constraint the forces generated by the four propellers to the interval $[0, f_{max}]$ in all the operative conditions, where $f_{max} = 5\text{N}$ represents the maximum thrust that a single motor-propeller can generate. In order to satisfy these requirements the coefficients p_j of matrix $\mathbf{A}$ in (8.28) are such that:

$$\text{eig}(\mathbf{A}) = \{-1, -5, -8, -15 \pm 15i\},$$

while, for orientation, a ND has ben considered such that:

$$\text{eig}(\mathbf{A}) = \{-1, -5, -10\}.$$

8.4.1 Data fusion of IMU and GPS measurements

In this subsection it is discussed the problem of data fusion between measurements given by IMU (sampled at 100Hz) and measurements given by GPS (sampled at 10Hz). Since the measurement systems used in this field have a low update rate with respect to the control algorithm, a suitable observer with sampled measurements have to be designed. Moreover, also the fact that the time between two consecutive measurements is not constant, but it can vary randomly, have to be considered. For these reasons the data fusion process between data given by the above mentioned devices is governed by the hybrid nonlinear observer described in [1], based on the strap-down model of the IMU. In particular, the observer combines a Nonlinear Luenberger-Type Observer (NLTO), which gives the state estimate, and an Updating Mechanism (UM) which carries out a corrective action on the NLTO when new position and velocity vector measurements arrive from GPS. The NLTO is realized so that its update avoids discontinuities in the estimate. For further details the reader is addressed to [1].

8.4.2 Simulation results

The control system is tested by means of simulations in Matlab/Simulink environment. The mathematical model implemented for simulation, is the standard one considered in the literature. The implementation of the controller is carried out taking into account the hybrid nonlinear observer briefly described above. The comparison is carried out for nominal parameters and in presence of aerodynamic disturbances. The results are shown

in Figs. 8.4-8.11, and they are referred to two control laws, i.e. the FL based on the analytical computation of the disturbance vector, and the proposed control law based on the estimation of the disturbance vector, carried out by means of the ESOs. In particular, Figs. 8.4-8.7 show the waveforms of the tracking errors of the coordinates of the center of gravity and the yaw angle with respect to the corresponding reference values. The forces generated by the four propellers corresponding to the considered experiment are shown in Figs. 8.8-8.11. The results show the effectiveness of the proposed approach over the standard FL. In fact, with the proposed robust control law, the quadcopter tracks the trajectory with a high dynamic accuracy, whereas the standard FL control law produces tracking errors greater than those corresponding to the proposed robust control law. The simulation results, confirm that the forces required to the propellers are effectively feasible. Finally, The simulation results show that the aerodynamic forces and moments are correctly estimated and compensated by the proposed control technique. Obviously, in absence of aerodynamic forces and moments or other unknown endogenous disturbances, such as parametric uncertainties, the two control laws give the same results since the estimated disturbance vector is equal to the computed one.

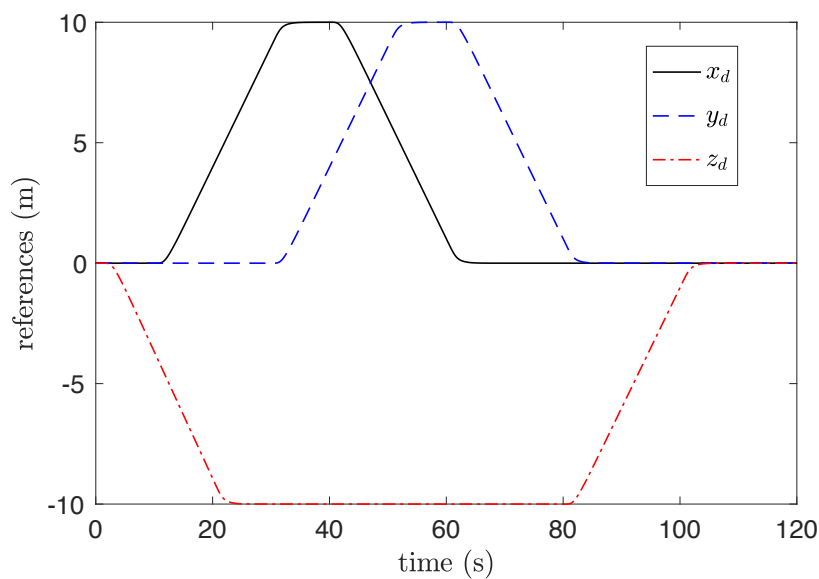


Figure 8.2: x , y and z reference components of the quadrotor during the simulation experiment.

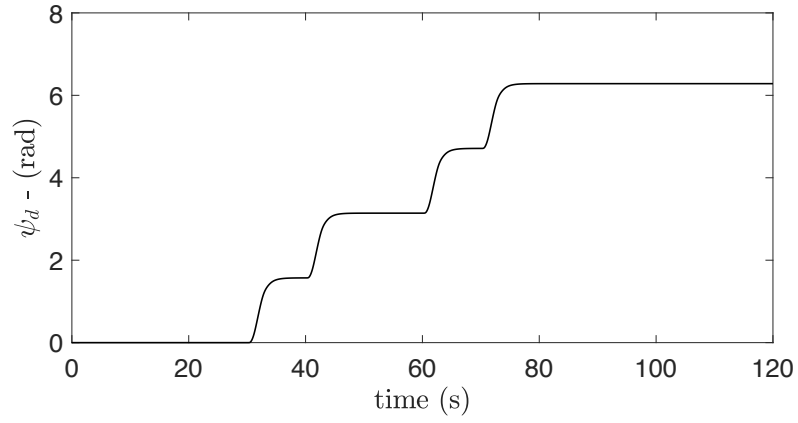


Figure 8.3: Reference yaw angle of the quadrotor during the simulation experiment.

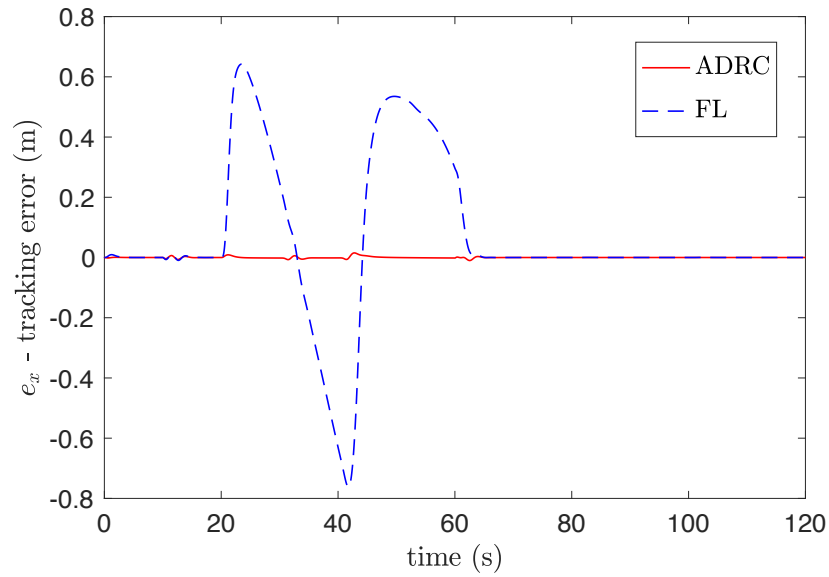


Figure 8.4: Tracking errors $e_x = x_d - x$ relative to ADRC and standard FL; nominal parameters and presence of aeronautic disturbances.

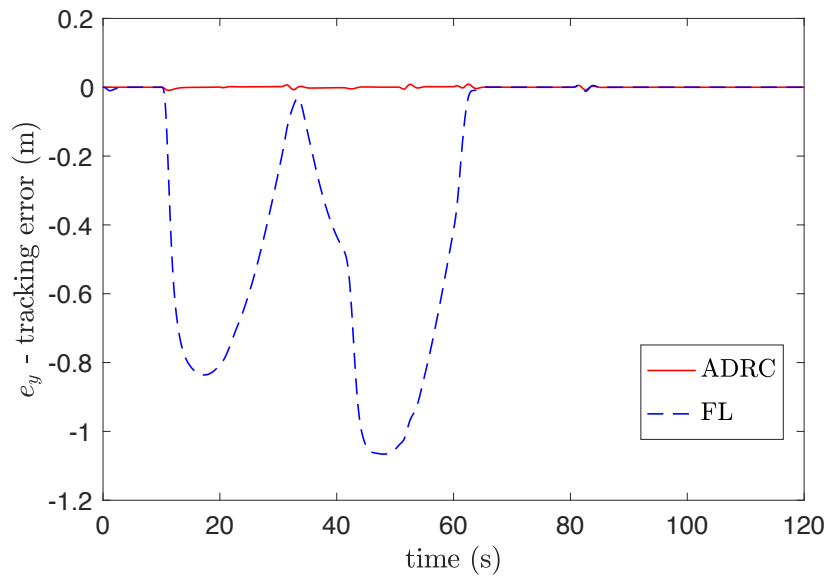


Figure 8.5: Tracking errors $e_y = y_d - y$ relative to ADRC and standard FL; nominal parameters and presence of aeronautic disturbances.

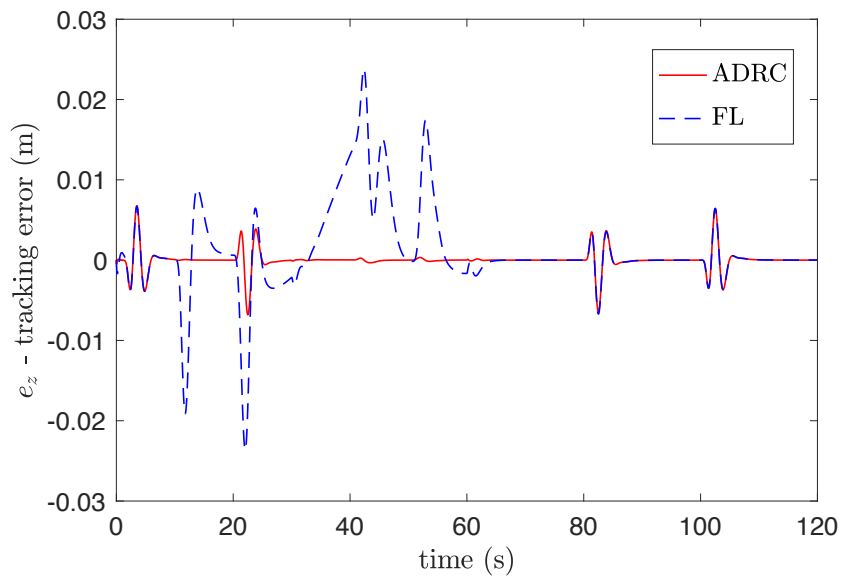


Figure 8.6: Tracking errors $e_z = z_d - z$ relative to ADRC and standard FL; nominal parameters and presence of aeronautic disturbances.

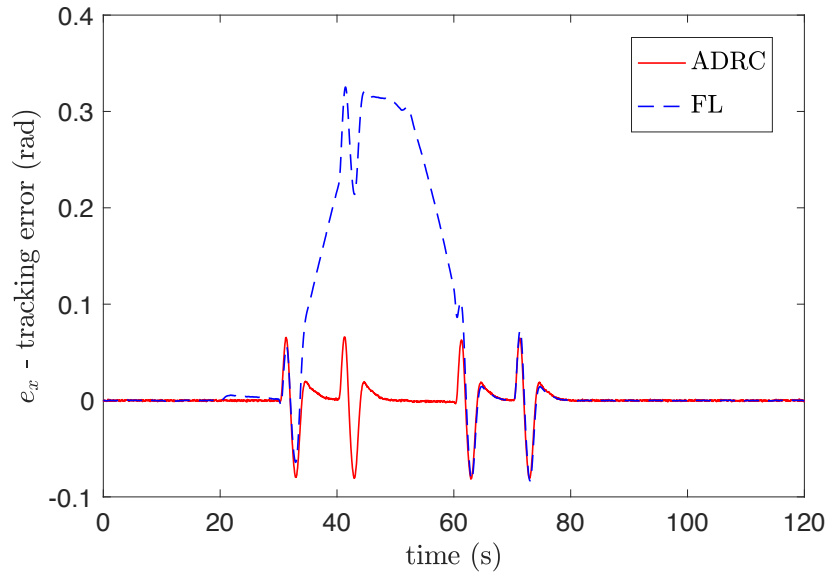


Figure 8.7: Tracking errors $e_\psi = \psi_d - \psi$ relative to ADRC and standard FL; nominal parameters and presence of aeronautic disturbances.

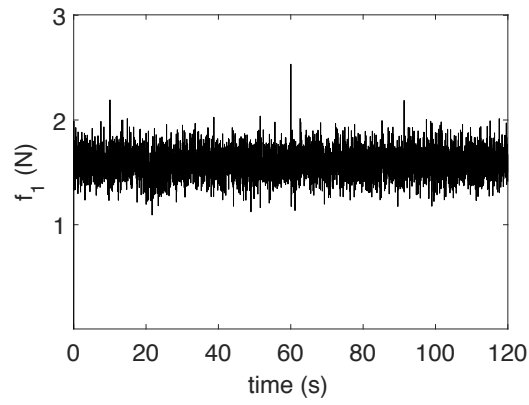


Figure 8.8: Force generated by the propeller 1 during the ADRC simulation experiment.

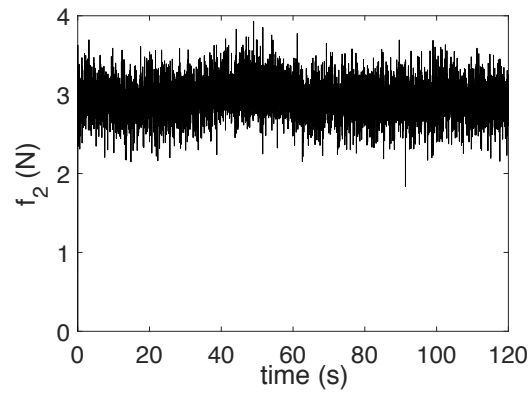


Figure 8.9: Force generated by the propeller 2 during the ADRC simulation experiment.

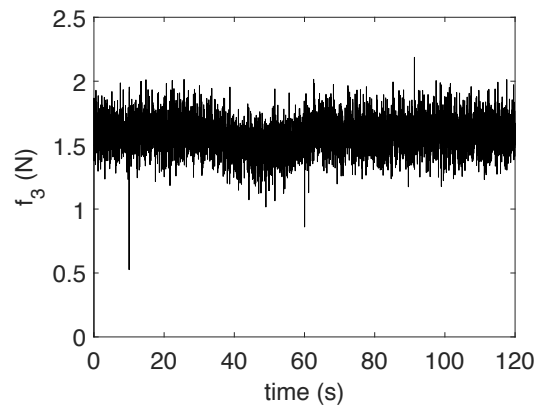


Figure 8.10: Force generated by the propeller 3 during the ADRC simulation experiment.

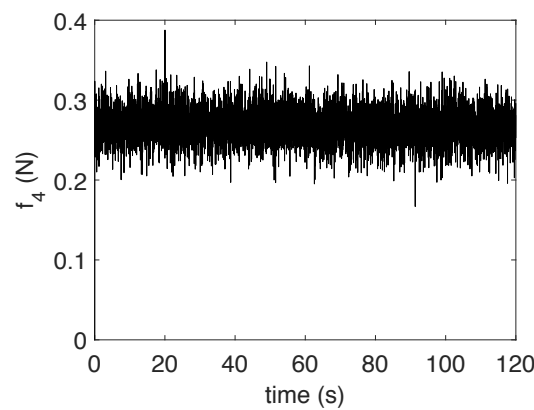


Figure 8.11: Force generated by the propeller 4 during the ADRC simulation experiment.

APPENDIX

The elements of $\mathbf{b}(\mathbf{x})$ are given by:

$$\mathbf{b}(\mathbf{x}) = \begin{bmatrix} b_1(\mathbf{x}) \\ b_2(\mathbf{x}) \\ b_3(\mathbf{x}) \\ b_4(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \frac{B_{11}+B_{12}+B_{13}}{mI_xI_y} \\ \frac{B_{21}+B_{22}+B_{23}}{mI_xI_y} \\ \frac{B_{31}+B_{32}+B_{33}}{mI_xI_y} \\ \frac{B_{41}+B_{42}+B_{43}}{c_\theta I_y I_z} \end{bmatrix}, \quad (\text{A-1})$$

with:

$$B_{11} = s_\theta c_\psi I_y \left(s_\phi \left(\zeta(a_p - (I_x - I_y + I_z)qr) + 2I_x \xi p \right) + I_x c_\phi \zeta(p^2 + q^2) \right) + a_x^{(2)} I_x I_y, \quad (\text{A-2})$$

$$B_{12} = c_\theta c_\psi I_x \left(-\zeta(a_q - (I_x - I_y - I_z)pr) - 2I_y \xi q \right), \quad (\text{A-3})$$

$$B_{13} = s_\psi I_y \left(c_\phi \left(-\zeta(a_p - (I_x - I_y + I_z)qr) - 2I_x \xi p \right) + I_x s_\phi \zeta(p^2 + q^2) \right), \quad (\text{A-4})$$

$$B_{21} = s_\theta s_\psi I_y \left(s_\phi \left(\zeta(a_p - (I_x - I_y + I_z)qr) + 2I_x \xi p \right) + I_x c_\phi \zeta(p^2 + q^2) \right) + a_y^{(2)} I_x I_y, \quad (\text{A-5})$$

$$B_{22} = c_\theta s_\psi I_x \left(\zeta(a_q - (I_x - I_y - I_z)pr) + 2I_y \xi q \right), \quad (\text{A-6})$$

$$B_{23} = c_\psi I_y \left(c_\phi \left(\zeta(a_p - (I_x - I_y + I_z)qr) + 2I_x \xi p \right) - I_x s_\phi \zeta(p^2 + q^2) \right), \quad (\text{A-7})$$

$$B_{31} = c_\theta s_\phi I_y \left(\zeta(a_p - (I_x - I_y + I_z)qr) + 2I_x \xi p \right), \quad (\text{A-8})$$

$$B_{32} = c_\phi c_\theta I_x I_y \zeta(p^2 + q^2) + a_z^{(2)} I_x I_y, \quad (\text{A-9})$$

$$B_{33} = s_\theta I_x \left(\zeta(a_q - (I_x - I_y + I_z)pr) + 2I_y \xi q \right), \quad (\text{A-10})$$

$$B_{41} = s_\phi I_z \left(a_q - (I_x + I_y - I_z)pr \right), \quad (\text{A-11})$$

$$B_{42} = c_\phi I_y \left(a_r + (I_x + I_y - I_z)pq \right), \quad (\text{A-12})$$

$$B_{43} = t_\theta I_y I_z \left(s_{2\phi}(q^2 - r^2) + 2c_{2\phi}qr \right). \quad (\text{A-13})$$

The elements of $\Delta(\mathbf{x})$ are given by:

$$\Delta(\mathbf{x}) = \begin{bmatrix} \delta_{11}(\mathbf{x}) & \delta_{12}(\mathbf{x}) & \delta_{13}(\mathbf{x}) & \delta_{14}(\mathbf{x}) \\ \delta_{21}(\mathbf{x}) & \delta_{22}(\mathbf{x}) & \delta_{23}(\mathbf{x}) & \delta_{24}(\mathbf{x}) \\ \delta_{31}(\mathbf{x}) & \delta_{32}(\mathbf{x}) & \delta_{33}(\mathbf{x}) & \delta_{34}(\mathbf{x}) \\ \delta_{41}(\mathbf{x}) & \delta_{42}(\mathbf{x}) & \delta_{43}(\mathbf{x}) & \delta_{44}(\mathbf{x}) \end{bmatrix}, \quad (\text{A-14})$$

with:

$$\delta_{11}(\mathbf{x}) = -\frac{c_\phi s_\theta c_\psi + s_\phi s_\psi}{m}, \quad (\text{A-15})$$

$$\delta_{12}(\mathbf{x}) = \frac{d\zeta(s_\phi s_\theta c_\psi - c_\phi s_\psi)}{mI_x}, \quad (\text{A-16})$$

$$\delta_{13}(\mathbf{x}) = -\frac{d\zeta c_\theta c_\psi}{mI_y}, \quad (\text{A-17})$$

$$\delta_{14}(\mathbf{x}) = 0, \quad (\text{A-18})$$

$$\delta_{21}(\mathbf{x}) = -\frac{-c_\phi s_\theta s_\psi + s_\phi c_\psi}{m}, \quad (\text{A-19})$$

$$\delta_{22}(\mathbf{x}) = \frac{d\zeta(s_\phi s_\theta s_\psi + c_\phi c_\psi)}{mI_x}, \quad (\text{A-20})$$

$$\delta_{23}(\mathbf{x}) = -\frac{d\zeta c_\theta s_\psi}{mI_y}, \quad (\text{A-21})$$

$$\delta_{24}(\mathbf{x}) = 0, \quad (\text{A-22})$$

$$\delta_{31}(\mathbf{x}) = -\frac{c_\phi c_\theta}{mI_y}, \quad (\text{A-23})$$

$$\delta_{32}(\mathbf{x}) = \frac{d\zeta s_\phi c_\theta}{mI_x}, \quad (\text{A-24})$$

$$\delta_{33}(\mathbf{x}) = \frac{d\zeta s_\theta}{mI_y}, \quad (\text{A-25})$$

$$\delta_{34}(\mathbf{x}) = 0, \quad (\text{A-26})$$

$$\delta_{41}(\mathbf{x}) = 0, \quad (\text{A-27})$$

$$\delta_{42}(\mathbf{x}) = 0, \quad (\text{A-28})$$

$$\delta_{43}(\mathbf{x}) = \frac{ds_\phi}{I_y c_\theta}, \quad (\text{A-29})$$

$$\delta_{44}(\mathbf{x}) = \frac{dc_\phi}{I_z c_\theta}. \quad (\text{A-30})$$

Chapter 9

Conclusions

In this Thesis the problem of Real Time Localization System observer design has been addressed and thoroughly discussed using tools from the Hybrid Framework and non linear system theory. After an introduction in Chapter 1 on the state of art regarding the ranging systems, wireless communication technologies, modeling and observing techniques, in Chapter 2 the implementation of the experimental setup for both indoor and outdoor localization applications has been addressed.

In Chapter 3 an indoor position estimation algorithm has been proposed. The position has been measured by means of a sensor network composed by fixed beacons placed on the indoor environment and a mobile beacon mounted on the object to be tracked. Moreover also inertial measurements have been used when the position measurements are not available. Two main problems have been considered in the proposed architecture: the fact that the beacons work with a lower update rate than the IMU, and that the mobile beacon can communicate with only one fixed beacon at a time. The experimental results presented shown the effectiveness of the proposed technique, and its capability to deal with the two above cited problems.

Since the measurement systems used in the RTLS field have a low update rate with respect to the control algorithm, a hybrid nonlinear observer with sampled measurements has been proposed in Chapter 4. Moreover, also the fact that the time between two consecutive measurements is not constant, but it can vary randomly, has been considered. The theoretical treatment given, constitutes an important tool for the tuning of the observer. Moreover the sensitivity analysis has been carried out, and it permits to quantify the upper bound of the estimation error when bounded errors of the acceleration measurement and of the angular velocity are considered. Simulation and experimental results showed the feasibility of the proposed observer and the good behavior, in terms of

estimation error, that can be achieved. In particular it was shown how the proposed observer merges properly the measurements with different update rates, and how the model is suitably used so that all the available information is exploited in order to provide an estimate as close as possible to the true value.

In Chapter 5 a new scheme for a range-based localization system has been given. The proposed approach is very suitable for applications in which the GPS fails, like indoor or urban environments. It has been shown that the method based on the direct kinematics problem for parallel robot is very appealing because: a) it is a deterministic method, easily understandable and easy to implement; b) it is an iterative method based on a closed loop control scheme; c) it is possible to prove the exponential convergence to zero of the estimation error as well as an explicit tuning rule to ensure a well defined convergence speed. In particular, the possibility to fix a-priori the convergence speed of the estimation error represents an appealing result from the practical point of view compared with existing methods proposed in literature based on trilateration, where these types of theoretical certificates cannot be given.

In Chapter 6 a novel estimation strategy for inertial navigation in indoor/outdoor environments has been proposed with specific attention to the sporadic nature of the sampled-data measurements. After introducing the inertial navigation model, an observer providing exponentially converging position and velocity estimates has been proposed. A hybrid dynamical systems framework has been used for presenting the results. The main theorem proves uniform exponential convergence to zero of the estimation error, together with an intuitive selection of the observer gains. Hardware-in-the-loop experiments confirmed the effectiveness of the proposed approach and the capability of the described observer to also deal with measurement noise.

In Chapter 7 a hybrid observation scheme based on a dynamic augmentation to cope with position-only sporadic measurements for inertial navigation in indoor/outdoor environments have been proposed. A dynamic model of the system subject to a (continuous-time) unknown disturbance and sampled-data measurements subject to (continuous-time) measurement noise have been proposed. The resulting continuous-discrete disturbance/noise inputs characterize a finite gain ISS bound on the estimation error. One advantage of the proposed approach is that the observer gains can be conveniently tuned, as they appear explicitly in the established bounds. Experimental results are reported, that confirm the effectiveness of the proposed approach.

Finally in Chapter 8 a systematic procedure is given for determining a robust motion control law for autonomous quadcopter vehicles. This procedure considers advanced control techniques, such as the input-output linearization and the active disturbance re-

jection control. These techniques justify the use of the Brunovsky canonical form as the starting point for designing a controller for the motion control of the autonomous quadcopter. Then, it is possible to split the nonlinear augmented model into four decoupled models, three of which are of fourth order and one of second order. Starting from each of these models, it is possible to construct the relative ESO and to impose the dynamics to the model by means of pole assignment technique. As a final result, a robust control law is obtained which outperforms the standard feedback linearization control. While a quantitative expression of this performance improvement is not easy to be provided, the obtained simulation results highlight that a high decrease of the tracking errors during transients has been achieved. The reason for this is that the endogenous and exogenous disturbances have been considered as further state variables estimated by means of extended state observers, permitting the non-linearity of the plant and disturbances to be compensated. It follows that the proposed methodology could be considered as an "adaptive robust version" of the feedback linearization control technique, since the state feedback terms are estimated on-line. In this way, not only the problems due to the uncertainties on the parameters have been addressed, but it is also possible to cope with unmodelled dynamics and exogenous disturbances (such as wind and aerodynamic disturbances). Simulation experiments, carried out on a Rapsberry-PI board by means of the hardware-in-the-loop method, validated the approach followed in the work, and has showed the better performance achieved with the proposed controller compared with standard feedback linearization. The experimental validation of this methodology represents a future development in order to show the effectiveness of the proposed approach.

In conclusion, a set of mathematical tools from control theory have been successfully used in Real Time Localization System to design observer and control law suitable for the autonomous navigation. Certainly this work is not a complete treatment on the topic but some contribution to improve the performance of this kind of system has undoubtedly been made.

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