Electron Structure, Ultra-Dense Hydrogen and Low Energy Nuclear Reactions

2 authors:

Antonino Oscar Di Tommaso
Università degli Studi di Palermo

Giorgio Vassallo
Università degli Studi di Palermo

J. Condensed Matter Nucl. Sci. 29 (2019) 525–547

Research Article

Electron Structure, Ultra-dense Hydrogen and Low Energy Nuclear Reactions

Antonino Oscar Di Tommaso and Giorgio Vassallo*,†
Università degli Studi di Palermo, Dipartimento di Ingegneria (DI), Viale delle Scienze, 90128 Palermo, Italy

Abstract

In this paper, a simple Zitterbewegung electron model, proposed in a previous work, is presented from a different perspective based on the principle of mass−frequency equivalence. A geometric−electromagnetic interpretation of mass, relativistic mass, De Broglie wavelength, Proca, Klein−Gordon, Dirac and Aharonov−Bohm equations in agreement with the model is proposed. A non-relativistic, Zitterbewegung interpretation of the 3.7 keV deep hydrogen level found by J. Naudts is presented. According to this perspective, ultra-dense hydrogen can be conceived as a coherent chain of bosonic electrons with protons or deuterons located in the center of their Zitterbewegung orbits. This approach suggests a possible role of ultra-dense hydrogen in some aneutronic and many-body low energy nuclear reactions.

Keywords: Aharonov–Bohm equations, Aneutronic and many-body low energy nuclear reactions, Compact structures, De Broglie wavelength, Electron structure, Dirac equation, ESR, Heisenberg's uncertainty principle, Klein–Gordon equation, Josephson constant, LENR, natural units, Proca equation, relativistic mass, Ultra-dense hydrogen, Zitterbewegung

Nomenclature

$\gamma_x^2 = \gamma_y^2 = \gamma_z^2 = -\gamma_t^2 = 1$ were $\{\gamma_x, \gamma_y, \gamma_z, \gamma_t\}$ are the four basis vectors of $Cl_{3,1}(\mathbb{R})$ Clifford algebra, isomorphic to Majorana matrices algebra [1]

$\gamma_i\gamma_j = -\gamma_j\gamma_i$ with $i \neq j$ and $i, j \in \{x, y, z, t\}$;

$\boldsymbol{\partial} = \gamma_x \frac{\partial}{\partial x} + \gamma_y \frac{\partial}{\partial y} + \gamma_z \frac{\partial}{\partial z} + \gamma_t \frac{1}{c}\frac{\partial}{\partial t}$

$I = \gamma_x\gamma_y\gamma_z\gamma_t$

$I_\Delta = \gamma_x\gamma_y\gamma_z$

1. Introduction

According to Carver Mead, mainstream physics literature has a long history of hindering fundamental conceptual reasoning, often *"involving assumptions that are not clearly stated"* [2]. One of these is the unrealistic assumption of

*Corresponding author. E-mail: giorgio.vassallo@unipa.it.
†Also at: International Society for Condensed Matter Nuclear Science (ISCMNS).

Nomenclature

Symbol	Name	SI units	Natural units (NU)
$\boldsymbol{A}_\square$	Electromagnetic four-potential	$\mathrm{V\ s\ m^{-1}}$	eV
$\boldsymbol{A}_\triangle$	Electromagnetic vector potential	$\mathrm{V\ s\ m^{-1}}$	eV
A_t	Time component of electromagnetic four potential	$\mathrm{V\ s\ m^{-1}}$	eV
A	Electromagnetic vector potential module	$\mathrm{V\ s\ m^{-1}}$	eV
m	Mass	kg	eV
$\boldsymbol{F}$	Electromagnetic field bivector	$\mathrm{V\ s\ m^{-2}}$	$\mathrm{eV^2}$
$\boldsymbol{B}$	Flux density field	$\mathrm{V\ s\ m^{-2}} = T$	$\mathrm{eV^2}$
$\boldsymbol{E}$	Electric field	$\mathrm{V\ m^{-1}}$	$\mathrm{eV^2}$
V	Potential energy	$\mathrm{J{=}kg\ m^2\,s^{-2}}$	eV
$\boldsymbol{J}_\square$	Four current density field	$\mathrm{A\ m^{-2}}$	$\mathrm{eV^3}$
$\boldsymbol{J}_\triangle$	Current density field	$\mathrm{A\ m^{-2}}$	$\mathrm{eV^3}$
ρ	Charge density	$\mathrm{A\ s\ m^{-3} = C\ m^{-3}}$	$\mathrm{eV^3}$
x, y, z	Space coordinates	m^*	$\mathrm{eV^{-1}}$
t	Time variable	$\mathrm{s}^\dagger$	$\mathrm{eV^{-1}}$
c	Light speed in vacuum	$2.997\,924\,58 \times 10^8\ \mathrm{m\ s^{-1}}$	1
$\hbar$	Reduced Planck constant	$1.054\,571\,726 \times 10^{-34}\ \mathrm{J\ s}$	1
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}\ \mathrm{V\ s\ A^{-1}\ m^{-1}}$	4π
ϵ_0	Dielectric constant of vacuum	$8.854\,187\,817{\times}10^{-12}\ \mathrm{A\ s\ V^{-1}\ m^{-1}}$	$\frac{1}{4\pi}$
e	Electron charge	$1.602\,176\,565 \times 10^{-19}\ \mathrm{A\ s}$	$0.085\,424\,546$
α	Fine structure constant	$7.2973525664 \times 10^{-3}$	$7.2973525664 \times 10^{-3}$
m_e	Electron rest mass	$9.10938356 \times 10^{-31}\mathrm{kg}$	$0.5109989461 \times 10^6\ \mathrm{eV}$
λ_c	Electron Compton wavelength	$2.426\,310\,2389 \times 10^{-12}\ \mathrm{m}$	$1.229\,588\,259 \times 10^{-5}\ \mathrm{eV^{-1}}$
K_J	Josephson constant	$0.4835978525 \times 10^{15}\ \mathrm{Hz\ V^{-1}}$	$2.71914766 \times 10^{-2}$
r_e	Reduced Compton electron wavelength (Compton radius)	$r_\mathrm{e} = \frac{\lambda_\mathrm{c}}{2\pi}$	
r_c	Electron charge radius	$r_\mathrm{c} = \alpha r_\mathrm{e}$	
T_e	Zitterbewegung period	$T_\mathrm{e} = \frac{2\pi r_\mathrm{e}}{c}$	

$^*1.9732705{\times}10^{-7}\ \mathrm{m} \simeq 1\ \mathrm{eV^{-1}}$.
$^\dagger\ 6.5821220 \times 10^{-16}\ \mathrm{s} \simeq 1\ \mathrm{eV^{-1}}$.

point-like shaped elementary particles with *intrinsic* properties as mass, charge, angular momentum, magnetic moment and spin. According to the laws of mechanics and electromagnetism, a point-like particle cannot have an “intrinsic angular momentum". Moreover, a magnetic moment must necessarily be generated by a current loop, that cannot exist in a point-like particle. Furthermore, the electric field generated by a point-like charged particle should have an infinite energy. Therefore, an alternative realistic approach that fully addresses these *very basic* problems is indispensable. A possibility is given by a *Zitterbewegung* interpretation of quantum mechanics, according to which charged elementary particles can be modeled by a current ring generated by a massless charge distribution rotating at light speed along a circumference whose length is equal to particle Compton wavelength [3,4]. As a consequence, every elementary charge is always associated with a magnetic flux quantum and every charge is coupled to all other charges on its light cone by time-symmetric interactions [2]. The aim of this paper is to present a gentle introduction to an electron Zitterbewegung model together with some observations that deems to reinforce its plausibility.

The present paper is structured in the following way. In Section 2 the deep connection between some basic concept as space, time, energy, mass, frequency, and information is exposed. In Section 3 an introduction to a Zitterbewegung electron model is presented, together with a geometric-electromagnetic interpretation of Proca, Klein–Gordon, Dirac and Aharonov–Bohm equations. In Section 4 a simple geometric interpretation of relativistic mass and De Broglie wavelength is proposed. In Section 5 the relation of Electronic Spin Resonance (ESR) frequency with Larmor precession frequency of the Zitterbewegung orbit is presented. Finally, in Section 6 some hypotheses on the structure of ultra-dense hydrogen are formulated, whereas Section 7 deals with the possible role of ultra-dense hydrogen in low energy nuclear reactions.

N.B. In this paper all equations enclosed in square brackets with subscript "NU" have dimensions expressed in natural units. The mathematical notation used in Sections 3.3–3.5, based on real Clifford algebra $Cl_{3,1}(\mathbb{R})$, is introduced in [1].

2. Energy, Mass, Frequency and Information

The concept of measurement plays a fundamental role in all scientific disciplines based on experimental evidence. The most used measurement units (such as the international system, SI) are based mainly on human conventions not directly related to fundamental constants. To simplify the conceptual understanding of certain physical quantities it is convenient to adopt in some cases a measurement system based on universal constants, such as the speed of light c and the Planck's quantum $\hbar$.

Considering that a measure is an event localized in space and time, the quantum of action can be seen, in some cases, as an objective entity in some respects analogous to a bit of information located in the space-time continuum. In accordance with Heisenberg's uncertainty principle, the result of the measurement of some values (such as angular momentum) cannot have an accuracy less than half a single Planck's quantum. Therefore, to simplify the interpretation of physical quantities, it may be useful to adopt a system in which both the speed of light and the quantum of action are dimensionless quantities (pure numbers) having a unit value, i.e.: $c = 1$ and $\hbar = 1$. In this system, the constancy of light speed makes possible to use a single measurement unit for space and time, simplifying, in many cases, the conceptual interpretation of physical quantities. The energy of a photon, a "particle of light", is equal to Planck's quantum multiplied by the photon angular frequency. By using the symbol T to indicate the period of a single complete oscillation and λ the relative wavelength, it is, therefore, possible to write

$$E = \hbar\omega = \frac{2\pi\hbar}{T} = \frac{2\pi\hbar c}{\lambda}. \tag{1}$$

By using natural units, period and wavelength coincide and the above expression is simplified in

$$\left[E = \omega = \frac{2\pi}{T} = \frac{2\pi}{\lambda}\right]_{\text{NU}}. \tag{2}$$

The subscript NU highlights the use of natural units for expressions contained within square brackets. This equation indissolubly links some fundamental concepts, as space, time, energy and mass, giving the possibility to express an energy value simply as a frequency or as the inverse of a time, or even as the inverse of a length. Vice versa, it allows to use as a measurement unit of both space and time a value equal to the inverse of a particular energy value as the electron-volt. Therefore, to compute photon wavelength in vacuum with natural units it is sufficient to divide the constant 2π by its energy. This value will correspond exactly to the period of a complete oscillation. Hence, in natural units the inverse of an eV can be used as a measurement unit for space and time:

$$L_{(1\,\mathrm{eV})} = 1\ \mathrm{eV}^{-1} \approx 1.9732705\ \times\ 10^{-7}\ \mathrm{m} \approx 0.2\ \mu\mathrm{m},$$

$$T_{(1\,\mathrm{eV})} = 1\ \mathrm{eV}^{-1} \approx 6.582122\ \times\ 10^{-16}\ \mathrm{s} \approx 0.66\ \mathrm{fs}.$$

Consequently, an angular frequency can be measured in electron volts:

$$1\ \mathrm{eV} \approx 1.519268\ \times\ 10^{15}\ \mathrm{rad\,s}^{-1}.$$

Following these concepts, it is possible to define a link between fundamental concepts of information, space, time, frequency and energy. A “quantum of information” carried by a single photon will have a “necessary reading time” and a “spatial dimension” inversely proportional to its energy. A simple example is given by radio antennas (dipoles), whose length is proportional to the received (or transmitted) “radio photons” wavelength and inversely proportional to their frequency and to the number of bits that can be received in a unit of time. In this perspective, the concept of energy is closely linked to the “density” of information in space and in time.

3. Electron Structure

The famous Einstein’s formula $E = mc^2$ becomes particularly explanatory if expressed in natural units:

$$[E = m]_{\mathrm{NU}}\,.$$

Mass is energy and it is, therefore, possible to associate a precise amount of energy to a particle having a given mass. Taking up the considerations made on the deep bond existing between the concepts of space, time, frequency and energy, it is interesting trying to associate the electron rest mass m_{e} to an angular frequency ω_{e}, a length r_{e} and a time T_{e}. In fact Einstein’s formula can be expressed as

$$E_{\mathrm{e}} = m_{\mathrm{e}}c^2 = \hbar\omega_{\mathrm{e}} = \frac{\hbar c}{r_{\mathrm{e}}} = \frac{h}{T_{\mathrm{e}}} \tag{3}$$

or adopting natural units

$$\left[E_{\mathrm{e}} = m_{\mathrm{e}} = \omega_{\mathrm{e}} = \frac{1}{r_{\mathrm{e}}} = \frac{2\pi}{T_{\mathrm{e}}}\right]_{\mathrm{NU}}. \tag{4}$$

These constants have a simple and clear interpretation if one accepts a particular electron model consisting of a current ring generated by a massless charge rotating at the speed of light along a circumference whose radius is equal to the electron reduced Compton wavelength, defined as $r_{\mathrm{e}} = \frac{\lambda_{\mathrm{c}}}{2\pi} \approx 0.38616\ \times\ 10^{-12}$ m [3–6]. According to the model described in [4] the charge is not a point-like entity, but it is distributed on a spherical surface whose radius is equal to the electron classical radius $r_c \approx 2.8179 \times 10^{-15}$ m . In Eq. (4) ω_e is the angular frequency of the rotating charge, r_e is its orbit radius and T_e its period. The current loop is associated with a quantized magnetic flux Φ_M equal to Planck's constant ($h = 2\pi\hbar$) divided by the elementary charge e (see Eq. (34) p. 84 [4])

$$\Phi_\mathrm{M} = h/e$$

or in natural units

$$[\Phi_\mathrm{M} = 2\pi/e]_\mathrm{NU} .$$

The rotation is caused by the centripetal Lorentz force due to the magnetic field associated with the current loop generated by the elementary rotating charge (Eq. (36)).The value of this elementary charge, in natural units, is a pure number and is equal to the square root of the ratio between the charge radius r_c and the the orbit radius r_e (see Eqs. (39) and (40) p. 85 [4]:

$$\left[e = \sqrt{\frac{r_\mathrm{c}}{r_\mathrm{e}}} = \sqrt{\alpha} \approx 0.0854245\right]_\mathrm{NU} . \tag{5}$$

Similar models, based on the concept of "current loop", have been proposed by many authors, but have often been ignored for their incompatibility with the most widespread interpretations of Quantum Mechanics [3,5–10]. It is interesting to remember how, already in his Nobel lecture of 1933, P.A.M. Dirac referred to an internal high-frequency oscillation of the electron: "*It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small*". In the scientific literature, the German word *Zitterbewegung* (ZBW) is often used to indicate this rapid oscillation/rotation of the electron charge. The rotating charge is characterized by a momentum p_c of purely electromagnetic nature:

$$p_\mathrm{c} = eA = e\frac{\Phi_\mathrm{M}}{2\pi r_\mathrm{e}} = \frac{\hbar\omega_\mathrm{e}}{c} = \frac{\hbar}{r_\mathrm{e}} = m_\mathrm{e}c.$$

In this formula the variable $A = \hbar/er_\mathrm{e}$ indicates the vector potential seen by the rotating charge (see Eq. (25), p. 82 [4]. Multiplying the charge momentum p_c by the radius r_e we obtain the "intrinsic" angular momentum $\hbar$ of the electron:

$$p_\mathrm{c}r_\mathrm{e} = \hbar. \tag{6}$$

Using natural units the momentum p_c has the dimension of energy and it is exactly equal to the electron mass–energy at rest m_e:

$$\left[p_\mathrm{c} = eA = E_\mathrm{e} = \frac{1}{r_\mathrm{e}} = m_\mathrm{e} = \omega_\mathrm{e}\right]_\mathrm{NU} .$$

3.1. Aharonov–Bohm equations and Zitterbewegung model

The magnetic Aharonov–Bohm effect is described by a quantum law that gives the phase variation φ of the "electron wave function" starting from the integral of the vector potential $\boldsymbol{A}_\triangle$ along a path [11], i.e.

$$\varphi = \frac{e}{\hbar} \int \boldsymbol{A}_\triangle \cdot \mathrm{d}\boldsymbol{l}. \tag{7}$$

In the proposed Zitterbewegung model, the electron "wave function phase" has a precise geometric meaning: the charge rotation phase. By using (7), a possible counter-test consists in verifying that the phase shift φ along the circumference of the Zitterbewegung orbit is equal exactly to 2π radians. In fact

$$\varphi = \frac{e}{\hbar} \oint \boldsymbol{A}_\triangle \cdot \mathrm{d}\boldsymbol{l} = \frac{e}{\hbar} \int_0^{2\pi r_\mathrm{e}} A \,\mathrm{d}l = \frac{e}{\hbar} \int_0^{2\pi r_\mathrm{e}} \frac{\hbar}{e r_\mathrm{e}} \,\mathrm{d}l = \frac{e}{\hbar} \frac{\hbar}{e r_\mathrm{e}} 2\pi r_\mathrm{e} = 2\pi, \tag{8}$$

because vectors $\boldsymbol{A}_\triangle$ and $\mathrm{d}\boldsymbol{l}$ have the same direction tangent to the elementary charge trajectory. This result is also consistent with the prediction of the *electric* Aharonov–Bohm effect, a quantum phenomenon that establishes the variation of phase φ as a function of the integral of electric potential V in a time interval T, i.e.:

$$\varphi = \frac{e}{\hbar} \int_T V \,\mathrm{d}t. \tag{9}$$

Applying the electric Aharonov–Bohm effect formula to compute the phase shift φ within a time interval $T_\mathrm{e} = \frac{2\pi}{\omega_\mathrm{e}}$ equal to a Zitterbewegung period we obtain the expected result, *i.e.* $\varphi = 2\pi$. In fact, the electric potential of the electron rotating charge can be expressed as

$$V = \frac{e}{4\pi\varepsilon_0 r_\mathrm{c}} = \left[\frac{e}{r_\mathrm{c}}\right]_{\mathrm{NU}}$$

and its period as

$$T_\mathrm{e} = \frac{2\pi r_\mathrm{e}}{c} = \left[2\pi r_\mathrm{e}\right]_{\mathrm{NU}}.$$

A simple calculation, applying (9) and (5), yields the same results:

$$\varphi = \frac{e}{\hbar} \int_0^{T_\mathrm{e}} V \,\mathrm{d}t = \frac{e}{\hbar} V T_\mathrm{e} = \frac{e}{\hbar} V \frac{2\pi r_\mathrm{e}}{c} = \left[\frac{e^2}{r_\mathrm{c}} 2\pi r_\mathrm{e}\right]_{\mathrm{NU}} = 2\pi. \tag{10}$$

Now, by equating the fifth term of (8) and the fourth term of (10) it is possible to demonstrate that

$$A_t = \frac{V}{c} = A = |\boldsymbol{A}_\triangle|,$$

$$\left[A_t = V = A = |\boldsymbol{A}_\triangle|\right]_{\mathrm{NU}},$$

$$\boldsymbol{A}_\square^2 = (\boldsymbol{A}_\triangle + \gamma_t A_t)^2 = \boldsymbol{A}_\triangle^2 - A_t^2 = 0. \tag{11}$$

By introducing the differential form of (9) we obtain

$$\mathrm{d}\varphi = \frac{e}{\hbar} V \,\mathrm{d}t$$

and this yields the phase speed

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \omega_\mathrm{e} = \frac{e}{\hbar} V = \frac{e^2}{4\pi\varepsilon_0 \hbar r_\mathrm{c}} = \frac{c\alpha}{r_\mathrm{c}} = \frac{c}{r_\mathrm{e}} = \frac{m_\mathrm{e} c^2}{\hbar} = \frac{ce}{\hbar} A,$$

$$\left[\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \omega_\mathrm{e} = m_\mathrm{e} = eV = eA\right]_\mathrm{NU}. \tag{12}$$

3.2. Proca equation and Zitterbewegung electron model

A deep connection of Maxwell's equations (see Eq. (97), p. 121 [1])

$$\boldsymbol{\partial}\left(\boldsymbol{\partial} \wedge \boldsymbol{A}_\square\right) + \mu_0 \boldsymbol{J}_\square = 0 \tag{13}$$

with Proca equation for a particle of mass m

$$\boldsymbol{\partial}\left(\boldsymbol{\partial} \wedge \boldsymbol{A}_\square\right) + \left(\frac{mc}{\hbar}\right)^2 \boldsymbol{A}_\square = 0, \tag{14}$$

$$\left[\boldsymbol{\partial}\left(\boldsymbol{\partial} \wedge \boldsymbol{A}_\square\right) + m^2 \boldsymbol{A}_\square = 0\right]_\mathrm{NU} \tag{15}$$

emerges if we prove that equation $\left[\mu_0 \boldsymbol{J}_\square = m^2 \boldsymbol{A}_\square\right]_\mathrm{NU}$ can be applied to the electron Zitterbewegung model introduced in [4]. In this model the electron's charge orbit delimits a disc-shaped volume with radius r_e and height $2r_\mathrm{c}$. Inside this volume the *average* Zitterbewegung current density $\bar{\boldsymbol{J}}_\mathrm{e}$ can be computed dividing the Zitterbewegung current by one half the disc vertical section $\mathcal{A}$:

$$\bar{\boldsymbol{J}}_\mathrm{e} = \frac{\boldsymbol{I}_\mathrm{e}}{\mathcal{A}},$$

where

$$\mathcal{A} = 2 r_\mathrm{e} r_\mathrm{c} = 2\alpha r_\mathrm{e}^2,$$

$$\bar{\boldsymbol{J}}_\mathrm{e} = \frac{\boldsymbol{I}_\mathrm{e}}{\mathcal{A}} = \frac{\boldsymbol{I}_\mathrm{e}}{2\alpha r_\mathrm{e}^2}. \tag{16}$$

From [4], p. 82, we have that

$$\left[\boldsymbol{I}_\mathrm{e} = \frac{\alpha \boldsymbol{A}_\triangle}{2\pi}\right]_\mathrm{NU}$$

and substituting it in (16) we get

$$\left[\bar{\boldsymbol{J}}_{\mathrm{e}} = \frac{\boldsymbol{A}_{\vartriangle}}{4\pi r_{\mathrm{e}}^2}\right]_{\mathrm{NU}},$$

$$\left[\mu_0 \bar{\boldsymbol{J}}_{\mathrm{e}} = 4\pi \bar{\boldsymbol{J}}_{\mathrm{e}} = \frac{\boldsymbol{A}_{\vartriangle}}{r_{\mathrm{e}}^2} = \omega_{\mathrm{e}}^2 \boldsymbol{A}_{\vartriangle} = m_{\mathrm{e}}^2 \boldsymbol{A}_{\vartriangle}\right]_{\mathrm{NU}}.$$

Remembering that the electron's electromagnetic four potential $\boldsymbol{A}_{\square} = \boldsymbol{A}_{\vartriangle} + \gamma_t A_t$ associated to the rotating charge is a light-like vector (i.e. $\boldsymbol{A}_{\square}^2 = 0$, see Eq. (11)) we can write the following relations:

$$\left[\mu_0 J_{et} = \frac{A_t}{r_{\mathrm{e}}^2} = \omega_{\mathrm{e}}^2 A_t = m_{\mathrm{e}}^2 A_t\right]_{\mathrm{NU}},$$

$$\left[\mu_0 \bar{\boldsymbol{J}}_{e\square} = \mu_0 \left(\bar{\boldsymbol{J}}_{\mathrm{e}} + \gamma_t J_{et}\right) = m_{\mathrm{e}}^2 \left(\boldsymbol{A}_{\vartriangle} + \gamma_t A_t\right) = m_{\mathrm{e}}^2 \boldsymbol{A}_{\square}\right]_{\mathrm{NU}},$$

and consequently (QED):

$$\left[\mu_0 \bar{\boldsymbol{J}}_{e\square} = m_{\mathrm{e}}^2 \boldsymbol{A}_{\square}\right]_{\mathrm{NU}}. \tag{17}$$

3.3. Proca and electromagnetic Klein–Gordon equations

In this paragraph and in the next one we will use only natural units, omitting the subscript NU. The aim is to show the connection of Proca equation with an "electromagnetic version" of Klein–Gordon equation. By applying the operator $\boldsymbol{\partial}\wedge$ to Proca equation

$$\boldsymbol{\partial}\left(\boldsymbol{\partial} \wedge \boldsymbol{A}_{\square}\right) + m^2 \boldsymbol{A}_{\square} = 0, \tag{18}$$

$$\boldsymbol{\partial} \boldsymbol{F} + m^2 \boldsymbol{A}_{\square} = 0,$$

we get

$$\boldsymbol{\partial} \wedge \boldsymbol{\partial} \boldsymbol{F} + m^2 \boldsymbol{\partial} \wedge \boldsymbol{A}_{\square} = 0,$$

$$\boldsymbol{\partial} \wedge \boldsymbol{\partial} \boldsymbol{F} + m^2 \boldsymbol{F} = 0.$$

Now, by writing Maxwell's equations considering an averaged four-current vector density

$$\boldsymbol{\partial} \boldsymbol{F} = -4\pi \bar{\boldsymbol{J}}_{\square}, \tag{19}$$

and by applying to both members the operator $\boldsymbol{\partial}\cdot$ we obtain the following expression

$$\boldsymbol{\partial} \cdot \boldsymbol{\partial} \boldsymbol{F} = -4\pi\, \boldsymbol{\partial} \cdot \bar{\boldsymbol{J}}_{\square} = 0,$$

that is equal to zero as a consequence of the charge–current conservation law. For this reason, the term $\boldsymbol{\partial} \wedge \boldsymbol{\partial F}$ can be safely substituted by the term $\boldsymbol{\partial}^2 \boldsymbol{F}$:

$$\boldsymbol{\partial} \wedge \boldsymbol{\partial F} = \boldsymbol{\partial}^2 \boldsymbol{F} - \boldsymbol{\partial} \cdot \boldsymbol{\partial F} = \boldsymbol{\partial}^2 \boldsymbol{F}.$$

As a result we obtain a Klein–Gordon-like equation where the electromagnetic bivector $\boldsymbol{F}$ substitutes the "wavefunction" ψ:

$$\boldsymbol{\partial}^2 \boldsymbol{F} + m^2 \boldsymbol{F} = 0. \tag{20}$$

A similar equation for the electromagnetic four potential can be obtained simply by applying the Lorenz gauge condition $\boldsymbol{\partial A}_\square = \boldsymbol{\partial} \wedge \boldsymbol{A}_\square$ to Proca equation:

$$\boldsymbol{\partial}^2 \boldsymbol{A}_\square + m^2 \boldsymbol{A}_\square = 0 \tag{21}$$

or

$$\boldsymbol{\partial}^2 \boldsymbol{A}_\square + \omega^2 \boldsymbol{A}_\square = 0. \tag{22}$$

It is important to note that the Lorenz gauge condition can been applied to Maxwell's equations (19) only when *an averaged* four current density vector value is used. In this case the electromagnetic four potential is also an averaged value and no more an harmonic function of space–time [1].

3.4. The electromagnetic Dirac equation

By following the same conceptual pattern of the previous paragraph, an electromagnetic–geometric version of the Dirac equation (23),

$$i\not{\partial}\boldsymbol{\psi} - m\boldsymbol{\psi} = 0 \tag{23}$$

should have the form

$$\boldsymbol{\partial F} - \boldsymbol{mF} = 0. \tag{24}$$

Here $\boldsymbol{m}$ cannot be a scalar, being $\boldsymbol{\partial F}$ a vector and $\boldsymbol{F}$ a bivector, respectively, but rather a space-like vector with module m. A possible candidate for $\boldsymbol{m}$ is a vector that has the same direction of the Zitterbewegung radius $\boldsymbol{r}$ and a module $m = \frac{1}{r} = \omega$. Calling $\boldsymbol{r}_u$ a unit vector in the same direction of $\boldsymbol{r}$, Eq. (24) becomes

$$\boldsymbol{\partial F} - \omega \boldsymbol{r}_u \boldsymbol{F} = 0, \tag{25}$$

where the operator $\boldsymbol{\partial}$ of $Cl_{3,1}(\mathbb{R})$ substitutes the Dirac operator $i\not{\partial}$, the Zitterbewegung angular frequency ω the electron mass and the electromagnetic bivector $\boldsymbol{F}$ the wave function $\boldsymbol{\psi}$. The unit vector $\boldsymbol{r}_u$ is always orthogonal to the vector potential and therefore:

$$\boldsymbol{r}_u^2 = 1,$$

$$\omega \boldsymbol{r} = \boldsymbol{r}_u,$$

$$\boldsymbol{r} \cdot \boldsymbol{A}_\square = 0.$$

By applying (19) to (25) we can write

$$4\pi \boldsymbol{J}_\square + \omega \boldsymbol{r}_u \boldsymbol{F} = 0 \tag{26}$$

whereas, by applying (17) to (26) and remembering that $\boldsymbol{F} = \boldsymbol{\partial} \boldsymbol{A}_\square$, we obtain:

$$\omega^2 \boldsymbol{A}_\square + \omega \boldsymbol{r}_u \boldsymbol{\partial} \boldsymbol{A}_\square = 0,$$

that can be written as

$$\boldsymbol{r}_u \boldsymbol{\partial} \boldsymbol{A}_\square + \omega \boldsymbol{A}_\square = 0.$$

Now, by left multiplying the last equation for the unit vector $\boldsymbol{r}_u$ we obtain a Dirac-like equation for the electromagnetic four potential

$$\boldsymbol{\partial} \boldsymbol{A}_\square + \omega \boldsymbol{r}_u \boldsymbol{A}_\square = 0. \tag{27}$$

Multiplying for the elementary charge e (27) becomes

$$e \boldsymbol{\partial} \boldsymbol{A}_\square + e\omega \boldsymbol{r}_u \boldsymbol{A}_\square = 0. \tag{28}$$

Moreover, by multiplying the electromagnetic four-potential for the ratio $\frac{e}{\omega}$, we obtain a light-like vector that can be interpreted as the charge four-velocity $\boldsymbol{c} - \boldsymbol{\gamma_t}$ (see Eq. (60) of [1] and Eq. (12))

$$\frac{e}{\omega} \boldsymbol{A}_\square = \boldsymbol{c} - \gamma_t, \tag{29}$$

that left multiplying by $\boldsymbol{r}_u$ becomes

$$\frac{e}{\omega} \boldsymbol{r}_u \boldsymbol{A}_\square = \boldsymbol{r}_u \boldsymbol{c} - \boldsymbol{r}_u \gamma_t. \tag{30}$$

Now, by applying (30) to (28) and remembering that $\boldsymbol{\partial} \boldsymbol{A}_\square = \boldsymbol{F}$, (28) becomes

$$e\boldsymbol{F} = -\omega^2 \left(\boldsymbol{r}_u \boldsymbol{c} - \boldsymbol{r}_{\boldsymbol{u}} \gamma_t \right). \tag{31}$$

Applying the identity $\boldsymbol{F} = (\boldsymbol{E} + I\boldsymbol{B})\, \gamma_t$ (see Eq. (73) of [1], Eq. (31) becomes

$$e\left(\boldsymbol{E} + I\boldsymbol{B} \right) \gamma_t = -\omega^2 \left(\boldsymbol{r}_u \boldsymbol{c} - \boldsymbol{r}_{\boldsymbol{u}} \gamma_t \right). \tag{32}$$

This last equation can be split in two equations. The first one deals with the electric field $\boldsymbol{E}$:

$$e\boldsymbol{E}\gamma_t = \omega^2 \boldsymbol{r}_u \gamma_t.$$

Applying the identity $eA = \omega$, the square ω^2 can be written as $eA\omega$, namely a term that is equal to the module of the force generated on an elementary electric charge by the time derivative of a rotating vector potential:

$$e\boldsymbol{E} = eA\omega\mathbf{r}_{\boldsymbol{u}} = -e\frac{\mathrm{d}\boldsymbol{A}_\triangle}{\mathrm{d}t}. \tag{33}$$

This electric force has the same value of the centrifugal force acting on a mass m rotating with angular frequency ω at distance r from its orbit center:

$$\boldsymbol{r}_u = \omega\boldsymbol{r} = m\boldsymbol{r},$$

$$e\boldsymbol{E} = m\omega^2\boldsymbol{r}.$$

The second part of (32) deals with the magnetic flux density field $\boldsymbol{B}$:

$$eI\boldsymbol{B}\gamma_t = -\omega^2\boldsymbol{r}_u\boldsymbol{c},$$

$$eI_\triangle\boldsymbol{B} = -\omega^2\boldsymbol{r}_u\boldsymbol{c}$$

that right multiplying for $\boldsymbol{c}$ becomes:

$$eI_\triangle\boldsymbol{B}\boldsymbol{c} = -\omega^2\boldsymbol{r}_u.$$

As $\boldsymbol{B}$ and $\boldsymbol{c}$ are orthogonal vectors in the Zitterbewegung model, it is possible to write also:

$$eI_\triangle\boldsymbol{B} \wedge \boldsymbol{c} = -\omega^2\boldsymbol{r}_u$$

that, using ordinary vector algebra, becomes:

$$e\boldsymbol{c} \times \boldsymbol{B} = -\omega^2\boldsymbol{r}_u. \tag{34}$$

Finally, merging (33) with (34) we obtain an equation that tell us that the mass-less rotating charge, with momentum $\boldsymbol{p} = e\boldsymbol{A}_\triangle$, is subjected to a centripetal magnetic force $-\omega^2\boldsymbol{r}_u$:

$$e\boldsymbol{c} \times \boldsymbol{B} = e\frac{\mathrm{d}\boldsymbol{A}_\triangle}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}, \tag{35}$$

$$e\boldsymbol{c} \times \boldsymbol{B} = -m\omega^2\boldsymbol{r} = -\omega^2\boldsymbol{r}_u. \tag{36}$$

These easy to interpret equations confirm the correctness of the original choice of $\omega\boldsymbol{r}_u$ for the vector $\boldsymbol{m}$ in the electromagnetic version of Dirac equation (24).

3.5. Proca equation, electric charge quantization and Josephson constant

An interesting consequence of Eq. (22) is the magnetic flux and electric charge quantization. In this paragraph we call "wave amplitude" the module A of vector potential $\boldsymbol{A}_{\vartriangle}$ in Eq. (22)

$$\boldsymbol{A}_{\square} = \boldsymbol{A}_{\vartriangle} + \gamma_t A_t, \qquad A = |\boldsymbol{A}_{\vartriangle}| = A_t$$

Substituting ω with eA in Eq. (22) we obtain *a non-linear wave equation for the electromagnetic four potential, where the wave angular frequency is proportional to the wave amplitude and the proportionality coefficient is the "electric charge quantum", i.e. the elementary charge e.*

$$\left[\boldsymbol{\partial}^2 \boldsymbol{A}_{\square} + e^2 A^2 \boldsymbol{A}_{\square} = 0\right]_{\mathrm{NU}}, \tag{37}$$

$$\left[\boldsymbol{\partial}^2 \boldsymbol{A}_{\square} + \alpha A^2 \boldsymbol{A}_{\square} = 0\right]_{\mathrm{NU}}. \tag{38}$$

In this equation the ratio frequency/amplitude, ν/A, expressed in natural units is a pure number equal to half the value of Josephson constant K_{J} :

$$\left[\frac{v}{A} = \frac{1}{2} K_{\mathrm{J}}\right]_{\mathrm{NU}}.$$

The product of wave amplitude and wave period T is equal to another constant exactly equal to a magnetic flux Φ_{M}, a value two times the magnetic flux quantum Φ_{o} (see Fig. 1). It is a reasonable conjecture to consider (37) also valid for other charged elementary particles. In natural units we have

$$\left[AT = \frac{\omega T}{e} = \frac{h}{e} = \Phi_{\mathrm{M}} = 2K_{\mathrm{J}}^{-1}\right]_{\mathrm{NU}},$$

where

$$\Phi_{\mathrm{M}} = 2\Phi_{\mathrm{o}} = 4.13566766 \times 10^{-15}\ \mathrm{V\,s},$$

$$[\Phi_{\mathrm{M}} = 73.55246018]_{\mathrm{NU}}.$$

4. Geometric Interpretation of Relativistic Electron Mass and De Broglie Wavelength

If an electron moves along an axis z orthogonal to its charge rotation plane, it will describe an helical trajectory whose length is $L = c\Delta t$ and whose z-axis length is $l = v_z \Delta t$. The electron mass is exactly equal to the inverse of the helix radius r if expressed in NU, i.e. $m = r^{-1}$. An acceleration along z, implies a smaller radius and, hence, a mass increase. Using the Pythagorean theorem it is possible to write the value of the radius r as a function of v_z [4,5]:

$$r = r_{\mathrm{e}} \sqrt{1 - \frac{v_z^2}{c^2}}$$

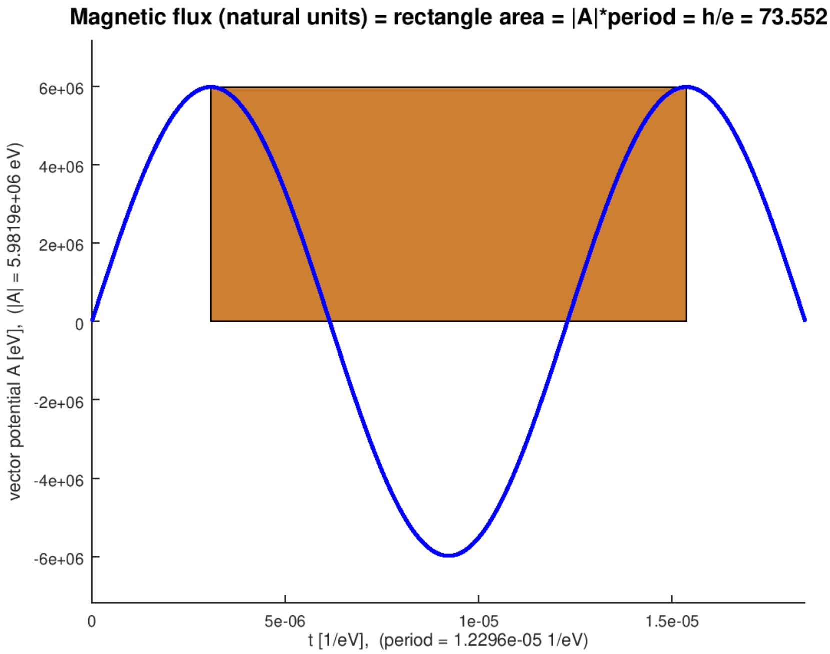

Figure 1. A possible explanation of magnetic flux and electric charge quantization: in electromagnetic Klein–Gordon/Proca equation vector potential amplitude time wave period is a constant $\Phi_m = h/e$.

and the related mass variation

$$m = \frac{\hbar\omega}{c^2} = \frac{m_\mathrm{e}}{\sqrt{1 - \frac{v_z^2}{c^2}}}.$$

The charge momentum is proportional to the angular frequency and it has a direction tangent to the helical path. The relativistic momentum of charge is, then,

$$p_\mathrm{c} = eA = \frac{\hbar\omega}{c} = \frac{\hbar}{r} \tag{39}$$

or, using natural units,

$$\left[p_\mathrm{c} = \omega = \frac{1}{r} = m\right]_\mathrm{NU}.$$

Equation (39) suggests a particular interpretation of the Heisenberg uncertainty principle: an electron, whose charge has a momentum p_c, cannot be confined within a spherical space of radius R less than r. This means that it must be

$$R > r = \frac{\hbar}{p_\mathrm{c}}.$$

Now, the charge momentum vector $\boldsymbol{p}_\mathrm{c} = e\boldsymbol{A}_{\vartriangle}$ can be decomposed into two components: $\boldsymbol{p}_\perp$, that is orthogonal to electron velocity and another one, $\boldsymbol{p}_\parallel$, that is parallel, i.e. in the z-direction. Therefore the charge momentum can be expressed as

$$\boldsymbol{p}_\mathrm{c} = \boldsymbol{p}_\perp + \boldsymbol{p}_\parallel.$$

The magnitude of component $\boldsymbol{p}_\perp$ is a constant, independent from velocity v_z, and is proportional to the charge angular speed ω_e in the xy-plane [12]. Therefore,

$$p_\perp = \frac{\hbar\omega_\mathrm{e}}{c} = m_\mathrm{e}c$$

or in natural units

$$[p_\perp = \omega_\mathrm{e} = m_\mathrm{e}]_\mathrm{NU},$$

whereas the component $p_\parallel$ is the momentum of the electron and is proportional to the *instantaneous* angular frequency $\omega_z = v_z/r$

$$p_\parallel = \frac{\hbar\omega_z}{c} = \frac{\hbar v_z}{cr} = \frac{\hbar\omega}{c^2}v_z = mv_z$$

or in natural units

$$\left[p_\parallel = \omega_z = \frac{v_z}{r} = mv_z\right]_\mathrm{NU}.$$

Using again the Pythagorean theorem it is possible to write the following equations

$$\omega_\mathrm{e} = \frac{v_\perp}{r} = \frac{\sqrt{c^2 - v_z^2}}{r} = \frac{\sqrt{c^2 - v_z^2}}{r_\mathrm{e}\sqrt{1 - \frac{v_z^2}{c^2}}} = \frac{c}{r_\mathrm{e}}. \tag{40}$$

and, as a consequence of (40), also

$$\omega = \frac{c}{r}.$$

But

$$\omega_z = \frac{v_z}{r}$$

and, therefore, the sum of squares of the angular frequencies yields the following relations

$$\omega^2 = \omega_\mathrm{e}^2 + \omega_z^2, \qquad p_\mathrm{c}^2 = p_\perp^2 + p_\parallel^2,$$

and, finally,

$$m^2c^2 = m_\mathrm{e}^2c^2 + m^2v_z^2. \tag{41}$$

For the sake of simplicity we will use the symbol p to indicate the electron momentum $p_{\|}$

$$p = p_{\|} = mv_z.$$

According to De Broglie hypothesis, ω_z is the instantaneous angular frequency associated to a particle with rest mass m_e, relativistic mass m and velocity $v_z = \omega_z r$. As a consequence

$$p = mv_z = \frac{\hbar\omega}{c^2}v_z = \frac{\hbar}{cr}v_z = \frac{\hbar\omega_z}{c} = \hbar\frac{2\pi}{\lambda} = \hbar k \tag{42}$$

or

$$\left[p = mv_z = \omega v_z = \frac{v_z}{r} = \omega_z = \frac{2\pi}{\lambda} = k\right]_\mathrm{NU}.$$

Equation (42) yields

$$\frac{p}{k} = p\frac{\lambda}{2\pi} = \hbar. \tag{43}$$

where the term $k = 2\pi/\lambda$ is the wave number of the electron and λ the related De Broglie wavelength. Of course, if we observe the electron at a spatial scale much larger than its Compton wavelength and at a time scale much higher than the very short period $T \approx 8.1 \times 10^{-21}$ s of the Zitterbewegung rotation period, for a constant speed v_z, the electron can be approximated to a point particle, provided with "mass" and charge, which moves with a uniform motion along the z-axis of the helix. Particularly, Fig. 2 represents the helical trajectories of electrons moving at different speeds.

5. ESR, NMR, Spin and *Intrinsic* Angular Momentum

As shown in the previous paragraph, in the proposed model, the electron has an angular momentum $\boldsymbol{\hbar}$ and a magnetic moment $\boldsymbol{\mu}_\mathrm{B}$, equal to Bohr magneton. It is, therefore, reasonable to assume that, in presence of an external magnetic field, the electron is subjected, as a small gyroscope, to a torque $\boldsymbol{\tau}$ and to a Larmor precession with frequency ω_p. The only difference with a classical gyroscope is the quantization of the $\hbar_{\|}$ component of the angular momentum $\boldsymbol{\hbar}$ along the external flux density field $\boldsymbol{B}_\mathrm{E}$. This component can take only two possible spin values, namely $\hbar_{\|} = \pm\frac{1}{2}\hbar$ (see [4], p. 83). The two spin values will correspond to two possible values for the angle θ formed between the angular momentum vector and the external magnetic field vector: $\theta \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$:

$$\hbar_{\|}^2 + \hbar_{\perp}^2 = \hbar^2, \quad \hbar_{\|} = \pm\frac{1}{2}\hbar.$$

The torque exerted by the external flux density field $\boldsymbol{B}_\mathrm{E}$ is

$$\tau = |\boldsymbol{\mu}_\mathrm{B} \times \boldsymbol{B}_\mathrm{E}| = \mu_\mathrm{B} B_\mathrm{E} \sin(\theta)$$

and the related Larmor precession angular frequency is

$$\omega_\mathrm{p} = \frac{B_\mathrm{E}\mu_\mathrm{B}}{\hbar}. \tag{44}$$

The precession angular frequency will correspond to two possible energy levels:

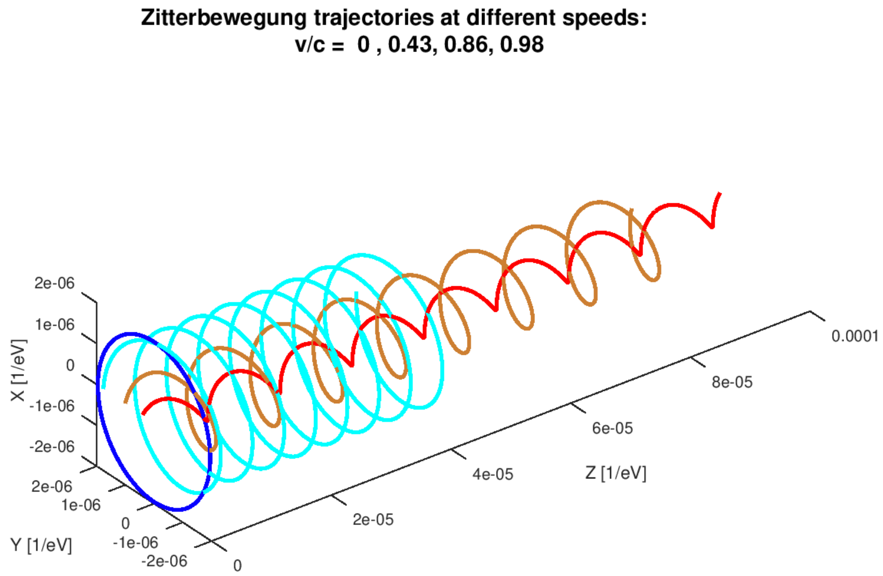

Figure 2. Zitterbewegung trajectories for different speeds.

$$E_{\mathrm{H}} = \hbar\omega_{\mathrm{p}} \quad \text{if} \quad \theta = \frac{2\pi}{3}$$

and

$$E_{\mathrm{L}} = -\hbar\omega_{\mathrm{p}} \quad \text{if} \quad \theta = \frac{\pi}{3}.$$

The difference of energy levels corresponds to the Spin Electronic Resonance (ESR) frequency ν_{ESR}:

$$\Delta E = E_{\mathrm{H}} - E_{\mathrm{L}} = 2\hbar\omega_{\mathrm{p}} = \hbar\omega_{\mathrm{ESR}} = h\nu_{\mathrm{ESR}}. \tag{45}$$

From (44) and (45) it is possible to determine the ESR frequency as

$$\nu_{\mathrm{ESR}} = 2\frac{B_{\mathrm{E}}\mu_{\mathrm{B}}}{h}. \tag{46}$$

For instance, an external magnetic flux density field equal to $B_{\mathrm{E}} = 1.5$ T yields a frequency $\nu_{\mathrm{ESR}} \approx 42$ GHz. By calling s the spin value and μ the nuclear magnetic moment we can also generalize (46) for particles other than the electron. In this case the term used is Nuclear Magnetic Resonance (NMR) frequency, which is equal to

$$\nu_{\mathrm{NMR}} \approx \frac{B_{\mathrm{E}}\mu}{hs}. \tag{47}$$

For instance, for isotope ${}^{7}_{3}\mathrm{Li}$, with $s = 3/2$, $\mu \approx 1.645 \times 10^{-26}$ and $B_{\mathrm{E}} = 1.5\,\mathrm{T}$, the NMR frequency is $\nu_{\mathrm{NMR}} \approx 24.8$ MHz, whereas for isotope ${}^{11}_{5}\mathrm{B}$ we have $s = 3/2$, $\mu \approx 1.36 \times 10^{-26}$ $\mathrm{J\,T^{-1}}$ and NMR frequency is $\nu_{\mathrm{NMR}} \approx 20.5$ MHz. Another example deals with isotope ${}^{87}_{38}\mathrm{Sr}$ with $s = 9/2$ and $\mu \approx 5.52 \times 10^{-27}$ $\mathrm{J\,T^{-1}}$. In this case NMR frequency is $\nu_{\mathrm{NMR}} \approx 278\,\mathrm{kHz}$ for $B_{\mathrm{E}} = 0.15$ T with a Larmor frequency $\frac{\omega_{\mathrm{p}}}{2\pi} = \frac{1}{2}\nu_{\mathrm{NMR}} \approx 139\,\mathrm{kHz}$.

5.1. Electron spin and coherent systems

In the proposed model, the electron, in presence of an external magnetic field, is subjected to Larmor precession and its spin value $\pm\hbar/2$ is interpreted as the intrinsic angular momentum component parallel to the magnetic field. It is interesting to note that a hypothetical technology, able to align the intrinsic angular momentum of a sufficient number of electrons, could favor the formation of a coherent superconducting and super-fluid condensate state. In this state, the electrons would behave as particles with whole spin $\hbar$ and would no longer be subject to the Fermi–Dirac statistic. The compression effect (pinch) of an electrical discharge, accurately localized in a very small "capillary" volume, inside which a very rapid and uniform variation of the electric potential occurs, could favor the formation of a superconducting plasma. The conjecture is based on the possibility that, as a consequence of Aharonov–Bohm effect, a rapid, collective and simultaneous variation of the Zitterbewegung phase catalyzes the creation of coherent systems like those described by K. Shoulders and H. Puthoff [13]: "*Laboratory observation of high-density filamentation or clustering of electronic charge suggests that under certain conditions strong coulomb repulsion can be overcome by cohesive forces as yet imprecisely defined*".

6. Hypotheses on the Structure Of Ultra-dense Hydrogen

In relativistic quantum mechanics, the Klein-Gordon equation describes a charge density distribution in space and time. In this equation a term $m^2c^2/\hbar^2$ appears, whose interpretation becomes simple and intuitive if one uses natural units and the principle of mass–energy–frequency equivalence. In particular, it is possible to recognize this term as the square of the Zitterbewegung angular frequency ω:

$$\left[\frac{m^2c^2}{\hbar^2} = m^2 = \omega^2\right]_{\mathrm{NU}}.$$

In the paper "*On the hydrino state of the relativistic hydrogen atom*" [14], the author, by applying the Klein–Gordon equation to the hydrogen atom, finds a possible deep energetic level of $E_0 \approx 3.7$ keV (see Eqs. (16) and (17)) at a distance r_0 from the nucleus. In particular Naudts demonstrates that

$$E_0 \approx m_{\mathrm{e}}c^2\alpha \approx 3.7\,\mathrm{keV}$$

at a distance from nucleus equal to

$$r_0 \approx \frac{\hbar}{m_{\mathrm{e}}c} \approx 0.39 \times 10^{-12}\,\mathrm{m}.$$

According to the author, the E_0 level corresponds to the hypothetical state of a relativistic electron: "*The other set of solutions contains one eigenstate which describes a very relativistic particle with a binding energy which is a large* *fraction of the rest mass energy*". It is possible to formulate an alternative hypothesis according to which the radius r_0 is simply the radius r_{e} of the Zitterbewegung orbit, in the center of which the proton is located. Consequently the energy, E_0, can be interpreted as the electrostatic potential energy between the electron charge and the proton:

$$E_0 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_{\mathrm{e}}} = \frac{\hbar}{r_{\mathrm{e}}}\alpha c = m_{\mathrm{e}}c^2\alpha, \qquad \left[E_0 = \frac{e^2}{r_{\mathrm{e}}} = \frac{\alpha}{r_{\mathrm{e}}} = \omega_{\mathrm{e}}e^2 = m_{\mathrm{e}}\alpha\right]_{\mathrm{NU}}.$$

A series of numerous experiments conducted by Leif Holmlid of the University of Gothenburg, recently replicated by Sindre Zeiner–Gundersen [15], seems to demonstrate the existence of a very compact form of deuterium [16–18]. Starting from the kinetic energy (about $630\ \mathrm{eV}$) of the nuclei emitted in some experiments, achieved by irradiating this particular form of ultra-dense deuterium with a small laser, a distance between deuterium nuclei of about $2.3\ \times\ 10^{-12}\ \mathrm{m}$ has been computed, a value much smaller than the distance of about $74\ \times\ 10^{-12}\ \mathrm{m}$ that separates the nuclei of a normal deuterium molecule. Therefore, it is possible to advance an hypothesis on the structure of ultra-dense hydrogen (UDH) starting from the electron Zitterbewegung model. The proton is considerably smaller than Zitterbewegung orbit radius r_{e}, consequently an hypothetical structure formed by an electron with a proton (or a deuterium nucleus) in its center would have a potential energy of

$$\left[\frac{-e^2}{r_{\mathrm{e}}} \approx -3.7\ \mathrm{keV}\right]_{\mathrm{NU}},$$

a value corresponding to the energy in the X-ray range with a wavelength of about $3.3\ \times\ 10^{-10}\ \mathrm{m}$. The distance between the deuterium nuclei in the Holmlid experiment could be explained by an ordered linear sequence of ultra-dense particles in which the rotation planes of the electron charges are parallel and equidistant. In these hypothetical aggregates, the Zitterbewegung phases of two neighboring electrons differ by π radians and the distance d_{c} between the charges of the two electrons is equal to the distance traveled by light in a time equal to a rotation period T. This distance amounts to $d_{\mathrm{c}} = cT = \lambda_{\mathrm{c}} \approx 2.42\ \times\ 10^{-12}\ \mathrm{m}$. In this case, the distance between the nuclei d_i can be obtained by applying the Pythagorean theorem, as shown in Fig. 3, yielding the value

$$d_i = \sqrt{\lambda_{\mathrm{c}}^2 - \left(\frac{\lambda_{\mathrm{c}}}{\pi}\right)^2} \approx 2.3\ \times\ 10^{-12}\ \mathrm{m}.$$

This UDH model is in agreement with the third assumption of Carver Mead "Alternate World View": *"every element of matter is coupled to all other charges on its light cone by time-symmetric interactions"* [2].

6.1. Ultra-dense hydrogen and anomalous heat generation in metal–hydrogen systems

The combustion of a mole of hydrogen (about two grams) generates an energy of $286\ \mathrm{kJ}$ (or $240\ \mathrm{kJ}$ if we do not take into account the latent heat of vaporization of water), a value that corresponds to an energy of $1.48\ \mathrm{eV}$ per atom. The formation of an ultra-dense hydrogen atom would release an energy of 3.7 keV per atom, a value 2500 times higher. The conversion of only two grams of hydrogen into ultra-dense hydrogen would then be able to generate an energy of $715\ \mathrm{MJ} \approx 198\ \mathrm{kWh}$. Consequently, the hypothesis, according to which in some experiments the development of anomalous heat is partially or totally due to the formation of ultra-dense hydrogen, cannot be excluded. Following *an alternative* hypothesis, the $\alpha m_{\mathrm{e}}c^2 \approx 3.7\ \mathrm{keV}$ energy is not emitted as an X-ray photon but is stored in the electron mass–frequency–energy, with a consequent small Zitterbewegung orbit radius reduction. By defining m_{eu} and r_{eu} the mass and the radius, respectively, in this new state we have:

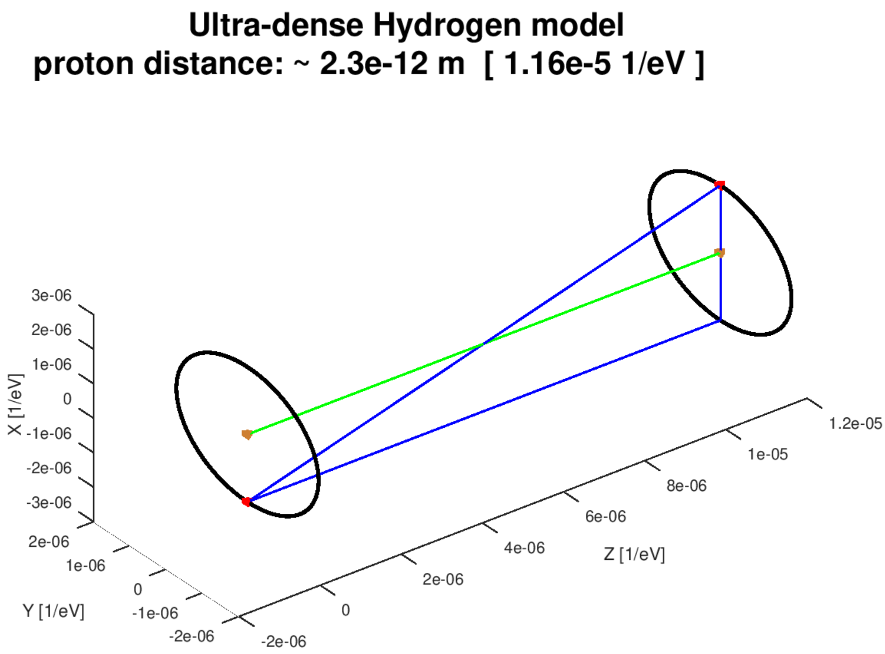

Figure 3. Ultra-dense hydrogen model..

$$m_{\mathrm{eu}}c^2 = m_{\mathrm{e}}c^2 + \alpha m_{\mathrm{e}}c^2 \approx 514.728\,\mathrm{keV}. \tag{48}$$

The mass increase implies a Zitterbewegung radius reduction. In fact

$$m_{\mathrm{e}}c^2 = \hbar\omega_{\mathrm{e}} = \frac{\hbar c}{r_{\mathrm{e}}},$$

$$m_{\mathrm{eu}}c^2 = m_{\mathrm{e}}\left(1+\alpha\right)c^2 = \hbar\omega_{\mathrm{eu}} = \frac{\hbar c}{r_{\mathrm{eu}}},$$

and therefore

$$r_{\mathrm{eu}} = \frac{\hbar}{m_{\mathrm{e}}\left(1+\alpha\right)c} = \frac{r_{\mathrm{e}}}{1+\alpha}.$$

This radius reduction generates a potential energy decrease:

$$\triangle E_{\mathrm{p}} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r_{\mathrm{e}}} - \frac{1}{r_{\mathrm{eu}}}\right) = \frac{e^2\alpha}{4\pi\varepsilon_0 r_{\mathrm{e}}} = \left[\frac{\alpha^2}{r_{\mathrm{e}}}\right]_{\mathrm{NU}} \approx 27.2\,\mathrm{eV}.$$

Following the Carver Mead "transactional" interpretation of photons, the eventual (or necessary?) emission of the ultraviolet $27.2\ \mathrm{eV}$ photon may be favored by a "Mills catalyst" [19,2].

Another Zitterbewegung model for deep electron states has been recently presented by A. Kovacs et al., aimed at explaining their impressive experimental results [20].

7. Ultra-dense Hydrogen and Low-energy Nuclear Reactions

In the proposed model the particles of hydrogen or ultra-dense deuterium are electrically neutral but have a magnetic moment almost equal to electron's one. This is a value 960 times higher than the neutron magnetic moment. A particle with magnetic moment $\boldsymbol{\mu}$ is subjected, in presence of a magnetic field $\boldsymbol{B}$, to a force $\boldsymbol{f}$ proportional to the gradient of $\boldsymbol{B}$

$$\boldsymbol{f} = \nabla\left(\boldsymbol{B} \cdot \boldsymbol{\mu}\right).$$

Therefore, the magnetic field $\boldsymbol{B}$ generated by a nucleus could exert a considerable "remote action" on the particles of ultra-dense hydrogen. This force could be the source of the "long range potential" mentioned in a theoretical work of Gullström and Rossi, "Nucleon polarizability and long range strong force from $\sigma I = 2$ meson exchange potential" [21]:

"A less probable alternative to the long range potential is if the e-N coupling in the special EM field environment would create a strong enough binding to compare an electron with a full nuclide. In this hypothesis, no constraints on the target nuclide are set, and nucleon transition to excited states in the target nuclide should be possible. In other words these two views deals with the electrons role, one is as a carrier of the nucleon and the other is as a trigger for a long range potential of the nucleon".

Hence, it is possible that, according to this scenario, electrons would have a fundamental dual role as catalysts of low-energy nuclear reactions (LENR): the first as neutralization-masking effect of the positive charge of hydrogen or deuterium nuclei, a necessary condition to overcome the Coulomb barrier, the second as the source of a relatively long-range magnetic force.

By using the Holmlid notation "H(0)" to indicate ultra-dense hydrogen particles, it is possible to hypothesize a LENR reaction involving the ${}^{7}_{3}\mathrm{Li}$, an isotope that constitutes more than 92% of the natural Lithium

$${}^{7}_{3}\mathrm{Li} + \mathrm{H}\,(0) \rightarrow 2\,{}^{4}_{2}\mathrm{He} + \mathrm{e}. \tag{49}$$

This reaction would produce an energy of about $17.34\ \mathrm{MeV}$ mainly in the form of kinetic energy of helium nuclei, without emission of neutrons or penetrating gamma rays. A similar reaction, able to release about $8.67\ \mathrm{MeV}$, could be hypothesized for the isotope ${}^{11}_{5}\mathrm{B}$

$${}^{11}_{5}\mathrm{B} + \mathrm{H}\,(0) \rightarrow 3\,{}^{4}_{2}\mathrm{He} + \mathrm{e}. \tag{50}$$

Emissions in the X-ray range would still be present in the form of braking radiation (*Bremsstrahlung*) generated by the deceleration caused by impacts of helium nuclei with other atomic nuclei.

The three "miracles" required by the low-energy nuclear reactions could therefore find, for example, in the reaction (49) a possible explanation:

(1) Overcoming the Coulomb barrier: the ultra-dense hydrogen particles are electrically neutral.
(2) No neutrons are emitted: the reactions products of (49) and (50) consist exclusively of helium nuclei and an electron.

(3) Absence of penetrating gamma radiation: the energy produced is mainly manifested as kinetic energy of the reaction products and as X-ray emission from *bremsstrahlung*. However a probability for gamma radiation from excited intermediate products and from secondary interaction of high energy alpha particles could not be completely dismissed.

The mechanical energy of the alpha particles produced by the reactions could be converted with a reasonable yield directly into electrical energy or into usable mechanical energy [22], avoiding the need for an intermediate conversion into thermal energy. None of the three miracles is required to justify the production of abnormal heat due to ultra-dense hydrogen formation.

In the Iwamura experiment the low-energy nuclear transmutation of elements deposited on a system formed by alternating thin layers of palladium (Pd) and calcium oxide (CaO) was observed. The transmutation occurs when the system is crossed by a flow of deuterium. The CaO layer, essential for the transmutation, is hundreds of atomic layers far from the surface where the atoms to transmute are deposited or implanted. It is, therefore, necessary to find a mechanism that explains the remote action, the role of the CaO and the overcoming of the Coulomb barrier by deuterium nuclei. An interesting hypothesis could derive from considering the formation of ultra-dense deuterium (UDD) at the interface between calcium oxide and palladium, an area in which the high difference in the work function between Pd and CaO favors the formation of a layer with high electron density (Swimming Electron Layer or SEL) [23]. The ultra-dense deuterium could subsequently migrate to the area where the atoms to transmute are present. Therefore, aggregates of neutral charged ultra-dense deuterium would be, according to this hypothesis, the probable responsible for the transmutation of Cs into Pr and Sr into Mo. It is possible that strontium oxide, with its very low work function, substitutes the calcium oxide role in Celani's experiments [24]. By using again the Holmlid notation "D(0)" to indicate "atoms" of ultra-dense deuterium, the hypothesized *many-body reactions* in Iwamura experiments [25] would be very simple:

$$^{133}_{55}\mathrm{Cs} + 4\mathrm{D}(0) \;\rightarrow\; ^{141}_{59}\mathrm{Pr} + 4\mathrm{e},$$

$$^{88}_{38}\mathrm{Sr} + 4\mathrm{D}(0) \;\rightarrow\; ^{96}_{42}\mathrm{Mo} + 4\mathrm{e},$$

$$^{138}_{56}\mathrm{Ba} + 6\mathrm{D}(0) \;\rightarrow\; ^{150}_{62}\mathrm{Sm} + 6\mathrm{e}.$$

In the above equations the symbols $4\mathrm{D}(0)$ and $6\mathrm{D}(0)$ represent picometric, coherent chains of ultra-dense deuterium particles. The short distance between deuterons in such hypothetical structures may favor these otherwise difficult to explain many-body nuclear transmutation. In this context, the electrons would have the precise role of deuterium nucleus vectors within the nucleus to be transmuted.

8. Conclusions

In this paper a simple Zitterbewegung electron model has been introduced, where the concepts of mass-energy, momentum, magnetic momentum and spin naturally emerge from its geometric and electromagnetic parameters, thus avoiding the obscure concept of "intrinsic property" of a "point-like" particle. An intuitive geometric interpretation of relativistic mass and De Broglie wavelength has been presented. Using only electromagnetic and geometric concepts an interpretation of Proca, Dirac, Klein–Gordon and Aharonov–Bohm equations based on this particular electron model has been presented. A non linear equation for electromagnetic four potential has been introduced that directly implies electric charge and magnetic flux quantization.

Electronic Spin Resonance (ESR) frequency has been computed starting from a spin model based on the Larmor precession frequency of Zitterbewegung rotation plane. A very simple model for ultra-dense hydrogen, where electron has only spin angular momentum, has been proposed, highlighting its possible role in many-body and aneutronic low energy nuclear reactions.

Acknowledgements

Authors wish to thank Francesco Celani and Giuliano Bettini for helpful discussions and suggestions.

References

[1] F. Celani and A.O. Di Tommaso and G. Vassallo, Maxwell's equations and Occam's razor, *J. Condensed Matter Nucl. Sci.* **25** (2017) 100–128.
[2] Carver Mead, The nature of light: what are photons? *Proc. SPIE*, 8832: 8832 – 8832 – 7, 2013.
[3] David Hestenes, Quantum mechanics from self-interaction, *Found. Phys.* 15(1) (1985) 63–87.
[4] F. Celani, A.O. Di Tommaso and G. Vassallo, The electron and Occam's razor, *J. Condensed Matter Nucl. Sci.* **25** (2017) 76–99.
[5] Oliver Consa, Helical model of the electron, *General Sci. J.* (2014) 1–14. https://www.gsjournal.net/Science-Journals/Research
[6] J. Paul Wesley and David L. Bergman, Spinning charged ring model of electron yielding anomalous magnetic moment, *Galilean Electrodynamics* **1** (1990) 63–67.
[7] A.L. Parson and Smithsonian Institution, *A Magneton Theory of the Structure of the Atom (with Two Plates)*, Vol. 65, in Publication (Smithsonian Institution), Smithsonian Institution, 1915.
[8] D.L. Bergman and C.W. Lucas, Credibility of common sense science, *Found. Sci.* (2003) 1–17. Reprinted by permission of *Galilean Electrodynamics* **1** (1990) 63—67. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.490.4984&rep=rep1&type=pdf.
[9] David Hestenes, The zitterbewegung interpretation of quantum mechanics, *Found. Phys.* **20**(10) (1990) 1213–1232.
[10] Richard Gauthier, The electron is a charged photon, Vol. 60, *APS April Meeting 2015*, 2015. http://meetings.aps.org/link/BAPS.2015.APR.Y16.4.
[11] Y. Aharonov and D. Bohm, Significance of electromagnetic potentials in the quantum theory, *Phys. Rev.* **115** (1959 485–491.
[12] G. Bettini, The Moebius Strip: a Biology of Elementary Particles, *viXra.org*, *Quantum Physics*, 2010. http://vixra.org/pdf/1004.0035v4.pdf.
[13] H.E. Puthoff and M.A. Piestrup, Charge confinement by casimir forces, arXiv:physics/0408114, 2004.
[14] J. Naudts, On the hydrino state of the relativistic hydrogen atom, *ArXiv Physics e-prints*, July 2005.
[15] S. Zeiner-Gundersen and S. Olafsson, Hydrogen reactor for Rydberg matter and ultra dense hydrogen, a replication of Leif Holmlid, *Int. Conf. Condensed Matter Nucl. Sci.*, ICCF-21, Fort Collins, USA, 2018.
[16] Shahriar Badiei, Patrik U. Andersson and Leif Holmlid, High-energy Coulomb explosions in ultra-dense deuterium: Time-of-flight-mass spectrometry with variable energy and flight length. *Int. J. Mass Spectrometry* **282**(1–2) (2009) 70–76.
[17] Leif Holmlid. Excitation levels in ultra-dense hydrogen p(-1) and d(-1) clusters: structure of spin-based Rydberg matter, *Int. J. Mass Spectrometry* **352** (2013) 1–8.
[18] Leif Holmlid and Sveinn Olafsson, Spontaneous ejection of high-energy particles from ultra-dense deuterium D(0), *Int. J. Hydrogen Energy* **40**(33) (2015) 10559–10567.
[19] R.L. Mills, J.J. Farrell and W.R. Good, *Unification of Spacetime, the Forces, Matter, and Energy*, Science Press, Ephrata, PA 17522, 1992.
[20] Andras Kovacs, Dawei Wang and Pavel N. Ivanov, Investigation of electron mediated nuclear reactions, *J. Condensed Matter Nucl. Sci.* **29** (2019).
[21] Carl-Oscar Gullström and Andrea Rossi, Nucleon polarizability and long range strong force from $\sigma_{I=2}$ meson exchange potential, arXiv 1703.05249, 2017.
[22] Alfonso Tarditi, Aneutronic fusion spacecraft architecture, 2012, NASA-NIAC 2001 PHASE I RESEARCH GRANT, Final

Research Activity Report (SEPTEMBER 2012), pp. 1–33,
https://www.nasa.gov/sites/default/files/atoms/files/niac_2011_phasei_tardittianeutronicfusionspacecraftarchitecture_tagged.pdf.

[23] H. Hora, G.H. Miley, J.C. Kelly and F. Osman, Shrinking of hydrogen atoms in host metals by dielectric effects and inglisteller depression of ionization potentials, *Proc. 9th Int. Conf. on Cold Fusion, (ICCF9)*, 2002, pp. 1–6.

[24] F. Celani, C. Lorenzetti, G. Vassallo, E. Purchi, S. Fiorilla, S. Cupellini, M. Nakamura, P. Boccanera, R. Burri, B. Ortenzi, L. Notargiacomo and A. Spallone, Steps to identify main parameters for ahe generation in sub-micrometric materials: measurements by isoperibolic and air-flow calorimetry, *ICCF21, Int. Conf. on Cold Fusion*, 3–8 June 2018, pp. 1–19, DOI: 10.13140/RG.2.2.16425.29287.

[25] Y. Iwamura, T. Itoh and N. Yamazaki, H. Yonemura, K. Fukutani and D. Sekiba, Recent advances in deuterium permeation transmutation experiments. *J. Condensed Matter Nucl. Sci.* **10** (2013) 76–99.