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A COHESIVE-FRICTIONAL INTERFACE MODEL SUBJECTED TO MIXED COMPLEX LOADING PATHS

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Abstract. *The paper presents a cohesive-frictional interface model based on surface damage mechanics. The proposed model is developed under the assumption that the fracture energies in mode I and in mode II are different values, as shown by several experimental evidences. At difference with the most spread available interface models, only one isotropic interface internal variable is adopted for the constitutive model. The interface constitutive model is developed in a Thermodynamic consistent framework with an Helmholtz free energy potential and the fulfillment of the thermodynamic principles is obtained enforcing the Clausius-Duhem inequality. The damage/friction activation functions and dissipative flow potentials are defined together with nonassociative flow rules and loading/unloading conditions. The latter loading/unloading conditions emerge directly from the nature of the proposed approach, which is framed in the mechanics dissipative process with internal variables, and then does not require any special ad-hoc unloading rule. Finally, some numerical examples of interface subjected to complex mixed loading/unloading/reloading paths are analyzed.*

1 Introduction

Mechanical interfaces are devices which can effectively reproduce decohesion processes along surfaces. Different approaches are available to derive constitutive relations for cohesive interfaces, in which surface traction and the correspondent displacement jump are related. The first available approaches were derived for studying the progressive dechoesion in the area in front a crack tip, as specific nonlinear elastic-softening relations [1, 2]. A further step was the introduction of specific constitutive potential functions [3, 4], which can handle mixed mode loading. However, because of their intrinsic reversibility nature (potential function implies non-linear reversible elastic behavior), it is necessary to introduce ad-hoc unloading and reloading rules, which may give unexpected response paths [5].

Since interface models are constructed for driving fracture processes (including possible closing and frictional sliding mechanisms) it seems reasonable to frame their constitutive relations in the area of mechanical dissipative processes, derived with the contribution of some internal variables. In this context two constitutive frameworks can be envisaged: the first based on surface plasticity laws [6, 7] and the other on surface damage mechanics (SDM) [8, 9, 10, 11, 12] Setting internal variables and constitutive parameters, either plasticity or damage models can properly reproduce proportional loading decohesion processes. However, they strongly differ as soon as unloading take place. Plasticity models unload keeping their original interface stiffness and display a permanent plastic displacement jump, whereas damage models unload with a reduced stiffness an do not develop permanent displacement jump. Because fracture is the main physical aspect to be reproduced, the damage approach seems to be the most suited, provided that a frictional law is also considered in order to describe compressive/sliding deformation modes. [8, 13, 9, 7, 10, 12]

A further topic to be considered is related to the experimental evidence that the energy dissipated in opening mode (Mode I) is typically smaller than the energy dissipated in a pure sliding mode (Mode II). Moreover it is olso important to correctly reproduce the mixed fracture mode propagation.

A different approach, which introduces micro-mechanic asperities, is given by Sacco and co-workers, [14, 15], where the difference between G_{II} and G_I is attributed to frictional dissipation for micro dilatancy deformation modes.

In [16] the potential based Xu-Needleman model [3] is analyzed for mixed mode delamination conditions, showing that it produces work of separation in mixed mode loading paths which is $W_T > G_{II} > G_I$, for material with mode II fracture energy greater than mode I fracture energy ($G_{II} > G_I$). In [16] this behavior is considered as physically inconsistent and certainly such results are in disagreement with experimental results proposed [17].

The same kind of deficiencies have been analyzed in [18], where the work of separation obtained by four different interface constitutive models [19, 20, 21], under mixed mode debonding process, are evaluated and compared to the pure mode I and pure mode II fracture energies ($G_I < G_{II}$). In [18] the authors show that the analyzed CZMs can produce physically inconsistent results. Moreover, the response of the model proposed by van den Bosch [4] results in agreement with the experimental data. In Dimitri et al [18], authors propose a model, with similar features of the van den Bosch model [4], but derived from a Helmholtz free energy functional in a thermodynamics consistent form and endowed with four scalar damage variables.

It is to remark that the well known experimental study of Benzeggagh and Kenane [17] on the composite delamination, devoted to the evaluation of the fracture energy under different mixed mode ratios loading, shows an almost linear law between the total fracture energy and the ratio G_{II}/G_T between the mode II energy release rate (ERR) and the total ERR. The latter experimental study confirms that the mixed mode fracture energy G_m is confined between the upper limit G_{II} and lower limit G_I. Delamination problems where a mixed mode fracture energy G_m is external to the interval (G_I, G_{II}) has never been experimentally found in author's best knowledge.

In the present contribution we analyse a cohesive-frictional interface model which is based on the SDM and we show that, in order to avoid possible unexpected response, the model is constructed possessing all the following features:

- It is defined with two different fracture energies G_I and G_{II} and can reproduce mixed proportional loading paths;
- Being based on SDM and since the mechanical phenomena to be described is the progressive decohesion (breaking of elastic links), only one isotropic scalar damage variable in adopted in the model;
- It is derived in a thermodynamics consistent setting, which means that first and second principle of thermodynamics are satisfied for any possible deformation processes;
- Beside damage other internal variables are introduced, which allow to describe the microstructure state in terms of softening;
- It has a well-defined damage/friction activation function (in mixed mode loading) and a well-defined damage/friction potential function. The last two functions being different, frame the proposed model in the class of non-associative SDM. Damage and kinematic internal variables flow rules are then derived, together with the Kuhn-Tucker loading/unloading damage conditions.

The features describe above allow the proposed model to be enough simple (just one scalar damage variable), but at the same time capable to properly reproduce all possible dissipative histories, included non-proportional and cyclic complex loadings. The proposed approach considers a zero-thickness interface, at difference with some interphase models which requires a finite initial thickness and therefore a layer with an internal stress state [22, 23, 15].

2 The cohesive-frictional model

Let us consider an interface Representative Surface Element (RSE) with local tangential and normal axises (T, N). $\boldsymbol{t}^+$ and $\boldsymbol{t}^-$ are the traction vectors on positive and negative edges with $\boldsymbol{t} = \boldsymbol{t}^+ = \boldsymbol{t}^-$, in order to satify equilibrium condition.

From the kinematic point of view, $\boldsymbol{u}^+$ and $\boldsymbol{u}^-$ are displacement vectors of the positive and negative edges respectively, and the separation displacement $\boldsymbol{u} = \boldsymbol{u}^+ - \boldsymbol{u}^-$ is an interface strain measure.

In order to set the problem, interface constitutive law, relating the traction $\boldsymbol{t}$ to the displacement jump $\boldsymbol{u}$, has to be defined. In damage mechanics theory the isotropic damage variable, say ω, can be defined (see [24]) as the local ratio between the area $\mathrm{d}A_d$ of the debonded fraction of RSE and the whole RSE area $\mathrm{d}A$; that is

$$\omega = \frac{\mathrm{d}A_d}{\mathrm{d}A}. \tag{1}$$

The interface constitutive model is developed in the same phenomenological setting proposed in [8], where the damage variable is also considered as a parameter which governs the transition from the initial nondamaged elastic behavior ($\omega = 0$) up to the fully damaged one ($\omega = 1$).

At the debonded fraction of the RSE, namely $\mathrm{d}A_d = \omega\,\mathrm{d}\mathrm{d}A$, the interface edges can mutually transmits only frictional traction $\boldsymbol{t}_f$; whereas at the nondamaged RSE fraction, of area $\mathrm{d}A_s = (1-\omega)\,\mathrm{d}A$, the two edges are still bonded and they can transmit cohesive traction $\boldsymbol{t}_c$.

Let us introduce the following Helmholtz free energy

$$\Psi = \frac{1}{2}(1-\omega)\,\boldsymbol{u}_c^{e\,T}\boldsymbol{K}_c\boldsymbol{u}_c^e + \frac{1}{2}\omega\boldsymbol{u}_f^{e\,T}\boldsymbol{K}_f\boldsymbol{u}_f^e + \Psi_{in}(\xi) \tag{2}$$

where: $\boldsymbol{u}_c^e = \boldsymbol{u}$ is the elastic separation displacement of cohesive fraction; $\boldsymbol{u}_f^e = \boldsymbol{u} - \boldsymbol{u}_f^p$ is the frictional elastic separation displacement of the nondamaged fraction, with $\boldsymbol{u}_f^p$ the plastic part, whose components are the frictional sliding and the frictional dilatancy; $\boldsymbol{K}_c = \lceil K_N^c\ K_T^c \rfloor$ and $\boldsymbol{K}_f = \left\lceil K_N^f\ K_T^f \right\rfloor$ are diagonal stiffness matrices respectively of cohesive and frictional fractions, respectively; and $\Psi_{in}(\xi)$ is the internal energy, function of the scalar kinematic internal variable ξ.

The state laws are then derived by standard arguments, that is:

$$\boldsymbol{t}_c = \frac{\partial\Psi}{\partial\boldsymbol{u}_c^e} = (1-\omega)\,\boldsymbol{K}_c\boldsymbol{u} \tag{3}$$

$$\boldsymbol{t}_f = \frac{\partial\Psi}{\partial\boldsymbol{u}_f^e} = \omega\boldsymbol{K}_f\left(\boldsymbol{u} - \boldsymbol{u}_f^p\right) \tag{4}$$

$$\chi(\xi) = \frac{\partial\Psi}{\partial\xi} \tag{5}$$

$$Y = -\frac{\partial\Psi}{\partial\omega} = \frac{1}{2}\boldsymbol{u}^T\boldsymbol{K}_c\boldsymbol{u} - \frac{1}{2}\left(\boldsymbol{u}-\boldsymbol{u}^p\right)^T\boldsymbol{K}_f\left(\boldsymbol{u}-\boldsymbol{u}^p\right) \tag{6}$$

Moreover, dissipation inequality can be derived in the form of Clausius-Duhem inequality, namely

$$D = \boldsymbol{t}^T\dot{\boldsymbol{u}} - \dot{\Psi} = \boldsymbol{t}^T\dot{\boldsymbol{u}} + Y\dot{\omega} - \boldsymbol{t}_c^T\dot{\boldsymbol{u}}_c^e - \boldsymbol{t}_f^T\dot{\boldsymbol{u}}_f^e - \chi\dot{\xi} \geq 0, \tag{7}$$

where Y is the strain energy release rate, power conjugate variable of the damage ω, and χ is the internal static variable, which governs hardening and softening phenomena. It can be easily shown that

$$\boldsymbol{t} = \boldsymbol{t}_c + \boldsymbol{t}_f \tag{8}$$

so that the overall interface elastic traction-displacement relation is

$$\boldsymbol{t} = (1-\omega)\boldsymbol{K}_c\boldsymbol{u} + \omega\boldsymbol{K}_f\left(\boldsymbol{u} - \boldsymbol{u}_f^p\right). \tag{9}$$

Damage activation is governed by the following yield function

$$\phi_d\left(Y,\chi;u_T,u_N\right) = Y - \frac{1}{2}A_N u_N^2 - \frac{1}{2}A_T u_T^2 - \chi\left(\xi\right) - Y_0 \leq 0, \tag{10}$$

where $Y_0 > 0$ is the initial damage threshold, u_N and u_T are normal and tangential components of the separation displacement, $A_N \geq 0$ and $A_T \geq 0$ are constitutive parameters.

The relevant flow rules and loading-unloading conditions read

$$\begin{aligned} \dot{\omega} &= \frac{\partial \phi_d}{\partial Y}\dot{\lambda}_d = \dot{\lambda}_d, \\ \dot{\xi} &= -\frac{\partial \phi_d}{\partial \chi}\dot{\lambda}_d = \dot{\lambda}_d, \\ \dot{\lambda}_d &\geq 0, \quad \phi_d\dot{\lambda}_d = 0, \quad \dot{\phi}_d\dot{\lambda}_d = 0, \end{aligned} \tag{11}$$

with $\dot{\lambda}$ scalar multiplier.

It is to remark that in [8] the damage activation function depends only on the energy release rate Y, producing the same separation work in pure mode I, in pure mode II and in any other mixed mode debonding condition.

Activation and evolution of the frictional displacement components u_N^p and u_T^p are defined in the framework of non-associtive plasticity theory, governed by the classic Mohr-Coulomb yield function

$$\phi_p = \left| t_t^f \right| + \alpha\, t_N^f \leq 0. \tag{12}$$

and by means of the following plastic potential

$$\Omega_p = \left| t_t^f \right| + \beta\, t_n^f, \tag{13}$$

where t_n^f and t_t^f are the components of frictional traction $\boldsymbol{t}_f$, α and β, with $\alpha \geq \beta$, are respectively the frictional coefficient and the dilatancy one. The plastic (or frictional) flow rules and loading/unloading conditions are

$$\begin{aligned} \dot{u}_T^p &= \frac{\partial \Omega_p}{\partial t_T^f}\dot{\lambda}_p = \mathrm{sgn}\left(t_T^f\right)\dot{\lambda}_p, \\ \dot{u}_N^p &= \frac{\partial \Omega_p}{\partial t_n^f}\dot{\lambda}_p = \beta\dot{\lambda}_p, \\ \dot{\lambda}_p &\geq 0, \quad \phi_p\dot{\lambda}_p = 0, \quad \dot{\phi}_p\dot{\lambda}_p = 0. \end{aligned} \tag{14}$$

The cohesive model is completed by the damage softening law, which allow to derive the actual damage threshold, namely $\chi + Y_0$, defined in the damage activation function of eq.(10)

$$\chi(\xi) := \frac{1}{2}K_n^s (u_e)^2 \left[\left(\frac{u_f}{u_f(1-\xi) + u_e\xi} \right)^2 - 1 \right] \tag{15}$$

$$Y_0 := \frac{1}{2}K_n^s\, u_e\, u_f \tag{16}$$

where K_n^c is the normal stiffness of interface cohesive fraction, u_e and u_f are limit values of normal separation displacement, respectively, at the elastic threshold and at the unitary damage condition, in a pure mode I opening condition.

In accordance with experimental data, only the case of mode II fracture energy greater then the mode I value $(G_{II} > G_I)$ is considered by assuming the following constitutive parameters of damage activation function (10): $A_N = 0$ and $A_T > 0$. Moreover, in order to prevent damage activation under pure compressive stress state, normal stiffness of the cohesive fraction and normal stiffness of the frictional fraction are imposed to be equal, that is $K_N^c = K_N^f$. In fact,

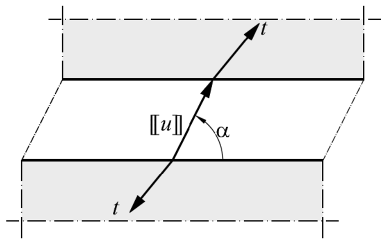

Figure 1: Complex mixed mode loading condition

for a displacement $u_N < 0$ and $u_T = 0$, and with null frictional plastic displacement $\mathbf{u}_c^p = \mathbf{0}$, the relevant energy release rate is

$$Y = \frac{1}{2} K_N^c u_N^2 - \frac{1}{2} K_N^f u_N^2 = 0, \tag{17}$$

than, damage activation function in eq.(10) is negative and damage does not evolve.

Finally in order to set all the material constants of the interface, we define the two different fracture energies G_I and G_{II} as

$$G_I = \frac{1}{2} k_N^c u_e u_f; \qquad G_{II} = \frac{1}{2} k_T^c \bar{u}_{II}^e \bar{u}_{II}^f \tag{18}$$

and since we are in the condition $G_I < G_{II}$, we set $A_{II} = 0$ and

$$A_I = K_T^c \frac{G_{II} - G_I}{G_{II}} \tag{19}$$

It can be mathematically proved that for any mixed mode debonding the total separation work G_m is bounded, namely $G_I \leq G_m \leq G_{II}$ and G_m monotonically increases from the pure mode I condition to the pure mode II condition. The latter condition, even if it is non a universal law, has been confirmed by several experimental investigations, see e.g. Benzeggagh and Kenane in [17], who measured the fracture energy of a unidirectional glass/epoxy composite for six different mixed mode conditions, by the mixed mode bending apparatus developed by Crews and Reeder in [25].

3 Numerical simulations

In the present section the response of the cohesive interface to complex mixed mode loading conditions is analysed. The quasi-static loading condition is represented in Fig.1 where the law of the imposed displacement is

$$u(t) = u_0 \sin\left(\frac{t}{T_0}\pi\right) \tag{20}$$

with $T_0 = 50$ and t a time-like variable. The numerical simulation have been performed for four different value of the loading angle, that is: $\alpha = 0, \alpha = 30, \alpha = 60, \alpha = 90$. The set of constitutive parameters of the proposed interface constitutive model are collected in Table 1. The results of the numerical simulations are plotted in Fig. 2 in terms of traction t vs imposed diplacement u for the four loading angles. It can be observed that the maximum traction and the relevant fracture energy increase from the pure mode I condition $\alpha = 0$ to the pure mode II condition $\alpha = 90$. Moreover, after the first loading cycle, the behavior is elastic.

Normal elastic stiffness	$k_n^c = 1\,500 N/mm^3$
Tangential elastic stiffness	$k_t^c = 1\,500 N/mm^3$
Mode I elastic displ.	$\bar{u}_I^e = u_e = 0.002mm$
Mode I debonding displ.	$\bar{u}_I^f = u_f = 0.05mm$
Tensile strength	$\bar{t}_I = 3N/mm^2$
Mode II elastic displ.	$\bar{u}_{II}^e = 0.0049mm$
Mode II debonding displ.	$\bar{u}_{II}^f = 0.1225mm$
Shear strength	$\bar{t}_{II} = 7.348N/mm^2$
Mode I Fracture energy	$G_I = 0.075N/mm$
Mode II Fracture energy	$G_{II} = 0.45N/mm$

Table 1: Constitutive parameters used for the numerical simulations.

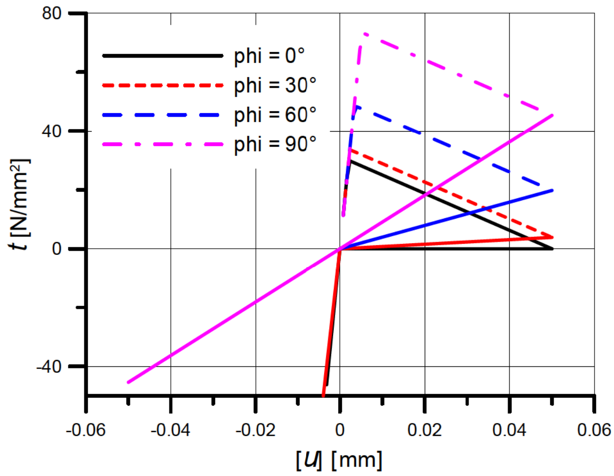

Figure 2: Response of the numerical simulations, for the complex mixed mode loading conditions, in terms of traction t vs imposed diplacement u

A second mixed mode loading test has been performed with two independent laws for the tangential and normal imposed displacement, which are plotted in Fig.3. The interface response to the second mixed mode loading condition is plotted in the Figures 4 and 5 respectively in terms of normal traction components vs time and in terms of tangential traction components vs time. The interface response shows that the unloading condition is linear elastic and any damage increment involves both the tangential and the normal components. At the full damage condition the interface cannot transmit neither normal traction or tangential traction.

4 Conclusions

The paper proposes an interface constitutive model based on a single scalar damage variable which produces effects either in normal or in tangential directions to the interface. The constitutive model enjoys thermodynamic consistency being defined through a free energy, the damage activation function and a non-associative frictional function. The evolution rules are derived in the context of dissipative mechanics with internal variables. The proposed interface model, produces two independent fracture energies, G_I in pure mode I opening condition and G_{II} in pure mode II sliding fracture. G_I and G_{II} are minimum and maximum values of the work-of-separation for any proportional loading path. The model describes also frictional trac-

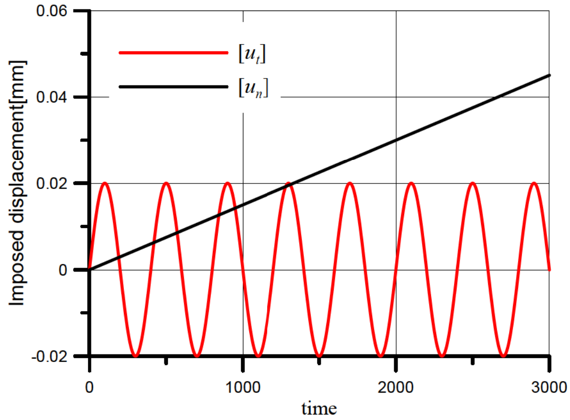

Figure 3: Laws of the normal and tangential components of imposed diplacement

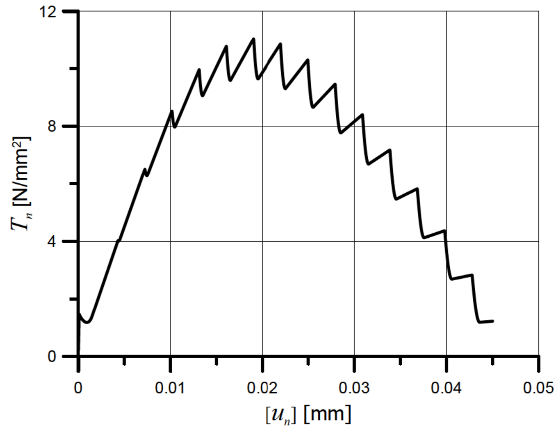

Figure 4: Response of the second complex mixed mode loading condition in terms of normal traction compontent

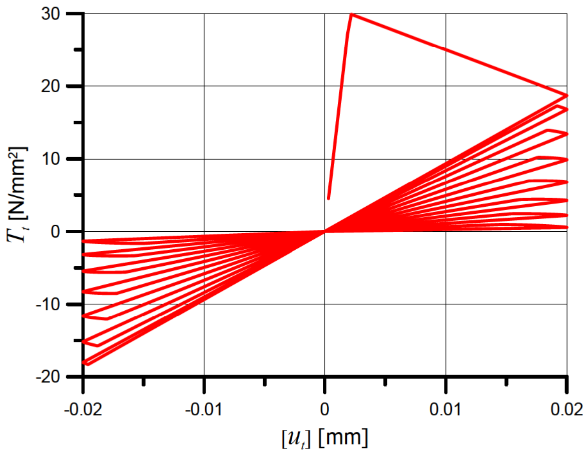

Figure 5: Response of the second complex mixed mode loading condition in terms of tangential traction compon-tent

tion both at the fully debonded zones and at the partially debonded ones under the combined action of sliding tangential and compressive forces. The proposed model is able to accurately reproduce with a unique set of few constitutive parameters, very different loading paths, either in opening mode or in sliding mode and in any mixed condition, recovering also closing conditions and frictional effects. In order to verify the actual capacities of the proposed model a non-proportional complex loading/unloading/reloading history has been applied to an interface element showing the features of the relevant interface response.

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