

PAPER: nesmcq18

New trends in nonequilibrium statistical mechanics: classical and quantum systems

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Received 22 July 2020

Accepted for publication 7 August 2020

Published XX XX XXXX

Online at stacks.iop.org/JSTAT/2020/000000

<https://doi.org/10.1088/1742-5468/abaed0>



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Abstract. The main aim of this special issue is to report recent advances and new trends in nonequilibrium statistical mechanics of classical and quantum systems, from both theoretical and experimental points of view, within an interdisciplinary context. In particular, the nonlinear relaxation processes in the dynamics of out-of-equilibrium systems and the role of the metastability and environmental noise will be overviewed. Three main areas of nonequilibrium statistical mechanics will be covered: slow relaxation phenomena and dissipative dynamics; long-range interactions and classical systems; quantum systems. New trends such as quantum thermodynamics and novel types of quantum phase transitions occurring in nonequilibrium steady states and topological phase transitions will be also highlighted.

Keywords: metastable states, quantum phase transitions, topological phases of matter

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1. Introduction

Nonlinear relaxation phenomena in the presence of metastable states are ubiquitous phenomena in condensed matter systems and often characterize the dynamics of nonequilibrium systems. Specifically, metastability is a universal phenomenon in many natural systems, ranging from physics to cosmology, chemistry, biology and ecology [1–3]. Indeed, metastability is an active research field in statistical physics [4–15]. Understanding metastability is a fundamental issue to understand and describe the dynamical behavior of many natural systems. In fact, metastability is relevant in the dynamical behavior of complex systems, such as spin glasses and glasses, and its relevance depends on the number of metastable states, their local stability and related basin of attraction in the presence of thermal or external fluctuations [16]. It is the time scale of the basin of attraction around the investigated stable point of the system that defines the stability and metastability of state under consideration. Metastability is a signature of a first order phase transition, often characterized by a long-living metastable state, whose dynamics is typical of out-of-equilibrium systems [4–6, 11, 12, 17]. Several theoretical investigations have focused on the positive effects of the noise on nonlinear systems [18–24], showing that, under suitable conditions, the addition of external, Gaussian and non-Gaussian fluctuations to nonlinear systems can induce many noise-induced phenomena such as stochastic resonance, resonant activation, noise enhanced stability [25–43], resulting in a less noisy response of the system in many interdisciplinary applications [44–50].

Here we give an introduction to the special issue ‘new trends in nonequilibrium statistical mechanics: classical and quantum systems’, related to the workshop held in the Ettore Majorana Centre in Erice (Trapani, Sicily, Italy) from 25 to 31 July 2018. The

discussion after each presentation made the workshop into a lively, interesting scientific event with a really stimulating and participating atmosphere. The aim of this meeting was to bring together scientists interested in the challenging problems connected with dynamics of out of equilibrium classical and quantum physical systems from both theoretical and experimental point of view, within an interdisciplinary context. Specifically, three main areas of out-of-equilibrium statistical mechanics have been covered: slow relaxation phenomena and dissipative dynamics; long-range interactions and classical systems; quantum systems. Moreover, the conference has been a discussion forum to promote new ideas in this fertile research field, and in particular new trends such as quantum thermodynamics and novel types of quantum phase transitions occurring in non-equilibrium steady states, and topological phase transitions. The 39 research articles in this special topic report recent advances in the field and showcase the diversity of this rich class of problems, which constitute a very interesting and rapidly expanding interdisciplinary research field.

2. Slow relaxation phenomena and dissipative dynamics

In nature, most of dynamical phenomena are far from equilibrium, and the theory of nonequilibrium systems is nowadays a challenging area of investigation. One of the most interesting breakthroughs came out from the nonequilibrium interface growth of the Kardar–Parisi–Zhang (KPZ) equation [51] described by a nonlinear stochastic differential equation. The KPZ equation is an archetypal equation useful for describing not only interface and surface growth problems but also stochastic heat diffusion with multiplicative noise, dynamics of polymers in random media, last passage percolation, interacting particle systems, and so on. In the paper [52], asymptotic KPZ properties have been investigated in the totally asymmetric simple exclusion process with a localized geometric defect.

Another interesting discovery concerns the glass transition. However, although Anderson in 1995 [53] expected a breakthrough by the year 2005 and despite intense worldwide activity ever since, ‘the deepest and most interesting problem in solid state theory’ [53], as he labeled the glass transition, remains as puzzling as ever. Different and often controversial views on experimental observations and phenomena near the glass transition continue to compete, and Anderson’s expectation has not been fulfilled, because universally applied phenomenological fits to glassy relaxation data have still not found a convincing model-free theoretical foundation [54]. In the context of dissipative dynamics another intriguing problem is the soliton dynamics in the cubic-quintic complex Ginzburg–Landau equation in one spatial dimension [55]. This paper has highlighted the possibility of modeling the soliton dynamics as a hidden Markov process.

Finally, we mention that the equilibrium and non-equilibrium steady-state distributions of the internal state variable of a memristive system [56], the influence of fluctuations on the resistive switching, and the relaxation time to the steady-state have been analyzed in [57]. The relaxation time shows a nonmonotonic dependence, with a minimum, on the intensity of the fluctuations, paving the way for using the intensity of fluctuations as a control parameter for switching dynamics in memristive devices.

Experimental analysis of the complex dynamics of memristors and their noise-induced resistive switching have been investigated in references [58, 59].

3. Long-range interactions and classical systems

Long-range interactions are ubiquitous in nature, the most common examples being gravitational and Coulomb interactions. Recently, long-range interacting systems have been extensively studied for their, often intriguing, statistical behaviour [60–62]. In fact, these systems exhibit inequivalence of ensembles, breakdown of ergodicity, long-lived quasi-stationary states and suppression of chaos, to name just a few anomalous phenomena. Due to the paradigmatic importance of the celebrated Fermi–Pasta–Ulam (FPU) model, it is interesting to explore the effect of long-range interactions in this context. In particular, the FPU model provides an appropriate arena to explore the behaviour of systems that persist out of equilibrium for long time depending on the features of the interactions [63]. Concerning transport processes the standard scenario of heat transport in nonlinear lattices, based on numerical investigations and nonlinear fluctuating hydrodynamics, has been reviewed in [64], taking into account that the role of finite size corrections is typically amplified by peculiar long-living nonlinear excitations. In [65] experimental analysis of heat transfer coefficients in phosphoric acid concentration process has been presented. In the same context, a discrete linear-stochastic Schrödinger equation, which conserves exactly both the total energy and the total norm, has been presented in [66]. Indeed, within the vast class of nonequilibrium classical and quantum phenomena, the physics of coupled transport is a growing field with potentially revolutionary technological innovations [67, 68]. Long-range interactions influence the nonequilibrium properties of ferromagnetic systems, as shown in the phase separation process of a one-dimensional Ising model in the presence of a power-law decaying coupling studied in [69]. Considering the non-conserved dynamics, the authors have discovered that the scaling of the autocorrelation function may be affected by the presence of long-range interactions. Furthermore, the dynamics of fluctuations in the Gaussian model with conserved stochastic dynamics, in which large deviations may display the phenomenon of condensation, has been investigated in detail in [70].

In the context of information theory and processing, unifying theories for nonequilibrium statistical mechanics have been presented in [71]; work extraction, information-content and the Landauer bound in the continuous Maxwell Demon have been analyzed in an experimental paper [72]. Concerning unstable and metastable systems, stochastic dynamics in highly unstable systems has been studied in [73]. Indeed, unstable stochastic dynamics is ubiquitous in many nonlinear models. Its characteristic feature is the fast divergence of trajectories that have left a metastable state [74]. In a paper of this special issue [73], the authors use ideas from the probabilistic theory of conditioned stochastic processes and derive generic behavior of non-diverging trajectories. Renyi and Tsallis entropy theory in the analysis of the multifractal nonequilibrium structures of dielectric breakdown has been applied in [75].

Mesoscopic systems, such as Josephson junctions (JJs) devices, have been investigated in references [76, 77]. In particular, magnetic vortex-ratchet effects in various

designs of asymmetric arrays of JJs ratchets have been simulated numerically and measured experimentally in [76]; the interplay between thermal transport and soliton dynamics in a temperature-biased long Josephson tunnel junction [78–81] operating in the flux-flow regime has been investigated in [77]. The out-of-equilibrium aspects of single strands in a DNA folding hairpin structure and the main features of DNA G4 thermal and mechanical stability have been analyzed in [82, 83]. Finally, in the paper [84] the generation-recombination processes of magnetic monopoles in spin ices have been theoretically investigated. Interestingly, the measurements of the spectral density of the magnetic flux noise in reference [85] gave experimental confirmation of the theoretical results obtained by the authors in [84].

4. Quantum systems

Open quantum systems, that is quantum systems that exchange energy or particles with their environment, are ubiquitous in a wide variety of fields including condensed matter physics, quantum information, quantum optics, and quantum thermodynamics [86, 87]. The coupling to the environment gives rise to dynamical processes such as fluctuations, dephasing, relaxation, thermalization, nonequilibrium excitation, and transport. The understanding of these processes is a major goal in the field of theoretical physics. In this context, quantum phase transitions play a fundamental and challenging role in quantum statistical physics and in quantum many-body system dynamics near the critical points. In particular, topological phase transitions (TPTs) have emerged as a new paradigm since they do not fall under the Landau theory description, where phases are characterized by local order parameters and symmetry breaking occurring across criticalities. Topological phases indeed are identified in the bulk by topological invariants, i.e. quantities that only depend on the topology, that are constructed out of ground states properties [88]. Moreover, topological systems have attracted intense interest because of their peculiar properties, such as topologically protected edge states or exotic statistics excitations [89]. New materials have been discovered and experiments are performed in the direction of probing anyonic excitations, which have promising applications in many other areas such as quantum computing. Indeed, the topological ordered phases (TOPs) of matter are a new and interesting field of research that has recently attracted the interest of many physicists. In particular, the role of temperature at thermal equilibrium or mixedness in out-of-equilibrium systems on the TOPs has recently received increasing attention [90, 91]. Among different proposed approaches, that by Uhlmann [92] to generalize the geometric Berry phase to mixed states seems to be one of the most promising [93]. The Uhlmann geometric phase and its related quantities have been recently used as a starting point to describe the topological phases of systems in a mixed-state configuration [94–99]. In this context, an interesting example of 2D topological insulator which can be described by the Haldane model [100] has been investigated in reference [101]. There, the phases of the Haldane model have been studied at finite temperature by means of the Uhlmann number, directly connected to the Uhlmann geometric phase. The critical properties of the Berry curvature in the Kitaev honeycomb model have been analyzed in [102]. Quantum metrology finds applications in a wide variety of fields, from

fundamental physics, to gravitational wave interferometry, thermometry, spectroscopy, imaging, to name a few [103]. In multi-parameter quantum estimation protocols, several variables are simultaneously evaluated, in a way which may outperform individual estimation strategies with equivalent resources, thereby motivating the use of such protocols in a variety of diverse contexts. Exploring this possibility plays a pivotal role in the current development of quantum technology [103, 104]. In the paper [103], a new approach has been introduced to quantitatively derive a measure of quantumness in quantum multi-parameter estimation problems by defining a figure of merit, that is, the ratio between the mean Uhlmann curvature and the Fisher information. This quantity estimates the amount of incompatibility arising from the quantum nature of the underlying physical system. In the same context, the ability to generate long-lived potentials on topological insulators has given the opportunity to manipulate the spin-momentum locked topological surface states using the time and angle-resolved photoemission spectroscopy (T-ARPES) [108]. Moreover, a systematic study of quantum statistics and dynamics of a pair of anyons in the lowest Landau level, of direct relevance to quasiparticle excitations in the quantum Hall bulk, has been presented in [109]. Dynamical correlations and a quantum glass phase in a random hopping Bose–Hubbard model has been investigated in [110].

In the context of quantum phase transitions, quenches and quantum thermodynamics, the out-of-equilibrium dynamics, after a quench, has been investigated in Ising chains with long-range interactions [105], in quantum harmonic chains [106] and in Motzkin and Fredkin spin chains [107]. Moreover, the maximal amount of energy that can be extracted from a quantum system via projective measurements has been considered in [111]; and the slow heating in a quantum coupled kicked rotors system has been studied [112].

As far as quantum dissipative systems are concerned, quantum metastability has been investigated in a micromaser model based on a solid-state setup with a double quantum dot coupled to a microwave cavity [113]; the quantum jump statistics with a shifted jump operator in a chiral waveguide has been presented in [114]; the superstatistics of energy for a free quantum Brownian motion has been considered in [115]; the transmission spectra of a flux qubit coupled to a dissipative resonator in the ultrastrong coupling regime has been calculated in [116]; the coherent trapping in small quantum networks has been considered in [117]; the charge carrier density noise in graphene has been investigated in [118]; the nonequilibrium electron spin relaxation in n-type doped GaAs bulk semiconductor has been investigated in [119]. In particular, in this last paper [119] a nonmonotonic behavior has been found with a maximum of both the spin lifetime and the depolarization length, as a function of the lattice temperature. This peculiar behavior, due to the nonlinear spin-lattice interaction through the phonon thermal bath, is akin to the noise-enhanced stability (NES) phenomenon, theoretically and experimentally investigated in classical and quantum systems [4, 5, 8, 11, 12, 14, 32–34]. The NES effect is one of the noise-induced phenomena relevant for understanding the effect of temperature on the transient dynamics of nonequilibrium complex systems.

Acknowledgments

This work was supported by the Government of the Russian Federation through Agreement No 074-02-2018-330 (2), and partially by Ministry of Education, University and Research of Italian Government.

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