DATA, METHODS, AND TECHNOLOGY

Editor

Sofia V. Dermisi

University of Washington
College of Built Environments
224 Gould Hall, Box 355740
Seattle, WA 98195-5726

206-543-0756 (Phone)
sdermisi@uw.edu

This section of the *Journal of Real Estate Literature* focuses on innovative use of real estate related data and methodologies. This is also a forum for the discussion of new data sources and standardization techniques in the real estate industry. If you have any suggestions or submissions to this section, please contact Sofia Dermisi.

Associate Editors

John S. Baen (2016)
University of North Texas

John Cirincione (2016)
JVI Solutions

Mark Kaboudan (2016)
University of Redlands

Isaac Megbolugbe (2016)
Johns Hopkins University

Stuart Michelson (2016)
Stetson University

Helen Neill (2016)
University of Nevada–Las Vegas

Stephanie Y. Rauterkus (2016)
University of Alabama–Birmingham

Mauricio Rodriguez (2016)
Texas Christian University

Bo Soderberg (2016)
University of Uppsala

Stephen L. Sperry (2016)
Clemson University

Susan M. Wachter (2016)
University of Pennsylvania

Ko Wang (2016)
Johns Hopkins University

MEASUREMENTS OF RATIONALITY FOR A SCIENTIFIC APPROACH TO THE MARKET-ORIENTED METHODS

Marco Simonotti

Palermo University

Francesca Salvo

Calabria University

Marina Ciuna

Palermo University

Manuela De Ruggiero

Calabria University

Abstract

Appraisals are based on regression models. Its major application limits are all in situations in which the real estate market doesn't offer a sufficient number of comparables to implement statistical tools. In low-transaction real estate markets, the most effective appraisal method is the market comparison approach (MCA). The application limits of MCA depend on the degree of similarity between the properties' characteristics and the reliability of the prices of comparables. In this paper, we introduce some rational measurements in order to guarantee a scientific approach to MCA is associated with the correct prices. Our method is able to restore appraisals of the market value of any type of property, even in situations in which the comparables are limited or when the reliability of the buying and selling prices could be better.

The International Valuation Standards (IVS) are internationally recognized appraisal methods used in the market-oriented approach, the income approach, and the cost approach. When a real estate market is active and all necessary market data are available, the market comparison approach (MCA) is the most direct, probative, and documented method used to appraise real estate market values (Simonotti, 2006); in particular, the MCA is the most important method and is comparable to the market-oriented approach. The MCA is known by several different names in the appraisal literature. In some of the older literature it is called the market data approach, while elsewhere it is referred to as the grid adjustment technique.

There are many appraisal procedures that can be used in the logic application of the MCA. These procedures are grouped into two categories: (1) quantitative techniques based on detection of objective markets data (paired data analysis, graphic analysis, trend analysis, secondary data analysis, adjustment grid method, etc.); and (2) qualitative techniques based on the analyst's subjective judgment (relative comparison analysis, personal interviews, etc.).

In particular, the adjustment grid method (AGM) represents a link between the MCA and the multiple regression analysis, aiming to formalize the adjustment step of the MCA, without reaching the formal and symbolic perfection of the multiple regression models. There are three methods documented in the literature used in the application of the AGM (Colwell, 1983): The first is the additive dollar adjustment method (ADAM), which is based on the assumption that the selling price is a linear function of the property attributes. The second, is the additive percentage adjustments method (APaM), which may be rationalized by a number of functional forms. In general, the percentage difference in prices between a subject and a comparable is found by adding functions of the two properties' attributes. The third is the multiplicative percentage adjustment method (MPaM), in which it is supposed that the ratio of the selling price to that of the comparable is equal to the product of functions of the subject and the comparable's attributes.

The evaluation of hedonic prices in applying AGMs can be done directly through paired data analysis and indirectly by acquiring hedonic prices obtained through the application of multiple regression analysis to a sufficiently large sample. There are situations in which both modes are not very applicable due to the lack of data suitable for the application of the paired analysis and the limited availability of sufficient data to the application of multiple regressions (Colwell, Cannaday, and Wu, 1983). Even when the data are available, a statistical analysis does not always lead to consistent results in terms of quantities because they achieved following generalizations that are not always able to take into account the specificities of each property.

To overcome these drawbacks, some authors provide an approach other than ADAM, APaM, and MPaM, in which hedonic prices have to be calculated through formulations arising from the application of the valuation criteria (market value, cost value, transformation value, complementary value, replacement value), designed to calculate the hedonic prices indirectly through established formulas (Simonotti, 1997). The application of these mathematical formulations resolves the issue in an objective evaluation of the hedonic price of real estate quantitative characteristics, although some limitations are attributable to: (1) the presence of duplication in formulas that can lead to overestimates or underestimates; (2) the degree of similarity between the properties characteristics; or (3) the reliability of the comparable prices.

The presence of abnormalities, related to the similarity and reliability in the process of reconciliation, requires the application of analytical criteria able to measure dissimilarity in the characteristics of the comparable sample and anomalies in their buying and selling prices. These measures, associated with the correct prices in the appraisal synthesis, lead to more efficient valuations.

Our aim is to introduce some rational measurements in order to guarantee a scientific approach to the MCA by simplifying the use of the theorems used in the procedure and correcting anomalies related to incidental dissimilarities or unreliable prices. The proposed implementation involves the introduction of a correction factor which, used in the reconciliation, takes into account the phenomenon of duplication, as well as introducing similarities and reliability coefficients intended to monitor anomalies and dissimilarities in the sample. The Appendix contains a numerical example, which

shows the use of the proposed methodology and the method of calculation of the correction coefficient proposed in this paper.

LITERATURE REVIEW

The MCA is known by several different names in the appraisal literature. In some of the older literature it is called the market data approach, while elsewhere it is referred to as the grid adjustment technique. The sales comparison approach is usually of interest to professionals and academicians. Some academicians address the study of the sales comparison approach in order to offer methods for reducing or eliminating the appraisers' subjective judgments. Much of this literature can be found in Lentz and Wang's (1998) review of some academic books and articles that deal with the sales comparison approach. Another important work that focuses on the sales comparison approach is that of Colwell, Cannaday, and Wu (1983), who demonstrated how to derive adjustment factors using the ordinary least squares (OLS) method rather than appraiser judgment. Manaster (1991) reported that when the sales comparison approach was followed correctly, different appraisers using slightly different methods derived final appraisals that were very similar. Some interesting academic studies focused on the sales comparison approach with greater detail. Isakson (1986, 1988) presented a technique called the nearest neighbors appraisal technique (NNAT) according to which the final estimate of value is calculated as a weighted average of the actual selling prices of the comparable properties. The weights in NNAT are determined by a multidimensional measure of similarity, so that comparable properties more similar to the subject have greater weight. The NNAT eliminates the need to calculate the adjusted selling prices of the comparable properties. Vandell (1991) presented a minimum variance (among the adjusted values of the comparable sales) approach for selecting and weighting comparable properties, while Gau, Lai, and Wang (1992, 1994) proposed a variation of Vandell's techniques where the coefficient of variation replaced Vandell's variation as the measure to be minimized. In both techniques, the adjusted values of the comparable properties were calculated using the dollar additive technique and OLS-derived adjustment factors. Isakson's (1986) NNAT, Vandell's minimum variance, and Gau, Lai, and Wang's minimum coefficient of variation are computationally complex and their techniques have not been given much attention in professional work for this reason. Pace (1998) presented the sales comparison approach in a mathematical format (matrix algebra). In doing this, Pace was better able to focus on some of the more interesting aspects of this approach. Unfortunately, his model does not completely capture the sales comparison approach as practiced by appraisers.

Isakson (2002) described the sales comparison approach to value as a system of linear algebra equations. The difference between the appraiser and academician views of this system of equations is shown to pivot on the one-price assumption. The need of appraisers to subjectively derive adjustment factors is shown to be an artefact of the common practice of appraisers to assume multiple indicated values. The use of multiple regression analysis and other statistical techniques by academicians is shown to be possible only given the one-price assumption. Suggestions are presented for improvement of both views.

MingYou and Chang (2009) discussed the weight model and forecast weights directly instead of only forecasting value. By elaborating on the core question of weights, in this paper we hope to assist the degree of science and objectivity of appraisal.

In Italy, the MCA has been introduced with substantial changes as regards the methodology used in the United States, particularly in relation to the approach used to estimate real estate characteristics' hedonic price (Simonotti, 2006). Essentially, the methodology developed was intended to quantify the hedonic prices of properties without resorting to approaches like ADAM, APAM, and MPAM, which are difficult to apply in Italy. Instead, the technique in this work was based on indications of a mathematical purpose, designed and formulated by applying the economic criteria of the market value, the cost value, the value of transformation, the complementary value, and the value of subrogation. Later studies conducted at a national level made it possible to simplify and improve the methodology proposed by Simonotti (2006) with regard to the duplication of characteristics and the reliability of sales data.

METHODS

MARKET COMPARISON APPROACH

The MCA is an assessment procedure that leads to the evaluation of the market value of a property on the basis of comparable properties that have been recently sold. The MCA is used to determine property value through a process of monetary adjustments. The MCA is suitable for the evaluation of any kind of property, provided that there are an adequate number of recent transactions of comparable properties. The MCA is a systematic procedure based on monetary adjustments to be made at market prices, to take into account the differences between the characteristics of comparable properties and those of the property being appraised. The MCA has the followings steps: (1) market analysis, designed to collect transaction data of comparable properties; (2) choice of the property characteristics that will drive the comparison; (3) compilation of the data table and drafting of technical documents (photos, plans, etc.); (4) hedonic price analysis; (5) calculation of the corrective factors; (6) calculation of the degree of similarity; (7) calculation of the degree of reliability; (8) calculation of composed indices; (9) sales adjustment grid; and reconciliation.

Phases 1 to 3 concern the collection of data and the choice of the driver of the property's characteristics; phases 4 to 10 relate to the preparation of the evaluation report.

The hedonic price analysis (phase 4) is the central phase of the process, as hedonic prices are the basic adjustments of the MCA. In particular, for each characteristic, an hedonic price is calculated and used to adjust the observed prices. The evaluation of the hedonic prices makes use of the substitution principle, the complementarity principle, and the evaluation criteria (market value and cost value). The hedonic price can assume positive values in case a variation in the increase of a characteristic corresponds to an increase in the price; it can assume negative values when an increase

in a characteristic corresponds to a decrease in the price; it can assume null values if a variation in a characteristic does not produce any change in the price.

HEDONIC PRICE FORMULAS

The formulation of the hedonic prices is carried out through the application of appraisal criteria such as price value, cost value, transformation value, complementary value, and replacement value. The hedonic price of a feature represents an adjustment of the MCA. The adjustments can be applied to real estate for comparison in terms of value or percentage (adjustment percentage).

The analysis of the hedonic prices of the main features of the properties can be carried out in detail for each characteristic. It has to clarify that theoretically the real estate characteristics that contribute to the formation of the price are very numerous and property-specific.

The application of MCA, however, is exclusively focused on properties in the sample. Although the equitable properties are not relevant in the valuation, as they do not entail price changes in the evaluation table. The primary application of the MCA is therefore to seek comparable properties of the same type, and above all in the same vicinity, in order to make void the locational factor (in terms of accessibility, services, infrastructures, etc.). Otherwise it would be difficult to evaluate in quantitative mathematical terms if there is a small number of data.

The qualitative characteristics (dummy variables) are: the panoramic views, brightness, the architectural quality, etc., which deserve a separate discussion. At the present time there are no direct quantitative appraising formulas for these variables, although they are quantified by different methods from MCA, even in the presence of few data (appraisal system approach, ASA, Simonotti, 2006).

For each quantitative real estate characteristic, it is possible to quantify the relative hedonic price with direct formulas deduced from the appraisal literature (Simonotti, 2006) or on the basis of score valuations given by the assessor in the appraisal report. By way of example, some formulas relating to residential property are shown. The presentation format lends itself to be easily extended to the characteristics of the property with other destinations and types.

Sale Date. The hedonic price of the contract date will update the prices at the valuation date, taking into account the increases or decreases through rate changes. The feature data are measured in terms of years (or months); the hedonic price expresses the price revaluation referable to the unitary time, measurable through the property price index numbers (Salvo, Simonotti, and Ciuna, 2014; Salvo, Simonotti, Ciuna, and De Ruggiero, 2014), the revaluation rate (u) is usually negative because the date of sale is measured in retrospect than at the time of evaluation. The hedonic price sale date can be expressed as:

$$p_{date} = Prz \cdot u, \quad (1)$$

where Prz is the total price of the comparable property, expressed in €; and u expresses the rate of devaluation/revaluation.

Surface. A really important characteristic is the surface one. Surface is the whole surface of the property, expressed in square meters, obtained by adding to the main surface (i.e., the inner one) and the secondary ones (balconies, attics, garages), each one weighted on the basis of a percentage ratio. The calculation of the hedonic prices for the surface characteristic can be resolved based on the average price:

$$p_1 = \frac{Prz}{S}, \quad (2)$$

where p_1 is the hedonic price of the surface, expressed in €/sqm; and S represents the surface of the property, expressed in square meters.

Exterior Surfaces (built-up). Exterior surfaces include the land used for patios, gardens, and service spaces. The hedonic price of the external surfaces can be evaluated by applying the transformation value: the external surface hedonic price can be set equal to the average price of the building areas minus the average unit cost of demolition, c_d , such that:

$$\bar{p} = p - c_d, \quad (3)$$

where p is the average price of the building areas, expressed in €/sqm; and c_d is the average unit cost of demolition in €/sqm. It is important to consider the cost of demolition due to the fact that there is a building on the property. Therefore the value of the land is calculated as the difference between the value of the land considered free of buildings and the expenses necessary to make it free.

Restrooms. The restroom feature refers to the number of toilets in the property. The hedonic price of the restrooms feature is evaluated according to the criteria of the depreciated cost of reconstruction, that is, as the cost of rebuilding again, usually depreciated according to a linear law:

$$p_{RES} = C \cdot \left(1 - \frac{t}{n}\right), \quad (4)$$

where p_{RES} is the hedonic price of the restrooms, expressed in €/n; C is the average cost for the installation of a new restroom, expressed in €; t is the past life, expressed in years; and n is the economic life of a toilet, in years. This approach it is usually considered an ordinary complete bathroom, with no need to express the surface consistency. It is observed that the total price of the property increases more than proportionally in the transition from one restroom to two, while it tends to increase less than proportionally when there are three or more.

Level Plan. The market price of a housing unit in a multi-storied building is linked to the level occupied inside the building, and then to its floor level. In principle, buyers prefer the intermediate levels compared to the first and the last, for environmental and

climate considerations. The presence of an elevator is one of the factors that most effects the appreciation of the level of the plan. It can be used as a level rate that expresses the percentage change in the total price going from one floor level to another. It is a quantitative parameter of the segment of the real estate market. It is generally presented as a percentage increase in the rise of the plan (in presence of the elevator) or decrease (in the absence of the elevator), or as a lump sum from one floor to another. These values are sometimes reported in the publications of the real estate industry.

The level plane hedonic price can be calculated by multiplying the total price of the property by the ratio percentage represented by the level rate:

$$p_{FLO} = \theta P, \quad (5)$$

where p_{FLO} is the hedonic price of the level plan, expressed in €/level; P is the total price of the property, expressed as a €; and θ is the level rate. In particular, if the subject level is higher than that of the comparable, it is: $p_{FLO} = \theta P$; if the subject level is less than that of the comparable, $p_{FLO} = \theta/(1 + n \cdot \theta) \cdot P$, where n is the difference between the floor level of the property being valued and the floor level of the comparable.

Technological Systems. An important factor in property appraisal is the presence of some technological systems, such as an elevator, air conditioning, heating system, and so on. The technological systems' hedonic price is calculated using the replacement depreciated cost, using a not linear law of depreciation:

$$p_{\text{technological system}} = C \cdot \left[1 - \sum_{s=1}^t \frac{2(n-s+1)}{n(n+1)} \right], \quad (6)$$

where C is the average cost for the installation of a new system, expressed in €; t is the life of the system in years; and n is the economic life of service in years. A different hedonic price has to be determined for each technological system.

Maintenance and Preservation. The hedonic price for maintenance and preservation state is the cost of repairs, maintenance or adjustment.

SALES ADJUSTMENT GRID

Once the hedonic prices have been calculated, the market value is calculated by monetary adjustments to the market prices. The market value of the subject property is appraised through a series of adjustments to the comparables' market prices; particularly, the market price of every comparable is corrected through a process of unitary adjustments (or hedonic prices) multiplied by the differences in the amount of the quantitative characteristics between the comparables and the subject. The correct prices represent the market prices that the comparables have if they possessed the same characteristics as the subject property. Thus, they represent different appraisals of the subject's value.

The adjustment for the j -th ($j = 1, 2, \dots, m$) property and for the i -th characteristic ($i = 1, 2, \dots, n$) is equal to the product of the hedonic price p_i (defined as the change in the total price corresponding to a unit change in the amount of the characteristic) and the difference between the characteristics of the subject x_{i0} and those of the comparables x_{ij} :

$$p_i \cdot (x_{i0} - x_{ij}). \quad (7)$$

The correct price of the generic j -th comparable P_{cj} is obtained by adding algebraically to the P_j price sale the adjustments corresponding to the n characteristics:

$$P_{cj} = P_j + \sum_{i=1}^n p_i \cdot (x_{i0} - x_{ij}). \quad (8)$$

The correct prices are then condensed into a single market value V , usually obtained as the expected value of the correct prices. In the case of equal probability, V is equal to:

$$V = \frac{1}{m} \cdot \sum_{j=1}^m P_{cj}. \quad (9)$$

CORRECTIVE FACTORS IN THE SALES ADJUSTMENT GRID

The mathematical formulations derived from appraisal criteria contain some duplication, which may lead to overvalue (or undervalue). The surface hedonic price p_{SUR} cannot be compared to the total sale price P , but rather to the rate of total price P_{SUR} , which is exclusively comparable to the main surface:

$$p_{SUR} = \frac{P_{SUR}}{x_{SUR}}, \quad (10)$$

where x_{SUR} is the main surface. On a practical level, it is not possible to a priori identify the predicted price rate P_{SUR} from the total sale price P . It is therefore necessary to resort to an approximation using the sale price P instead of the main surface price P_{SUR} :

$$p_{SUR} = \frac{P}{x_{SUR}}. \quad (11)$$

Using the total sale price P , the main surface hedonic price is calculated as:

$$p_{SUR} = \frac{P}{x_{SUR}} = \frac{P_{SUR} + P_{EXT}}{x_{SUR}}, \quad (12)$$

where P_{EXT} represents the rate of price due to all the characteristics. The approximation depends on the presence of the rate of price P_{EXT} , and significantly

affects the adjustments made to the comparable properties' characteristics in the sales adjustment grid, in which each adjustment related to the main surface is calculated as:

$$\left[\frac{P_{SUR} + P_{EXT}}{x_{SUR_j}} \right] \cdot (x_{SUR_0} - x_{SUR_j}), \quad (13)$$

and therefore:

$$\left[\frac{P_{SUR}}{x_{SUR_j}} + \frac{P_{EXT}}{x_{SUR_j}} \right] \cdot (x_{SUR_0} - x_{SUR_j}), \quad (14)$$

where the subscript j indicates the generic comparable, and the subscript 0 the subject. The exact adjustment to the main surface should be relative just to the rate of the total price P_{SUR} :

$$\frac{P_{SUR}}{x_{SUR_j}} \cdot (x_{SUR_0} - x_{SUR_j}), \quad (15)$$

whereas, in fact, it takes into account all of the other price rates:

$$\frac{P_{SUR}}{x_{SUR_j}} \cdot (x_{SUR_0} - x_{SUR_j}) + \frac{P_{SER}}{x_{SUR_j}} \cdot (x_{SUR_0} - x_{SUR_j}) + \dots \quad (16)$$

Note that in the adjustment shown there are overestimations (and/or underestimations) linked to the presence of price rates that differ from those closely linked to the main surface. If the only practical way is to use the total sale price, at least in theory the cited approximations will have to be determined in subsequent steps, appropriately purifying each one in the adjustment of the corresponding characteristic.

The effect of approximations can be mathematically identified and eliminated by observing the presence of repeated terms in each summand in the general adjustment formula. In particular, it is clear that each member in (16) is formed by the product of the single price rate and a term exclusively linked to main surface variables:

$$\left(\frac{x_{SUR_0} - x_{SUR_j}}{x_{SUR_j}} \right). \quad (17)$$

Such a term is independent of the specific price rate, depending on the subject's main surface area and that of the j -th comparable, so that it can be written as:

$$r_j = \left(\frac{x_{SUR_0} - x_{SUR_j}}{x_{SUR_j}} \right). \quad (18)$$

By introducing the r_j coefficient, (16) can be written as:

$$P_{SUR} \cdot r_j + P_{RES} \cdot r_j + \dots \quad (19)$$

or, in terms of hedonic prices:

$$p_{SUR} \cdot x_{SUR} \cdot r_j + p_{RES} \cdot x_{RES} \cdot r_j + \dots \quad (20)$$

In this way, each approximation can be compactly expressed in terms of the r_j coefficient, with the subscript j indicating the generic property; each overvaluation (or undervaluation) can be written as:

$$p_{ij} \cdot x_{ij} \cdot r_j, \quad (21)$$

where p_{ij} is the hedonic price of each characteristic that is different from the main surface. In light of what has been stated previously, p_{ij} can be referred to as the first price commercial theorem in any practical case: the hedonic price of the main surface is first calculated from the total sale price, as shown in equation (11); afterwards, the hedonic price of all the other characteristics is determined. Eventually, every possible error (over or undervaluation) in the determination of the hedonic price of the main surface is removed from the sales adjustment grid. Each adjustment can be calculated as:

$$p_{ij} \cdot (x_{0i} - x_{ij}) - p_{ij} \cdot x_{ji} \cdot r_j. \quad (22)$$

It can be shown that (22) can also be written as:

$$p_{ij} \cdot (x_{0i} - x_{ij} \cdot (1 + r_j)), \quad (23)$$

where (23) represents the i -th adjustment of the j -th comparable. The use of the first price commercial theorem is generalized so that the minimum of the calculated prices can be used, because in this kind of appraisal, only the key characteristics are considered and all the other characteristics are not.

The correct price P_{cj} of the generic purchase j -th is obtained by adding algebraically to the purchase price adjustments P_j , which correspond to the n characteristics in question:

$$P_{cj} = P_j + \sum_{i=1}^n p_{ij} \cdot (x_{0i} - x_{ij} \cdot (1 + r_j)). \quad (24)$$

The correct prices reported in the evaluation table are then combined into a single value or a range of values of the property being appraised: a point estimate is carried out by selecting the value corresponding to the mean, mode or median of the prices, corrected by considering the number of the measured data, the variability of real estate, the accuracy of the data, the reliability of the source of information, and the

likelihood of the estimated marginal prices. Otherwise, an interval encompassed by the minimum and maximum correct price is indicated.

MEASURE OF THE SIMILARITY AND RELIABILITY DEGREE

The central question of the application of MCA is the need to identify a comparison sample of properties similar to the subject. A first consideration is due to “the similarity degree” of the properties in the sample. A second consideration relates to price data which, although corresponding to the definition of market value, may also be affected by contingent circumstances and present a different “reliability degree” for the purposes of estimation. In the case of MCA, similarity and reliability of data are considered in the reconciliation that determines the expected market value of the correct prices and in the absence of other assumptions, assigns the same probability to each correct price. The hypothesis is that the values to be reconciled are derived from similar properties, characterized by sales prices that are equally reliable. The inevitable presence of anomalies requires the introduction in the reconciliation phase of analytical criteria able to measure the dissimilarity of the sample prices in terms of the characteristics and anomalies in the comparables. These measures are associated with the correct prices in the appraising synthesis.

MEASURE OF THE SIMILARITY DEGREE

A similarity coefficient provides a measure of the degree of similarity between the comparables, giving values between 0 and 1, the first corresponding to observations with no common elements, and the second relating to the surveys that perfectly meet the criterion used to measure the similarity. The similarity coefficients usually reported in literature can be divided into symmetrical coefficients and asymmetrical ones. While using symmetrical coefficients, null values are interpreted as the absence of the corresponding characteristic (e.g., the absence of a garage), and therefore the null data have the same comparative value of the others; in the case of asymmetric coefficients, the null value indicates the absence of information (e.g., it was not possible to measure the surface of the garage). A further distinction is between binary coefficients and semi-quantitative or quantitative coefficients: the former are useful in situations where it is sufficient to establish the presence or absence of the characteristic, while the latter are functional to interpret and numerically compare observations of the quantitative data. Quantitative coefficients are not numerous because, in the presence quantitative data, it is usually preferred that the use of distance measures returns a null value for identical observations and a variable value for different observations. The similarity measures can be obtained as 1 minus the distance coefficients. The distance coefficients are usually symmetrical and treat the 0 value as an actual measurement instead of a lack of information. In the real estate sector, the observations consist of quantitative and semi-quantitative data, so that it is possible to measure the similarity degree through symmetrical distance measurements. Therefore, a measure able to detect the similarity of individual properties has its foundation in the “closeness” between the amounts of property characteristics: greater comparability means greater proximity between the amounts of the characteristics, while less comparability means

more distant amounts. The measure of the degree of similarity of comparable properties may, therefore, be expressed as a function of the difference between the amounts of the i -th characteristic of the comparable and those of the subject. This difference can be expressed in terms of absolute value in the following way:

$$gs_a^{j*} = \frac{\sum_{j=1}^m \sum_{i=1}^n \left| \frac{x_{ij} - x_{i0}}{\bar{x}_i} \right| - \sum_{i=1}^n \left| \frac{x_{ij} - x_{i0}}{\bar{x}_i} \right|}{(m - 1) \cdot \sum_{j=1}^m \sum_{i=1}^n \left| \frac{x_{ij} - x_{i0}}{\bar{x}_i} \right|}, \quad (25)$$

where gs_a^{j*} is the indicator of the degree of similarity of a j^* generic comparable and $\bar{x}_i$ is the average of the considered characteristic (Colwell, Cannaday, and Wu, 1983). The difference between the amount of the i -th characteristic of the comparable and that corresponding to the subject property can also be expressed in terms of square standardized distances, as follows:

$$gs_q^{j*} = \frac{\sum_{j=1}^m \sum_{i=1}^n \left| \frac{x_{ij} - x_{i0}}{\bar{x}_i} \right|^2 - \sum_{i=1}^n \left| \frac{x_{ij} - x_{i0}}{\bar{x}_i} \right|^2}{(m - 1) \cdot \sum_{j=1}^m \sum_{i=1}^n \left| \frac{x_{ij} - x_{i0}}{\bar{x}_i} \right|^2}, \quad (26)$$

where gs_q^{j*} is the degree of similarity of a generic property of comparison j^* .

MEASURE OF THE DEGREE OF RELIABILITY

The need to assess the degree of reliability is related to the presence of any abnormal market prices of the comparables. The anomaly exists if the following assumptions hold: (1) prices were collected in the same market segment of the subject; (2) prices are recent and obey the definition of market value; and (3) prices were obtained following a consistent methodology and gathered from common and comparable sources. The methodological condition concerns the possibility for the evaluator to exclude, by a feedback, anomalous prices and related properties from the comparable sample when the corresponding correct prices exceed the threshold of divergence. Under these assumptions and in accordance with the methodological condition, the measurement of the degree of reliability of the test operates within the range of divergence and, possibly, in the neighborhood of the extreme threshold. It is the evaluator's task to decide whether to use the measure of the degree of reliability in the synthesis of the final estimate and justify this choice, within the limits imposed by the assumptions and methodological conditions. Data are obviously excluded from measurement (and from the appraisal) if they are extraneous to the evaluation or introduced to compensate for their presence with the use of the measure of reliability in the final synthesis.

In order to identify outliers, it is necessary to compute in advance all the adjustments to be made to the different characteristics and calculate prices, corrected by means of

the evaluation table. As it is known, the correct prices should theoretically converge perfectly, and therefore any error can be sought in the differences between the correct prices. The measure of the degree of reliability in each comparable, in relation to the correct price, is determined by the following:

$$ga^{j*} = \frac{\left(1 - \left| \frac{P_{cj*} - \bar{P}_{cj}}{\bar{P}_{cj}} \right| \right)^{m+1}}{\sum_{j=1}^m \left(1 - \left| \frac{P_{cj*} - \bar{P}_{cj}}{\bar{P}_{cj}} \right| \right)^{m+1}}, \quad (27)$$

where P_{cj*} is the correct price of the j^* sale and $\bar{P}_{cj}$ is the average correct price.

The variation function has an almost linear trend taking values between 0 and $1/(m-1)$: the greater the difference between the correct price and the correct price of the comparable average of the sample, the lower the reliability of the corresponding correct price.

COMPOSED INDICES

The presence of significantly divergent correct prices induces the evaluator to conclude that the comparable sample is affected by defects of similarity between comparable properties or anomalies in the comparable market prices. When the simultaneous presence of dissimilarities and anomalies is detected, it is useful to use simultaneously the similarity measure of the compared properties and the reliability measure of the buying and selling prices; the two different procedures are then combined into a single procedure. If gs_a^{j*} , gs_q^{j*} , and ga^{j*} are used to denote, respectively, the indicators of the measure of the degree of similarity, and reliability of trades, the overall indicator to be assigned to each transaction is provided by a second level index obtained as a combination of the similarity measures and reliability. The measures of the composite index, gc_a^{j*} , referred to the absolute value of the distances, and of the composite index gc_q^{j*} , referred to as the square of the distances, are the following:

$$gc_a^{j*} = \frac{gs_a^{j*} \cdot ga^{j*}}{\sum_{j=1}^m gs_a^{j*} \cdot ga^{j*}}, \quad (28)$$

$$gc_q^{j*} = \frac{gs_q^{j*} \cdot ga^{j*}}{\sum_{j=1}^m gs_q^{j*} \cdot ga^{j*}}, \quad (29)$$

RECONCILIATION

The final step of the MCA is represented by the evaluation synthesis during which, in the case of point estimation, all correct prices are combined into a single value. The correct prices are verified by calculating the percentage difference. If the difference falls within the eligibility threshold, the market value is assigned as the simple arithmetic average of the correct prices, implicitly considering the same level of probability to the correct prices and therefore acceptable conditions of similarity

and reliability. If, however, the difference exceeds the threshold for eligibility, it is possible to choose a fixed price based on the same considerations for the performance of the appraisal, or delete a comparable with apparent anomalies, or assign subjective probability functions to the correct prices on the basis of information external to the procedure. In quantitative terms, the basic principle of reconciliation lies in the assumption of ordinariness in the formulation of the statistical criterion, for which the prices represent a random variable that associates an event (the correct price) to a given probability. In this perspective, a probability function can be constructed by exploiting the properties of similarity and reliability measures as the probability measures associated with the correct prices. If the correct price divergence is related to the characteristics, the probability is proposed as a coefficient of similarity; if divergence of the correct prices is detected in the purchasing and selling prices, the probability is proposed as a reliability coefficient; finally, if there are anomalies in the characteristics and market prices, the probability is proposed as second-level indicators. In all the circumstances indicated, the more likely market value of the property being valued is calculated as expected value as follows:

$$V = \sum_{j=1}^n g_j \cdot P_{cj}, \quad (30)$$

where g_j is the probability associated to the j -th mode. The coefficients and the composite index play a dual role: in the first role, since they are built on the basis of statistical measures, they take into account the distortion of the data sample detected in the assessment; the second role can be interpreted as probabilities associated with correct prices that are considered as random variables. In the latter role, they are used in the synthesis assessments. In order to show how to use the proposed coefficients, in the Appendix we provide a numerical example.

RESULTS

We present an innovative procedure intended to apply in a more simply way the methodological approach known in the literature as MCA. First, the proposed method involves the exogenous appraisal of the hedonic prices of real estate properties through mathematical formulations generated by the application of fundamental economic principles of property valuation. Thus, the evaluator can apply the methodology without necessarily knowing in advance the hedonic prices with certain procedures such as mass appraisal.

Another new element is the definition of some rational coefficients that are able to improve the application of the MCA. We propose measures that can (1) control the possible presence of anomalies in the formulations, and (2) reduce the impact of any dissimilarity or anomalies in the comparable sample. As for the duplication, these are due to the MCA mathematical formulation itself, which in some cases may lead to overvaluation or undervaluation in many practical situations. An interesting possibility is the use of corrective coefficients related to each comparable: using corrective coefficients diminishes distortions in the hedonic price analysis, and simplifies the appraisal procedure.

The proposed approach starts by determining the hedonic price of the surface feature using the sale price and surface area. In order to take into account the impact on the price of all the property's characteristics, the method includes the use of some corrective factors during the drafting of the evaluation table, which are able to correct overestimates or underestimates incurred in the evaluation process.

Dissimilarities and anomalies are linked to the need to identify a sample of comparables similar to the property being valued. A first consideration is the degree of similarity of compared properties. A second consideration relates to the market prices recorded that, while meeting the definition of market value, can be constrained by the surrounding circumstances and present a different degree of reliability.

In appraisal methodologies based on adjustments, the similarities and the reliability of the data are reflected in the reconciliation, which determines the market value as the expected value of correct prices, and unless there are other assumptions it attributes the same probability to each correct price. The values to be reconciled are derived from similar properties, characterized by sale prices that are equally reliable. The inevitable presence of anomalies suggests the opportunity to identify a procedure that is able to detect and measure the dissimilarity and anomalies in the comparables. In the reconciliation, a probability measure is introduced that is associated with each correct price, constructed through quantitative measure of the comparability level between the comparables and the subject property.

We identify through the definition of analytical criteria measures to quantify the dissimilarity in the real estate sample in terms of property characteristics and anomalies in the comparables' selling prices. In order to take into account simultaneously the dissimilarity and anomalies in prices, we propose a second level index. The similarity and reliability coefficients and the second level index have been studied as measures of probability, and in this way they are introduced into the evaluation. These measurements can then be associated with the correct prices in the appraisal process that to the appraised market value of a property.

CONCLUSION

The MCA is without doubt one of the most reliable appraisal procedures. Compared to the market-oriented methodologies employing multiple regression models, the MCA provides precise and accurate evaluations, using just a few comparable properties in a very limited geographic area, without the use of extrapolations and generalizations. The MCA, moreover, being able to work with a reduced number of data, is well suited to evaluate properties in limited markets where there is not enough comparison data to implement statistical models.

It should be noted that the market approach, in order to provide reliable results, must work with data meeting criteria of similarity and reliability and based on mathematically and logically correct formulas. An interesting possibility is the use of corrective coefficients related to each comparable: using corrective coefficients diminishes distortions in the hedonic price analysis, and simplifies the appraisal

process. Results obtained using corrective coefficients can diverge considerably from those obtained using classic formulations, in greater measure as well as comparables' complexity and dissimilarities.

The application of the MCA can be further simplified by substituting the sales adjustment grid with an adjustment vector, considering mean values in the sample instead of individual comparables' features, comparing the subject with an ideal "average" comparable. The values obtained using the mean values converge with those obtained by comparing the subject with each comparable, or diverge in minimum measure in presence of characteristics for which mean values lose descriptive significance. However these divergences are negligible in view of the relevant level of simplification of the procedure, which increases as the number of comparables increases.

The identification of a comparables sample that meets the criteria of similarity and reliability is a prerequisite to the application of market-oriented procedures. The traditional formulation of the MCA assumes an equal probability distribution of the correct prices. This assumption reflects in the reconciliation in which the appraised value is calculated as a simple arithmetic average of the correct prices. The presence of correct prices diverging over the eligibility threshold indicates the presence of anomalies in the comparable sample, anomalies which may invalidate the hypothesis of same statistical probability.

The MCA can be used to construct a different probability distribution associated with the correct prices by calculating a series of measures to express quantitatively the degree of similarity and reliability of the comparables. These statistical measures have the dual purpose of detecting any distortions in the data and represent the probabilities associated with the correct prices. The market value is, therefore, resigned as the expected value of the correct prices, associating each comparable a probability measure consisting of: (1) the degree of similarity, if anomalies are traced in the distance in the amount of the property's characteristics; (2) the degree of reliability, if the fault lies in the matching of buying and selling prices; (3) from the second level indicators, if the differences relate to both the possible circumstances.

The market investigation, aimed at finding sales data meeting the necessary requirements of similarity and reliability, is the most important element of the evaluation since the credibility of the evaluation process is based on these data. A procedure to detect any anomalies and discrepancies in the sample represents a further investigative tool, allowing the quantitative measure of the homogeneity of the sample, and providing an answer to the problem of evaluation in circumstances when conditions are not ideal.

The approach based on the use of measures and numerical indices mathematically determined on the basis of available data, and the construction of an objective probability distribution give transparency and ultimately objectivity to the appraisal. Our aim is to propose some rational measures to improve and simplify the use of MCA, checking for duplication, dissimilarities, and anomalies in the data sample,

providing a methodology that may be operationally used by the individual practitioner who cannot rely on models whose results are often statistically correct but sometimes not satisfying the market evidence.

Our proposed method is aimed at professionals working in the field of real estate valuations, in every situation in which there is very limited data, not suitable for a statistical valuation. Interesting possibilities are looming in the field of mass appraisal. A procedure that, thanks to the presence of variables, formulas, and indices is well suited to be implemented in a programming language, entering the broader line of research of automated valuation methods.

APPENDIX

In order to test the accuracy of the proposed MCA, we provide a numerical example based on a concrete appraisal sample concerning flats in a condominium. We conduct a market analysis to collect data relating to contracts in recent sales of properties similar to the one being valued. In this way, the analysis identifies three properties similar to the one being valued, all located in the same neighborhood (Exhibit A1).

In the sample, we have identified driver characteristics, that is, the characteristics that are common to the entire sample but different in mode (Exhibit A2). The most difficult step in the application of the proposed methodology is the analysis of the hedonic prices of the property's characteristics. On the basis of the data table, the hedonic price of each characteristic is calculated as indicated in the main text.

SURFACE HEDONIC PRICE

The main surface's hedonic price is obtained for comparables A, B, and C, as follows:

$$p_{SUR_A}^A = \frac{SPR^A}{SUR_A} = \frac{€125,000.00}{mq73.91} = \frac{€}{sq} 1,691.25,$$

$$p_{SUR_B}^B = \frac{SPR^B}{SUR_B} = \frac{€175,000.00}{mq118.00} = \frac{€}{sq} 1,483.05,$$

$$p_{SUR_C}^C = \frac{SPR^C}{SUR_C} = \frac{€140,000.00}{mq105.00} = \frac{€}{sq} 1,333.33.$$

HEDONIC PRICE OF RESTROOMS

The hedonic price of restrooms is evaluated as indicated in the main text:

$$p_{RES} = C \cdot \left(1 - \frac{t}{n}\right) = €10,000.00 \cdot \left(1 - \frac{5}{15}\right) = \frac{€}{n^\circ} 6,666.67,$$

Exhibit A1
Sales Adjustment Grid (€)

Sale Price and Characteristic	A	B
Sale price	P_A	P_B
Characteristic	$P_{iA}(x_{0i} - x_{iA} \cdot (1 + r_A))$	$P_{iB}(x_{0i} - x_{iB} \cdot (1 + r_B))$
Correct price	$P_{cA} = P_A + \sum_{i=1}^n p_A(x_{0i} - x_{iA} \cdot (1 + r_A))$	$P_{cB} = P_B + \sum_{i=1}^n p_B(x_{0i} - x_{iB} \cdot (1 + r_B))$

Exhibit A2
Data Table

Sale Price and Characteristic	Comparable A	Comparable B	Comparable C	Subject
Sale price (SPR) €	125,000.00	175,000.00	140,000.00	?
Surface (SUR) mq	73.91	118.00	105.00	100.00
Restrooms (RES) n	2.00	1.00	1.00	2.00
Maintenance (MAI) point	2.00	4.00	2.00	4.00
Floor level (FLO) level	6.00	6.00	5.00	3.00

where C is the average cost for a new restroom, set equal to €10,000.00 obtained through a calculation of the costs required to install a new restroom; t is the past life, equal to five years; and n is the economic life of service set equal to 15 years.

STATE OF MAINTENANCE AND PRESERVATION OF HEDONIC PRICE

From a cost valuation done at a construction company in the area, we determined the cost of restoring a property from maintenance level 2 to level 4 was €10,000.00.

FLOOR LEVEL HEDONIC PRICE

The hedonic price for floor level is calculated by multiplying the total price of the property by the ratio percentage of the particular market segment:

$$p_{FLO} = \theta P,$$

where P is the total price of the property, expressed as a €; and θ is the 3% rate change in prices.

Exhibit A3
Hedonic Price Table

Hedonic Prices	Comparable A	Comparable B	Comparable C
<i>SUR</i> €/mq	1,691.25	1,483.05	1,333.33
<i>RES</i> €/n	6,666.67	6,666.67	6,666.67
<i>MAI</i> €/point	10,000.00	10,000.00	10,000.00
<i>FLO</i> €/level	3,750.00	5,250.00	4,200.00
r_j	0.35	-0.15	-0.05

$$p_{FLO}^A = 0,03 \cdot \text{€}125,000.00 = \frac{\text{€}}{\text{level}} 3,750.00,$$

$$p_{FLO}^B = 0,03 \cdot \text{€}175,000.00 = \frac{\text{€}}{\text{level}} 5,250.00,$$

$$p_{FLO}^C = 0,03 \cdot \text{€}140,000.00 = \frac{\text{€}}{\text{level}} 4,200.00.$$

Exhibit 3A reports the different hedonic prices. To take into account any duplication in the analysis, we calculate the coefficient r for each comparable as indicated in the main text. The r_j coefficient is calculated using equation (18):

$$r_A = \frac{SUR_{Subject} - SUR_A}{SUR_A} = \frac{\text{mq}100.00 - \text{mq}73.91}{\text{mq}73.91} = 0.35,$$

$$r_B = \frac{SUR_{Subject} - SUR_B}{SUR_B} = \frac{\text{mq}100.00 - \text{mq}118.00}{\text{mq}118.00} = -0.15,$$

$$r_C = \frac{SUR_{Subject} - SUR_C}{SUR_C} = \frac{\text{mq}100.00 - \text{mq}105.00}{\text{mq}105.00} = -0.05,$$

The r_j coefficients are significant in the hedonic price table (Table 3).

SURFACE ADJUSTMENTS

$$p_{SUR_A}[SUR_{Subject} - SUR_A] = \frac{\text{€}}{\text{sq}} 1,691.25 \cdot [\text{sq}(100.00 - 73.91)] = \text{€}44,124.61,$$

$$p_{SUR_B}[SUR_{Subject} - SUR_B] = \frac{\text{€}}{\text{sq}} 1,483.05 \cdot [\text{sq}(100.00 - 118.00)] = -\text{€}26,694.92,$$

$$p_{SUR_C}[SUR_{Subject} - SUR_C] = \frac{\text{€}}{\text{sq}} 1,333.33 \cdot [\text{sq}(100.00 - 105.00)] = -\text{€}6,666.67.$$

RESTROOM ADJUSTMENTS

$$p_{RES_A}[RES_{Subject} - RES_A \cdot (1 + r_A)] = \frac{€}{n^o} 6,666.67 \cdot [2.00 - 2.00 \cdot (1 + 0.35)]$$

$$= -€4,706.63,$$

$$p_{RES_B}[RES_{Subject} - RES_B \cdot (1 + r_B)] = \frac{€}{n^o} 6,666.67 \cdot [2.00 - 1.00 \cdot (1 - 0.15)]$$

$$= -€7,683.62,$$

$$p_{RES_C}[RES_{Subject} - RES_C \cdot (1 + r_C)] = \frac{€}{n^o} 6,666.67 \cdot [2.00 - 1.00 \cdot (1 - 0.05)]$$

$$= -€6,984.13.$$

FLOOR LEVEL ADJUSTMENTS

$$p_{FLO_A}[FLO_{Subject} - FLO_A \cdot (1 + r_A)] = \frac{€}{point} 3,750.00 \cdot [3.00 - 6.00 \cdot (1 + 0.35)]$$

$$= -€19,192.43,$$

$$p_{FLO_B}[FLO_{Subject} - FLO_B \cdot (1 + r_B)] = \frac{€}{point} 5,250.00 \cdot [3.00 - 6.00 \cdot (1 \pm 0.15)]$$

$$= -€10,944.92,$$

$$p_{FLO_C}[FLO_{Subject} - FLO_C \cdot (1 + r_C)] = \frac{€}{point} 4,200.00 \cdot [3.00 - 5.00 \cdot (1 - 0.05)]$$

$$= -€7,400.00.$$

In the sales adjustment grid, every adjustment is explicit; the last record reports the correct prices (Exhibit A4). It is possible to see the convergence of corrected prices, calculated using the corrective factors rj .

The percentage difference between the correct prices is 4.6% higher than the eligibility threshold. From the analysis of the data table, the degree of dissimilarity in the characteristics can be measured by calculating the coefficients of similarity (25 and 26) and the degree of reliability of the selling prices by calculating the coefficients of reliability (27). From these coefficients, it is possible to calculate the second level indices (28 and 29). Finally, the corresponding expected values were calculated (30) (Exhibit A5). The similarity coefficient and the coefficient of reliability, respectively, aim to correct the dissimilarity of the characteristics and anomalies of the prices, recorded in the hypothesis according to which equal correct prices should correspond to property characteristics equal to those of the property being valued. Therefore, the correction takes place in the calculation of the expected value and can be determined: without correction; adjusted for only the dissimilarity; corrected only for reliability; and corrected for both based on the index compound.

Exhibit A4
Sales Adjustment Grid (–€)

Sale Price and Characteristic	Comparable A	Comparable B	Comparable C
Sale price (<i>SPR</i>)	125,000.00	175,000.00	140,000.00
Surface (<i>SUR</i>)	$1,691.25 \cdot (100.00 - 73.91) = 44,124.61$	$1,483.05 \cdot (100.00 - 118.00) = -26,694.92$	$1,333.33 \cdot (100.00 - 105.00) = -6,666.67$
Restrooms (<i>RES</i>)	$6,666.67 \cdot [2.00 - 2.00 \cdot (1 + 0.35)] = -4,706.63$	$6,666.67 \cdot [2.00 - 1.00 \cdot (1 - 0.15)] = 7,683.62$	$6,666.67 \cdot [2.00 - 1.00 \cdot (1 - 0.05)] = 6,984.13$
Maintenance (<i>MAI</i>)	$10,000.00 \cdot [4.00 - 2.00 \cdot (1 + 0.35)] = 12,940.0$	$10,000.00 \cdot [4.00 - 4.00 \cdot (1 - 0.15)] = 6,101.69$	$10,000.00 \cdot [4.00 - 2.00 \cdot (1 - 0.05)] = 20,952.38$
Floor level (<i>FLO</i>)	$3,750.00 \cdot [3.00 - 6.00 \cdot (1 + 0.35)] = -19,192.43$	$5,250.00 \cdot [3.00 - 6.00 \cdot (1 - 0.15)] = -10,944.92$	$4,200.00 \cdot [3.00 - 5.00 \cdot (1 - 0.35)] = -7,400.00$
Correct price	158,165.62	151,145.48	153,869.84

Exhibit A5
Coefficients of Similarity and Reliability, Second-level Indicators and Expected Values

Price, Coefficient, and Index		Comparable A	Comparable B	Comparable C	Expected Value
Indices of similarity	gs^i_a	0.34	0.5	0.31	€154,385.83
	gs^i_q	0.35	0.35	0.29	€154,403.47
Reliability indices	ga^i	0.32	0.33	0.35	€154,363.97
Composite indices	gc^i_a	0.33	0.34	0.32	€154,356.79
	gc^i_q	0.34	0.35	0.31	€154,374.19

REFERENCES

Colwell, P.F., R.E. Cannaday, and C. Wu. The Analytical Foundations of Adjustment Grid Methods. *Journal of the American Real Estate and Urban Economics Association*, 1983, 11:1, 11–29.

Gau, G.W., T-Y. Lai, and K. Wang. Optimal Comparable Selection and Weighting in Real Property Valuation: An Extension. *Real Estate Economics*, 1992, 20:1, 107–24.

———. A Further Discussion of Optimal Comparable Selection and Weighting, and a Response to Green. *Real Estate Economics*, 1994, 22:4, 655–63.

Isakson, H.R. The Nearest Neighbors Appraisal Technique: An Alternative to the Adjustment Grid Methods. *Journal of the American Real Estate and Urban Economics Association*, 1986, 14:2, 274–86.

———. Valuation Analysis of Commercial Real Estate Using the Nearest Neighbors Appraisal Technique. *Growth and Change*, 1988, 19:2, 11–24.

———. The Linear Algebra of the Sales Comparison Approach. *Journal of Real Estate Research*, 2002, 24:2, 117–28.

Lentz, G. and K. Wang. Residential Appraisal and the Lending Process: A Survey of Issues. *Journal of Real Estate Research*, 1998, 15, 11–39.

Manaster, M.S. Sales Comparison Approach: A Comparative Analysis of Three Appraisal Techniques. *Growth and Change*, 1988, 19:2, 11–24.

Manaster, M.S. Sales Comparison Approach: A Comparative Analysis of Three Appraisal Reports on the Same Property. *The Real Estate Appraiser*, 1991, May, 12–26.

MingYou, S. and C-O. Chang. Weight Regression Model from the Sales Comparison Approach. *Property Management*, 2009, 27:5, 302–18.

Pace, R.K. Total Grid Estimation. *Journal of Real Estate Research*, 1998, 15:1/2, 101–14.

Salvo, F., M. Ciuna, and M. De Ruggiero. Property Prices Index Numbers and Derived Indices. *Property Management*, 2014, 32, 139–53.

Salvo, F. and M. De Ruggiero. Market Comparison Approach between Tradition and Innovation. A Simplifying Approach. *AESTIMUM*, 2013, 62, 585–94.

———. Adjustment Grid Methods. *AESTIMUM*, 2011, 58, 47–58.

Simonotti, M. La stima Immobiliare. Città Studi, Utet, Torino, 1997.

———. Moderni Strumenti di Analisi Estimativa. Quaderni del Dipartimento Patrimonio Architettonico e Urbanistico Gangemi Editore. 2006, 29–32, 441–44.

Simonotti, M., F. Salvo, and M. Ciuna. Multilevel Methodology Approach for the Construction of Real Estate Monthly Index Numbers. *Journal of Real Estate Literature*, 2014, 22:2, 281–302.

Vandell, K.D. Optimal Comparable Selection and Weighting in Real Property Valuation. *Real Estate Economics*, 1991, 19:2, 213–39.

Marco Simonotti, Palermo University, 90133 Palermo, Italy or marco.simonotti@unipa.it.

Francesca Salvo, Calabria University, 87036 Rende (Cpsenza), Italy or francesca.salvo@unical.it.

Marina Ciuna, Palermo University, 90133 Palermo, Italy or marina.ciuna@unipa.it.

Manuela De Ruggiero, Calabria University, 87036 Arcavacata di Rende, Italy.